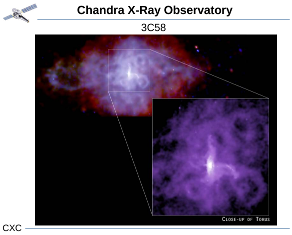
The Equation of State Problem

Recent advances in microscopic theories



Arnau Rios Huguet
Department of Physics
University of Surrey

on behalf of A. Polls (University of Barcelona)

Compstar Workshop
Catania, 10 May 2011

Outline

Outline

1 Motivation

2 Nuclear many-body problem

3 Review of many-body techniques

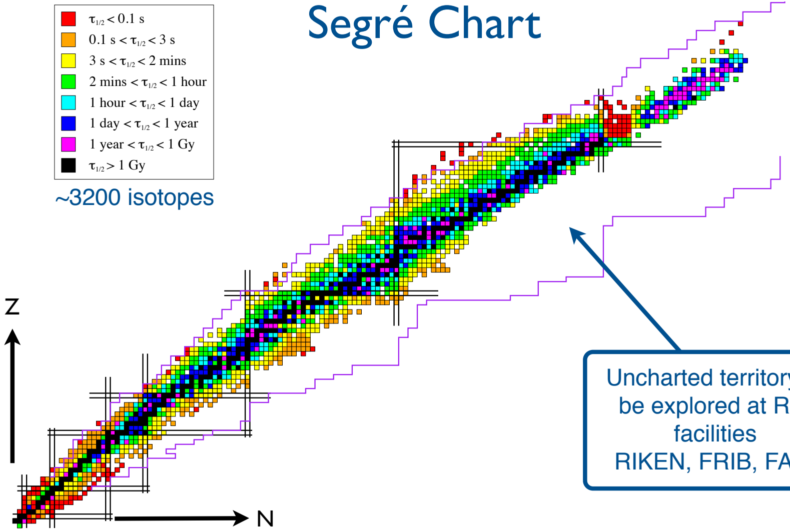
4 Three-body sector

5 Exotic phases of nuclear matter

6 Conclusions

UNIVERSITY OF SURREY

Segré Chart

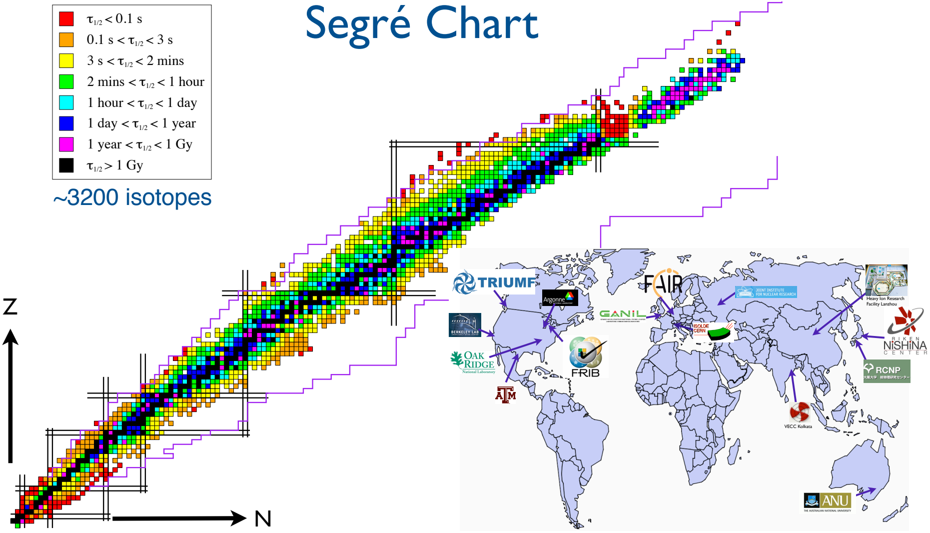


~3200 isotopes

Uncharted territory to be explored at RIB facilities RIKEN, FRIB, FAIR

UNIVERSITY OF SURREY

Segré Chart



~3200 isotopes

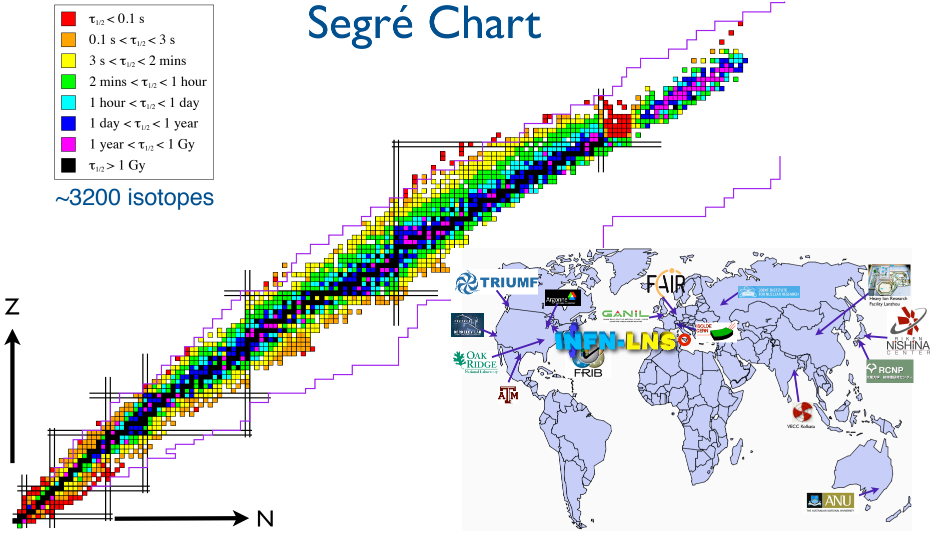
Isotope science

First principles in nuclear physics

Segré Chart

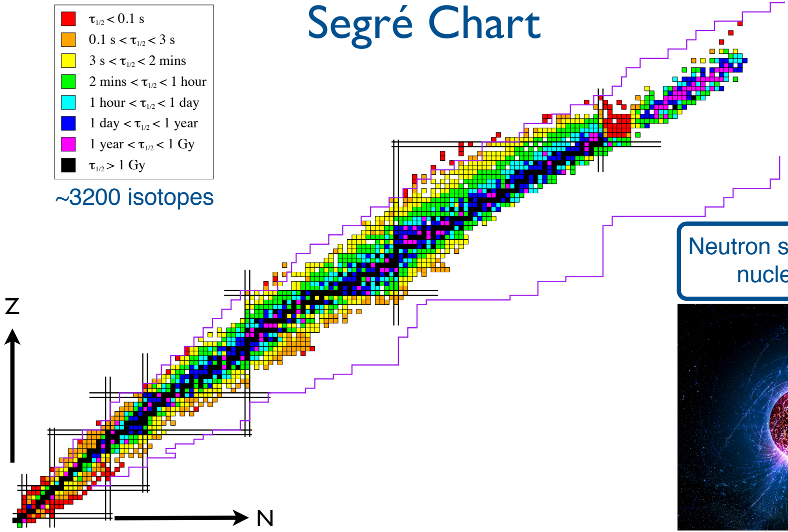
■ (red)	$\tau_{1/2} < 0.1$ s
■ (orange)	$0.1\text{ s} < \tau_{1/2} < 3$ s
■ (yellow)	$3\text{ s} < \tau_{1/2} < 2$ mins
■ (green)	$2\text{ mins} < \tau_{1/2} < 1$ hour
■ (cyan)	$1\text{ hour} < \tau_{1/2} < 1$ day
■ (blue)	$1\text{ day} < \tau_{1/2} < 1$ year
■ (magenta)	$1\text{ year} < \tau_{1/2} < 1$ Gy
■ (black)	$\tau_{1/2} > 1$ Gy

~3200 isotopes



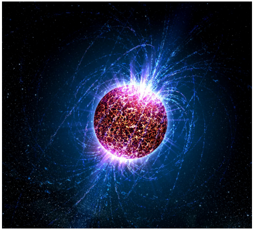
UNIVERSITY OF SURREY

Segré Chart



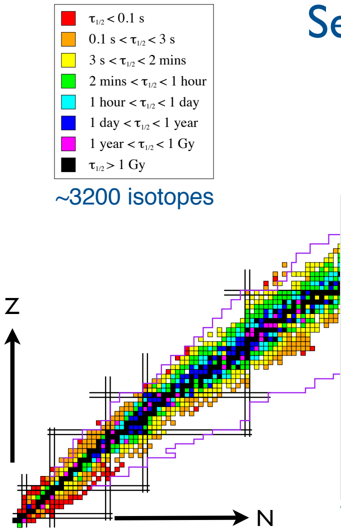
~3200 isotopes

Neutron stars are also nuclear labs!

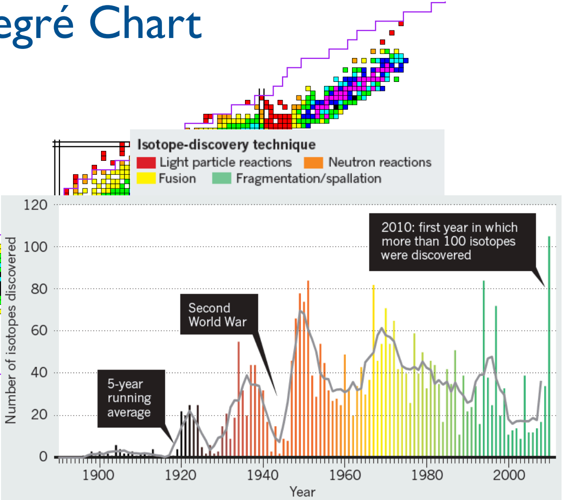


UNIVERSITY OF SURREY

Segré Chart



~3200 isotopes



Thoennessen & Sherrill, Nature (Comment) **473**, 25 (2011)

What do we know about the EoS?

Taylor expansion near symmetric matter

- EoS provides a characterization of bulk properties:

$$p(\varepsilon) = ? \qquad \varepsilon = \rho \frac{E}{A} \qquad p(\rho) = \rho^2 \frac{\partial E/A}{\partial \rho} \qquad \frac{E}{A}(\rho, \beta) = ?$$

- Taylor expansion
  - Minimum at saturation density, ρ_0
  - Minimum in asymmetry: $\beta = \frac{N-Z}{N+Z} = 0$
  - Isospin symmetry $\Rightarrow$ even powers of β
  - Give the coefficients a name!

What do we know about the EoS?

Taylor expansion near symmetric matter

- EoS provides a characterization of bulk properties:

$$p(\varepsilon) = ? \qquad \varepsilon = \rho \frac{E}{A} \qquad p(\rho) = \rho^2 \frac{\partial E/A}{\partial \rho} \qquad \frac{E}{A}(\rho, \beta) = ?$$

- Taylor expansion
  - Minimum at saturation density, ρ_0
  - Minimum in asymmetry: $\beta = \frac{N-Z}{N+Z} = 0$
  - Isospin symmetry $\Rightarrow$ even powers of β
  - Give the coefficients a name!

$$\begin{aligned}\frac{E}{A}(\rho, \beta) = & \frac{E}{A}(\rho_0, \beta) \\ & + 3\rho_0 \frac{\partial E/A}{\partial \rho}\Big|_{\rho_0} \left(\frac{\rho - \rho_0}{3\rho_0} \right) \\ & + \frac{9\rho_0^2}{2!} \frac{\partial^2 E/A}{\partial \rho^2}\Big|_{\rho_0} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 \\ & + \mathcal{O}(3)\end{aligned}$$

What do we know about the EoS?

Taylor expansion near symmetric matter

- EoS provides a characterization of bulk properties:

$$p(\varepsilon) = ? \qquad \varepsilon = \rho \frac{E}{A} \qquad p(\rho) = \rho^2 \frac{\partial E/A}{\partial \rho} \qquad \frac{E}{A}(\rho, \beta) = ?$$

- Taylor expansion
  - Minimum at saturation density, ρ_0
  - Minimum in asymmetry: $\beta = \frac{N-Z}{N+Z} = 0$
  - Isospin symmetry $\Rightarrow$ even powers of β
  - Give the coefficients a name!

$$\begin{aligned}
\frac{E}{A}(\rho, \beta) = & \frac{E}{A}(\rho_0, 0) + \frac{1}{2!} \frac{\partial^2 E/A}{\partial \beta^2}\Big|_{\rho_0, \beta=0} \beta^2 \\
& + \frac{3\rho_0}{2!} \frac{\partial^3 E/A}{\partial \beta^2 \partial \rho}\Big|_{\rho_0, \beta=0} \beta^2 \left(\frac{\rho - \rho_0}{3\rho_0} \right) \\
& + \frac{9\rho_0^2}{2!} \left\{ \frac{\partial^2 E/A}{\partial \rho^2}\Big|_{\rho_0, \beta=0} + \frac{1}{2!} \frac{\partial^4 E/A}{\partial \rho^2 \beta^2}\Big|_{\rho_0, \beta=0} \beta^2 \right\} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 \\
& + \mathcal{O}(3, 2)
\end{aligned}$$

What do we know about the EoS?

Taylor expansion near symmetric matter

- EoS provides a characterization of bulk properties:

$$p(\varepsilon) = ? \qquad \varepsilon = \rho \frac{E}{A} \qquad p(\rho) = \rho^2 \frac{\partial E/A}{\partial \rho} \qquad \frac{E}{A}(\rho, \beta) = ?$$

- Taylor expansion
  - Minimum at saturation density, ρ_0
  - Minimum in asymmetry: $\beta = \frac{N-Z}{N+Z} = 0$
  - Isospin symmetry $\Rightarrow$ even powers of β
  - Give the coefficients a name!

$$\begin{aligned} \frac{E}{A}(\rho, \beta) = &\, E_0 + E_{sym} \beta^2 \\ &+ L \beta^2 \left(\frac{\rho - \rho_0}{3\rho_0} \right) \\ &+ \frac{1}{2!} \left\{ K_0 + K_{sym} \beta^2 \right\} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 \\ &+ \mathcal{O}(3, 2) \end{aligned}$$

EoS from basic nuclear properties

An incomplete list

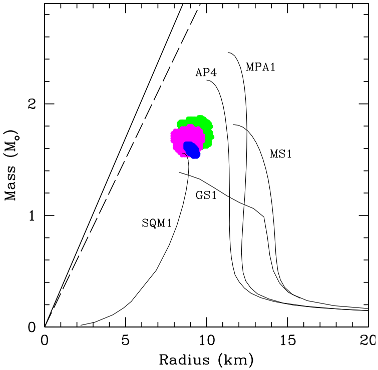
$$\frac{E}{A}(\rho,\beta) = E_0 + E_{sym}\beta^2 + L\beta^2\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2!}\left\{K_0 + K_{sym}\beta^2\right\}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2$$

Quantity	Experimental probes	Value	Ref.
ρ_0	(e,e') elastic scattering	$0.16\ \mathrm{fm}^{-3}$	[1]
E_0	$\frac{E}{A}$ bulk systematics	$-16\ \mathrm{MeV}$	[1]
K_0	GMR energy in $Z \sim N$	$240 \pm 20\ \mathrm{MeV}$	[2]
E_{sym}	$\frac{E}{A}$ bulk systematics + ID	$32 \pm 2\ \mathrm{MeV}$	[3]
L	ID, IVMR energies, δR	$88 \pm 25\ \mathrm{MeV}$	[3]
K_{sym}	?	?	

[1] Schuck & Ring, *The Nuclear Many-Body Problem* (Springer)
[2] Blaizot, Phys. Reps. 64, 171 (1981)
[3] Tsang *et al.*, Phys. Rev. Lett. 102, 122701 (2009)

EoS from astrophysical observations

Results from isolated NS

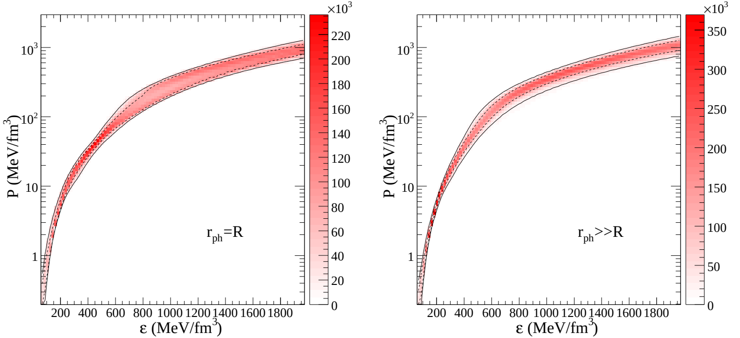


Özel, Baym & Güver, arxiv:1002.3153

- Mass-Radius relation from bursts
- Bayesian data analysis to get model-independent EoS
  1. 3 type-I X-ray bursts
  2. 3 transient low mass X-ray binaries
  3. 1 isolated cooling NS, RX J1856-3754

EoS from astrophysical observations

Results from isolated NS



Steiner *et al.*, ApJ 722, 33 (2010)

- Mass-Radius relation from bursts
- Bayesian data analysis to get model-independent EoS
  1. 3 type-I X-ray bursts
  2. 3 transient low mass X-ray binaries
  3. 1 isolated cooling NS, RX J1856-3754

Good nuclear parameters!

$$\frac{E}{A} = E_0 + \frac{K_0}{18}(u-1)^2 + \left[S_k u^{2/3} + S_p u^{\gamma}\right]\beta^2$$

$$K_0 = 180 - 280 \text{ MeV} \qquad u = \frac{\rho}{\rho_0}$$

$$E_{sym} = S_k + S_p = 28 - 38 \text{ MeV} \qquad \gamma = 0.2 - 1.2$$

Steiner *et al.*, ApJ 722, 33 (2010)

- Mass-Radius relation from bursts
- Bayesian data analysis to get model-independent EoS
  1. 3 type-I X-ray bursts
  2. 3 transient low mass X-ray binaries
  3. 1 isolated cooling NS, RX J1856-3754

Outline

UNIVERSITY OF SURREY

A philosophical matter

The EoS is unknown *a priori*

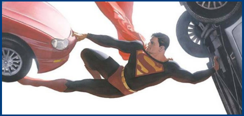
Ab-initio

Microscopic NN interaction

Use many-body theory

Build the EoS

Safest way to objective



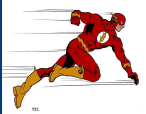
Phenomenological

Fit effective interaction

Rely on mean-field or DFT

Extrapolate the EoS

Fastest way to objective



A philosophical matter

The EoS is unknown *a priori*

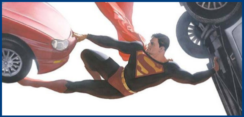
Ab-initio

Microscopic NN interaction

Use many-body theory

Build the EoS

Safest way to objective



Ab initio description of nuclear systems

UNIVERSITY OF SURREY

Many-body technique

- **BHF, FHNC, AFDMC, SCGF**

Empirical properties

- **Saturation density**
- **Binding energy**
- **Compressibility**

Hamiltonian

- **Phase-shift equivalent NN force**
- NNN force?

Degrees of freedom

- **Nucleons**
- Hyperons, quarks, mesons?

Framework

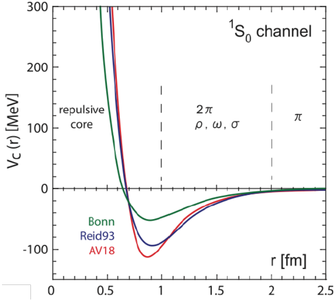
- **Non-relativistic & quantum**
- Relativistic, classical?

Complications

The hard life of nuclear many-body physicists

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij}$$

Different NN potentials



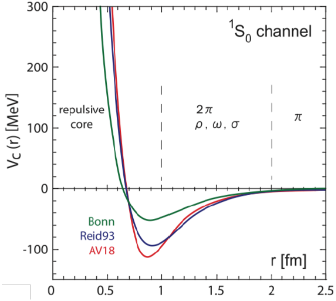
- NN interaction is not uniquely defined...
- Short-range core needs many-body treatment
- Complicated channel structure ⇒ tensor term coupling
- Very different techniques ...

Complications

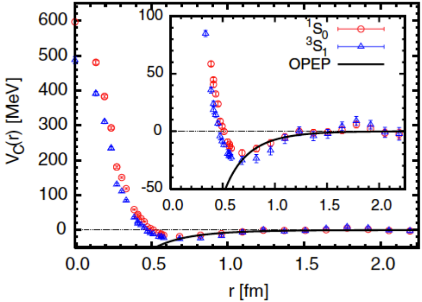
The hard life of nuclear many-body physicists

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij}$$

Different NN potentials



Lattice QCD potential



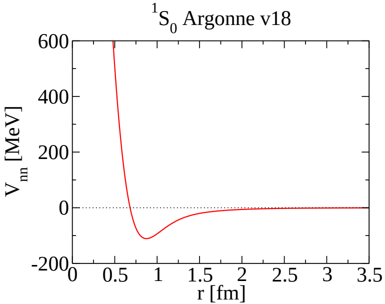
Ishii, Phys. Rev. Lett. **99**, 022001 (2007)

- NN interaction is not uniquely defined... Yet!
- Short-range core needs many-body treatment
- Complicated channel structure ⇒ tensor term coupling
- Very different techniques ...

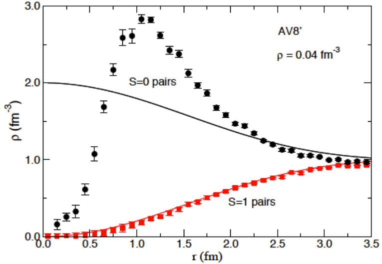
Complications

The hard life of nuclear many-body physicists

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij}$$



GFMC pair distribution function



Carlson *et al.*, Phys.Rev. C **68**, 025802 (2003)

- NN interaction is not uniquely defined... Yet!
- Short-range core needs many-body treatment
- Complicated channel structure $\Rightarrow$ tensor term coupling
- Very different techniques ...

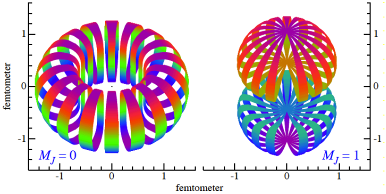
Complications

The hard life of nuclear many-body physicists

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij}$$

Deuteron wave-function: S-D mixing

Surfaces of density = 0.24 fm^{-3} in polarized deuteron states. The distinctive structures are induced by the strong tensor potentials which result from the pion-exchange component of the nucleon-nucleon interaction.



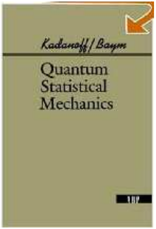
$$\delta E = \sum_{\substack{i,j<F \\ m,n>F}} \frac{\langle ijJ(LS)|V|mnJ(L'S)\rangle\langle mnJ(L'S)|V|ijJ(LS)\rangle}{E_i + E_j - E_m - E_n - i\eta}$$

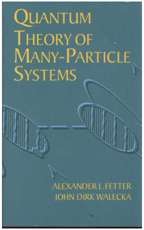
- NN interaction is not uniquely defined... Yet!
- Short-range core needs many-body treatment
- Complicated channel structure $\Rightarrow$ tensor term coupling
- Very different techniques ...

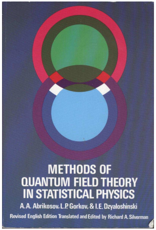
Complications

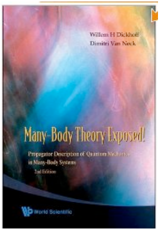
The hard life of nuclear many-body physicists

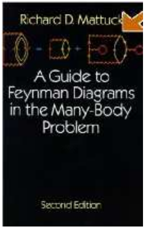
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij}$$











- NN interaction is not uniquely defined... Yet!
- Short-range core needs many-body treatment
- Complicated channel structure ⇒ tensor term coupling
- Very different techniques ...

Outline

Variational techniques

CBF, FHNC

- Trial many-body wave-function

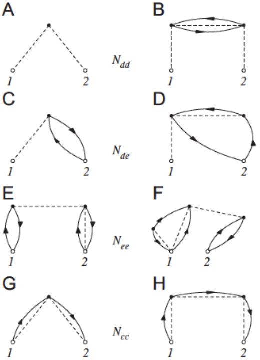
$$|\Psi\rangle = F|\Phi\rangle \qquad F = \mathcal{A}\left\{\prod_{i>j}\hat{F}_{ij}\right\}$$

$$\hat{F}_{ij} = \sum_{p=1}^{6} f^p(r_{ij})\hat{O}^p_{ij} \qquad h(r) = f^c(r)^2 - 1$$

- $g(x_1, x_2)$ from Hypernetted Chain expansion
- Massive resummation via integral equations
- Operatorial structure of correlations
- Minimization of the total energy

$$\min\left\{\frac{<\Psi|\hat{H}|\Psi>}{<\Psi|\Psi>}\right\} \geq E$$

Nodal diagrams in FHNC



Advantages

1. Access to several properties
2. Sums short- & long-range correlations
3. Applied to closed-shell nuclei

Limitations

1. Only local potentials
2. Difficulties with operatorial structure (SOC)
3. Treatment of elementary diagrams
4. Difficult to handle for asymmetric matter

Variational techniques

CBF, FHNC

- Trial many-body wave-function

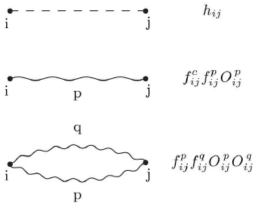
$$|\Psi\rangle = F|\Phi\rangle \qquad F = \mathcal{A}\left\{\prod_{i>j} \hat{F}_{ij}\right\}$$

$$\hat{F}_{ij} = \sum_{p=1}^{6} f^p(r_{ij})\hat{O}^p_{ij} \qquad h(r) = f^c(r)^2 - 1$$

- $g(x_1, x_2)$ from Hypernetted Chain expansion
- Massive resummation via integral equations
- Operatorial structure of correlations
- Minimization of the total energy

$$\min\left\{\frac{<\Psi|\hat{H}|\Psi>}{<\Psi|\Psi>}\right\} \geq E$$

Operatorial correlation bonds



Advantages

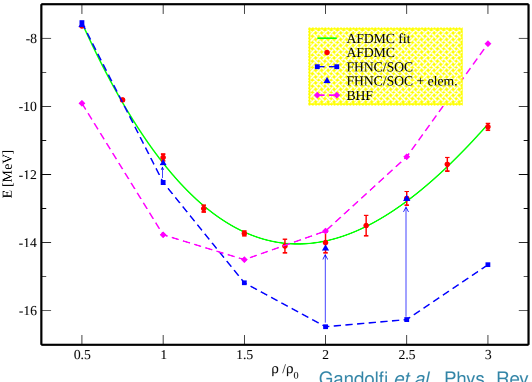
1. Access to several properties
2. Sums short- & long-range correlations
3. Applied to closed-shell nuclei

Limitations

1. Only local potentials
2. Difficulties with operatorial structure (SOC)
3. Treatment of elementary diagrams
4. Difficult to handle for asymmetric matter

Recent example: FHNC EoS

EoS of symmetric nuclear matter



Gandolfi *et al.*, Phys. Rev. Lett. **98**, 102503 (2007)

Classic references

Fantoni & Rosati, Nuov. Cim. A **20**, 179 (1974)
Benhar *et al.*, Nuc. Phys. A **550**, 201 (1992)
Pandharipande & Fantoni, PRC **37**, 1697 (1988)
Fantoni & Fabrocini, Lect. Not. Phys. **510**, 119 (1998)

Latest advances

Morales *et al.*, Phys. Rev. C **66**, 054308 (2002)
Arias de Saavedra *et al.*, Phys. Rep. **450**, 1 (2007)
Lovato *et al.*, arxiv:1011.3784

Monte-Carlo techniques

VMC, GFMC, AFDMC

- VMC: energy minimization

$$\min \left\{ \frac{< \Psi|\hat{H}|\Psi >}{< \Psi|\Psi >} \right\} \geq E$$

- DMC: Schroedinger equation in imaginary time

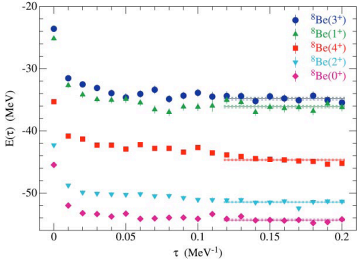
$$i\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle \Rightarrow -\frac{\partial}{\partial \tau}|\Psi\rangle = \hat{H}|\Psi\rangle$$

- GFMC: trial wave function

$$|\psi\rangle = \sum_{\alpha=0}^{N_\alpha} c_\alpha |\Psi_\alpha\rangle \Rightarrow |\Psi_0\rangle = \lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\psi\rangle$$

- AFDMC: Hubbard-Stratonovitch for spin-isospin operators

GFMC imaginary time evolution



Advantages

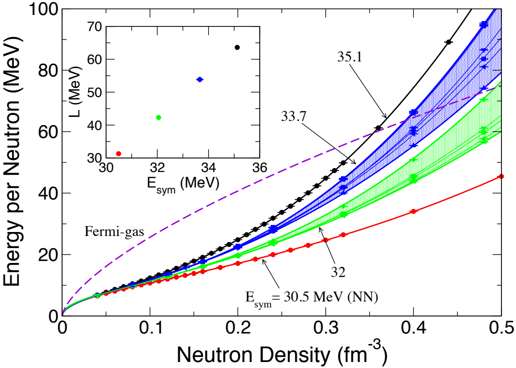
1. Symmetric & asymmetric matter
2. Applied to nuclei
3. Virtually exact

Limitations

1. Only local potentials
2. Fermion sign limitation
3. Finite-size effects?

Recent example: AFDMC EoS

EoS of neutron matter



Gandolfi, Carlson & Reddy, arxiv:1101.1921

Classic references

Pudliner, Pandharipande et al., PRL **74**, 4396 (1995)
Schmidt & Fantoni, Phys. Lett. B **446**, 99 (1999)
Pieper & Wiringa, Annu. Rev. Nucl. Part. Sci. **51**, 53 (2001)

Latest advances

Carlson et al., PRC **68**, 025802 (2003)
Gandolfi et al., MNRAS **404**, 35 (2010)
Gezerlis & Carlson, PRC **81**, 025803 (2010)
Wlazlowski & Majierski, PRC **83**, 012801 (2011)

Diagrammatic techniques: BHF

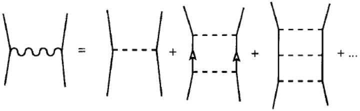
- Based on Bethe-Goldstone perturbation theory
- Infinite resummation of two-hole line diagrams
- pp Pauli blocked in-medium interaction (G-matrix)

$$G(\omega) = V + V \frac{Q}{\omega - \epsilon - \epsilon' + i\eta} G(\omega)$$

$$U(k) = \sum_{|\vec{k}'|<k_F} \langle \vec{k}\vec{k}' | G(\omega = \epsilon(k) + \epsilon(k')) | \vec{k}\vec{k}' \rangle_A$$

$$\epsilon(k) = \frac{\hbar^2 k^2}{2m_\tau} + Re[U(k)]$$

G-matrix diagrams



- Expansion for the energy

$$\frac{E}{A}(\rho, \beta) = \frac{1}{A} \sum_\tau \sum_{|\vec{k}|<k_{F_\tau}} \left(\frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} Re[U_\tau(\vec{k})] \right)$$

Energy diagrams

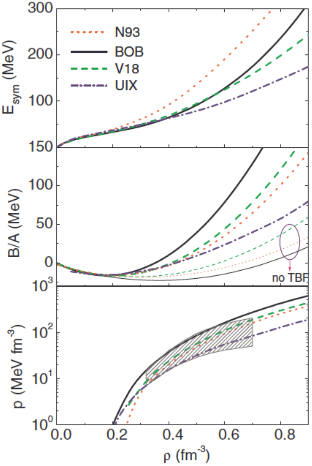
Advantages

1. Symmetric, asymmetric & exotic matter
2. Also non-local potentials
3. Systematic expansion

Limitations

1. Missing diagrams
2. Thermodynamical inconsistency

Recent example: BHF EoS



Li & Schulze, Phys. Rev. C **78**, 028801 (2008)

Classic references

Brueckner *et al.*, Phys. Rev. **95**, 217 (1954)
Brandow, Phys. Rev. **152**, 863 (1966)
Day, Rev. Mod. Phys. **39**, 719 (1967)
Jeukenne, Lejeune & Mahaux, Phys. Rep. **25**, 83 (1976)
Bombaci & Lombardo, Phys. Rev. C **44**, 1892 (1991)
Song, Baldo *et al.*, Phys. Rev. Lett. **81**, 1584 (1998)

Latest advances

Vidaña & Polls, Phys. Lett. B **666**, 232 (2008)
Li *et al.*, Phys. Rev. C **77**, 034316 (2008)
Baldo & Burgio, arxiv:1102.1364

Diagrammatic techniques: SCGF

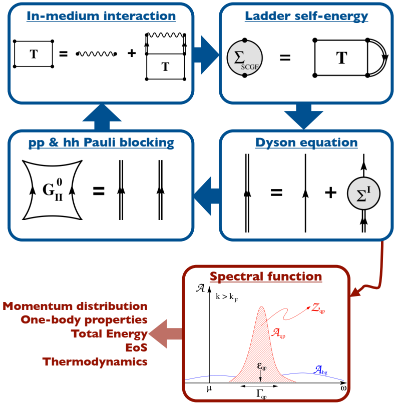
- Feynman diagrams for many-body propagators
- Truncate hierarchy & get pp+hh Pauli blocking
- Impose self-consistency at all levels
- Characterize medium with spectral function

$$\mathcal{A}^{<}(k,\omega) = \sum_{n,m} \frac{e^{-\beta(E_n - \mu A)}}{Z} \left|\langle m | a_{\boldsymbol{k}} | n \rangle\right|^2 \delta[\omega - (E_n^A - E_m^{A-1})]$$

- Energy from GMK sum rule

$$E = \sum_k \int \frac{\mathrm{d}\omega}{2\pi} \frac{1}{2} \left[\frac{k^2}{2m} + \omega\right] \mathcal{A}(k,\omega) f(\omega)$$

Ladder approximation within SCGF



Advantages

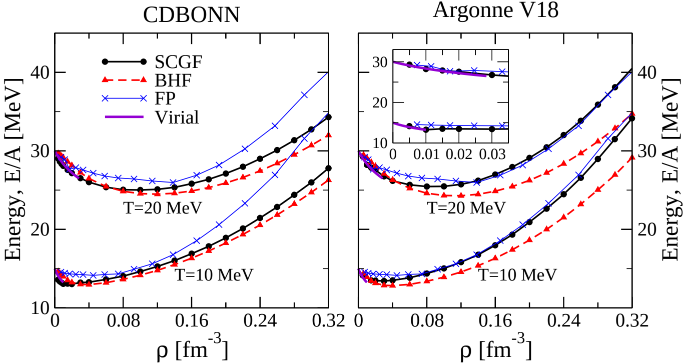
1. Symmetric, asymmetric & exotic matter
2. Also non-local potentials
3. Thermodynamically consistent

Limitations

1. Missing diagrams
2. T=0 instability, meaningful ground state?

Recent example: SCGF EoS

EoS of hot neutron matter



Rios, Polls & Vidaña, Phys. Rev. C **79**, 025802 (2009)

Classic references

Ramos, Polls & Dickhoff, Nucl. Phys. A **503**, 1 (1989)
Alm *et al.*, Nucl. Phys. A **551**, 45 (1993)
Dewulf *et al.*, Phys. Rev. Lett. **90**, 152501 (2003)
Frick & Muther, Phys. Rev, C **68**, 034310 (2003)
Dickhoff & Van Neck, *Many-Body theory exposed!*

Latest advances

Muether & Dickhoff, Phys. Rev. C **72**, 054313 (2005)
Frick *et al.*, Phys. Rev. C **71**, 014313 (2005)
Rios *et al.*, Phys. Rev. C **78**, 044314 (2008)
Somà & Bożek, Phys. Rev. C **78**, 054003 (2008)

Tempering the interaction...

Renormalization group inspired methods

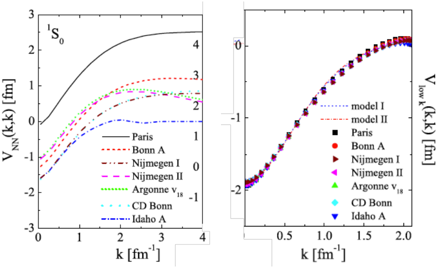
UNIVERSITY OF SURREY

- Use RG arguments to rebuild the NN interaction

$$T = V + VGT \Rightarrow \frac{d}{d\Lambda} T(k, k'; \Lambda) = 0$$

$$\frac{d}{d\Lambda} V^{\Lambda}(k, k') = \frac{2}{\pi} \frac{V^{\Lambda}(k', \Lambda) T(\Lambda, k')}{1 - (k/\Lambda)^2}$$

V_{lowk} evolved potentials



- Universal forces up to scale, Λ
- Softer potentials become perturbative
- Need of three-body forces!
- New technique: similarity renormalization group

$$\frac{dH_s}{ds} = [[T, H_s], H_s]$$

Advantages

1. Symmetric, asymmetric & exotic matter
2. Also non-local potentials
3. Applied to nuclei

Limitations

1. Many-body forces
2. Dressing of operators

Tempering the interaction...

Renormalization group inspired methods

- Use RG arguments to rebuild the NN interaction

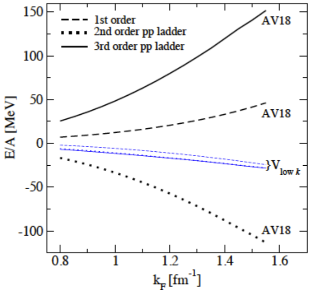
$$T = V + VGT \Rightarrow \frac{d}{d\Lambda} T(k, k'; \Lambda) = 0$$

$$\frac{d}{d\Lambda} V^{\Lambda}(k, k') = \frac{2}{\pi} \frac{V^{\Lambda}(k', \Lambda) T(\Lambda, k')}{1 - (k/\Lambda)^2}$$

- Universal forces up to scale, Λ
- Softer potentials become perturbative
- Need of three-body forces!
- New technique: similarity renormalization group

$$\frac{dH_s}{ds} = [[T, H_s], H_s]$$

Nuclear matter results



Advantages

1. Symmetric, asymmetric & exotic matter
2. Also non-local potentials
3. Applied to nuclei

Limitations

1. Many-body forces
2. Dressing of operators

Tempering the interaction...

Renormalization group inspired methods

- Use RG arguments to rebuild the NN interaction

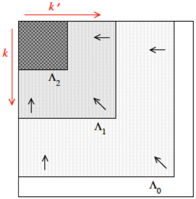
$$T = V + VGT \Rightarrow \frac{d}{d\Lambda} T(k, k'; \Lambda) = 0$$

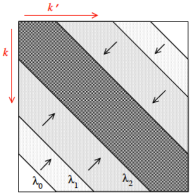
$$\frac{d}{d\Lambda} V^{\Lambda}(k, k') = \frac{2}{\pi} \frac{V^{\Lambda}(k', \Lambda) T(\Lambda, k')}{1 - (k/\Lambda)^2}$$

- Universal forces up to scale, Λ
- Softer potentials become perturbative
- Need of three-body forces!
- New technique: similarity renormalization group

$$\frac{dH_s}{ds} = [[T, H_s], H_s]$$

V_{lowk} vs. SRG flow





Advantages

1. Symmetric, asymmetric & exotic matter
2. Also non-local potentials
3. Applied to nuclei

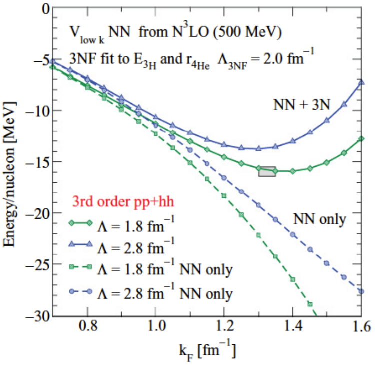
Limitations

1. Many-body forces
2. Dressing of operators

UNIVERSITY OF SURREY

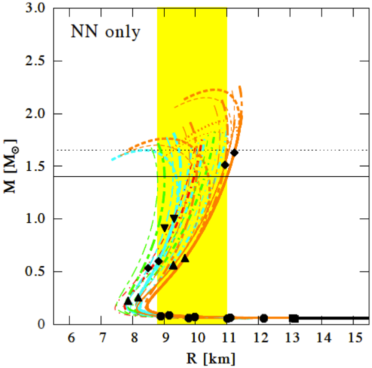
Recent example: EoS of neutron matter

3BF needed for saturation



Hebeler *et al.*, PRC **83**, 031301 (2011)

χPT allowed M vs. R



Hebeler *et al.*, PRL **105**,161102 (2010)

Classic references

Bogner *et al.*, Phys. Lett. B **576**, 265 (2003)
Bogner *et al.*, Phys. Rept. **386**, 1 (2003)
Bogner *et al.*, Phys. Lett. B **649**, 488 (2007)
Bogner *et al.*, , Prog. Part. Nucl. Phys. **65**, 94 (2010)

Latest advances

Tolós *at al.*, Nucl. Phys. A **806**, 105 (2008)
Hebeler & Schwenk, Phys. Rev. C **82**, 014314 (2010)

A benchmark calculation
Local interactions with operatorial structure

How can we gauge the quality of different many-body techniques?

Benchmark calculations with:

1. Non-relativistic quantum mechanics
2. Same degrees of freedom: nucleons
3. Same NN interactions

Argonne refitted NN potentials

Wiringa & Pieper, Phys. Rev. Lett. **89**, 182501 (2002)

$$V_{ij}^{M}(r) = \sum_{p=1}^{M} v_p(r)\hat{O}_{ij}^{p}$$

$$O_{ij}^{p=1,\ldots,8} = \{\mathbb{I}, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}\} \otimes \{\mathbb{I}, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}$$

A benchmark calculation

Local interactions with operatorial structure

How can we gauge the quality of different many-body techniques?

Benchmark calculations with:

1. Non-relativistic quantum mechanics
2. Same degrees of freedom: nucleons
3. Same NN interactions

Argonne refitted NN potentials

Wiringa & Pieper, Phys. Rev. Lett. **89**, 182501 (2002)

$$V_{ij}^{M}(r) = \sum_{p=1}^{M} v_p(r)\hat{O}_{ij}^{p}$$

$$O_{ij}^{p=1,\ldots,8} = \{\mathbb{I}, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}\} \otimes \{\mathbb{I}, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}$$

Argonne v4'

$$V_{ij}^{4} = v_1(r) + v_2(r)\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + v_3(r)\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + v_4(r)(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \otimes \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \quad \rightarrow \quad \text{Central}$$

A benchmark calculation

Local interactions with operatorial structure

UNIVERSITY OF SURREY

How can we gauge the quality of different many-body techniques?

Benchmark calculations with:

1. Non-relativistic quantum mechanics
2. Same degrees of freedom: nucleons
3. Same NN interactions

Argonne refitted NN potentials

Wiringa & Pieper, Phys. Rev. Lett. **89**, 182501 (2002)

$$V_{ij}^{M}(r) = \sum_{p=1}^{M} v_p(r) \hat{O}_{ij}^{p}$$

$$O_{ij}^{p=1,\dots,8} = \{\mathbb{I}, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}\} \otimes \{\mathbb{I}, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}$$

Argonne v6'

$$V_{ij}^{6} = v_1(r) + v_2(r)\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + v_3(r)\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + v_4(r)(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \otimes \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \quad \rightarrow \text{ Central}$$

$$+ v_5(r) S_{ij} + v_6(r)(S_{ij} \otimes \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \quad \rightarrow \text{ Tensor}$$

UNIVERSITY OF SURREY

How can we gauge the quality of different many-body techniques?

Benchmark calculations with:

1. Non-relativistic quantum mechanics
2. Same degrees of freedom: nucleons
3. Same NN interactions

Argonne refitted NN potentials

Wiringa & Pieper, Phys. Rev. Lett. **89**, 182501 (2002)

$$V_{ij}^{M}(r) = \sum_{p=1}^{M} v_p(r)\hat{O}_{ij}^{p}$$

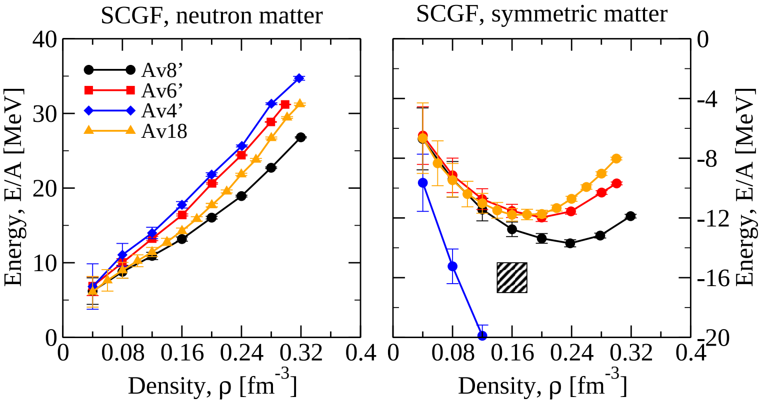
$$O_{ij}^{p=1,\dots,8} = \{\mathbb{I}, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}\} \otimes \{\mathbb{I}, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}$$

Argonne v8'

$$\begin{aligned} V_{ij}^{8} =& v_1(r) + v_2(r)\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + v_3(r)\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + v_4(r)(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j \otimes \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) \quad \rightarrow \ \text{Central} \\ &+ v_5(r)S_{ij} + v_6(r)(S_{ij}\otimes\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) \quad \rightarrow \ \text{Tensor} \\ &+ v_7(r)\mathbf{L}\cdot\mathbf{S} + v_8(r)(\mathbf{L}\cdot\mathbf{S}\otimes\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) \quad \rightarrow \ \text{Spin-orbit} \end{aligned}$$

UNIVERSITY OF SURREY

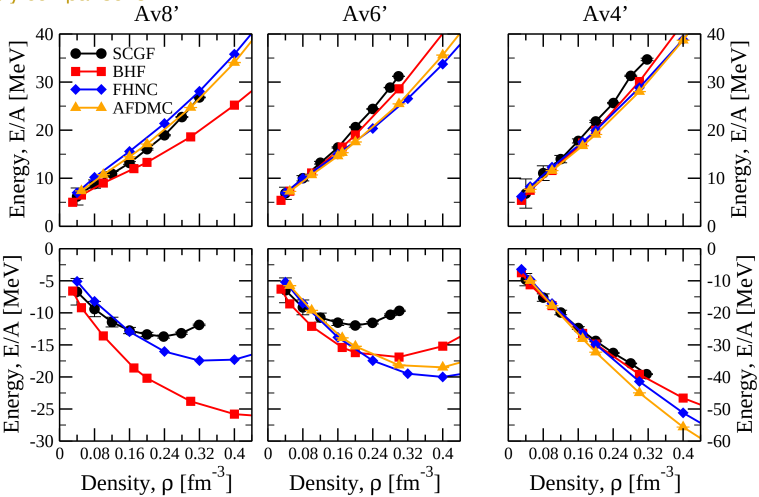
T=0 extrapolation of SCGF EoS



- NN interaction dependence
- Many-body dependence of EoS
- First comparison of non-relativistic approaches

The Compstar Equation of State

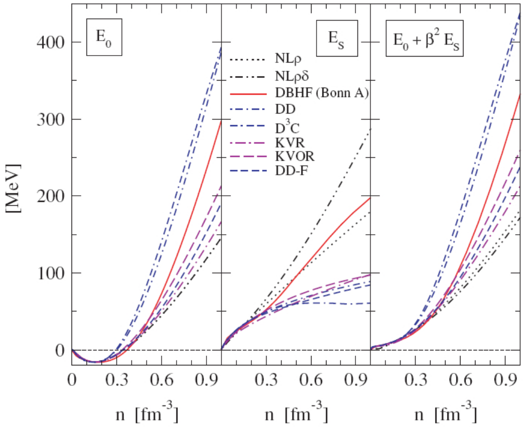
Preliminary comparisons



- NN interaction dependence
- Many-body dependence of EoS
- First comparison of non-relativistic approaches

The Compstar Equation of State

Preliminary comparisons



Klähn *et al.*, PRC 74 035802 (2006)

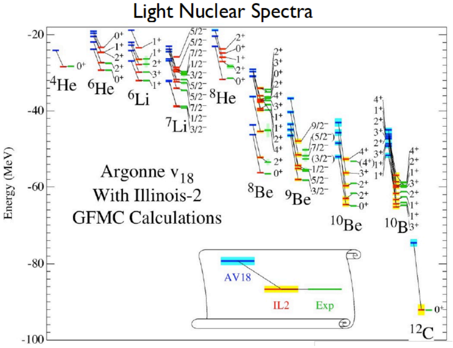
- NN interaction dependence
- Many-body dependence of EoS
- First comparison of non-relativistic approaches

Outline

UNIVERSITY OF SURREY

Why 3 body forces?

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$



Pieper & Wiringa, Annu. Rev. Nucl. Part. Sci. **51**, 53 (2001)

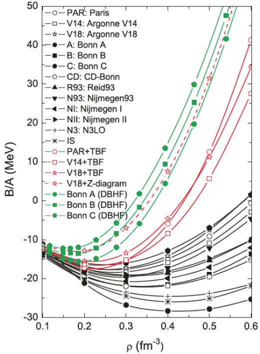
- Properties of light nuclei
- Saturation of nuclear matter
- Origin of 3BF

Why 3 body forces?

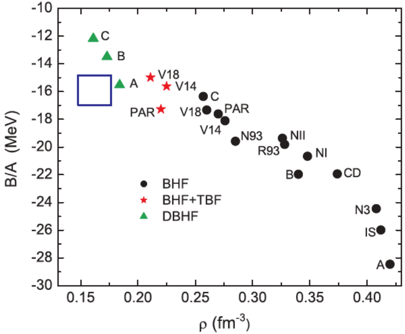
UNIVERSITY OF SURREY

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

SNM saturation BHF



Coester line with BHF



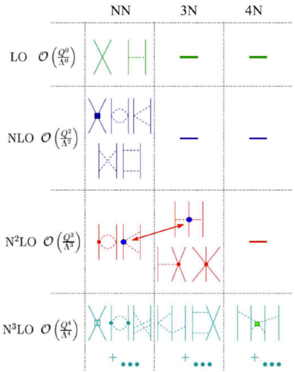
Li, Lombardo *et al.*, Phys.Rev. C **74**, 047304 (2006)

- Properties of light nuclei
- Saturation of nuclear matter
- Origin of 3BF

Why 3 body forces?

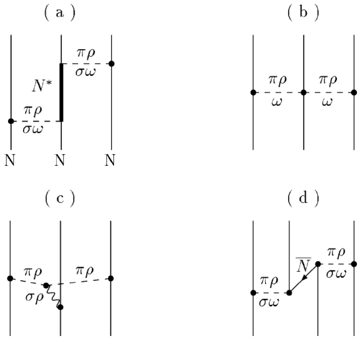
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

3NF in χPT



Epelbaum *et al.*, Phys. Rev. C **66**, 064001 (2001)

Phenomenological 3NF



Zuo *et al.*, Nucl. Phys. A **706**, 418 (2002)

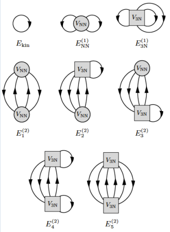
- Properties of light nuclei
- Saturation of nuclear matter
- Origin of 3BF

3BFs in many-body approaches

UNIVERSITY OF SURREY

3BFs in many-body calculations

Direct



FHNC

Carlson *et al.*, Nucl. Phys. A **401**, 59 (1983)

Monte Carlo

Gandolfi *et al.*, Phys. Rev. C **79**, 054005 (2009)

BHF

?

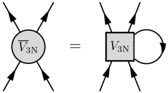
SCGF

?

RG

Tolos *et al.*, Nucl. Phys. A **806**, 105 (2008)

Average over 3rd particle



FHNC

Lovato *et al.*, arxiv:1011.3784

Monte Carlo

Gandolfi *et al.*, MNRAS **404**, 35 (2010)

BHF

Li & Schulze, Phys. Rev. C **78**, 028801(2008)
Vidaña *et al.*, Phys. Rev. C **80**, 045806 (2009)

SCGF

Soma *et al.*, Phys. Rev. C **78**, 054003 (2008)

RG

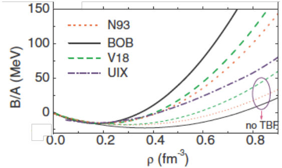
Holt *et al.*, Phys. Rev. C **81**, 024002 (2010)
Hebeler *et al.*, Phys. Rev. C **82**, 014314 (2010)

Recent examples

UNIVERSITY OF SURREY

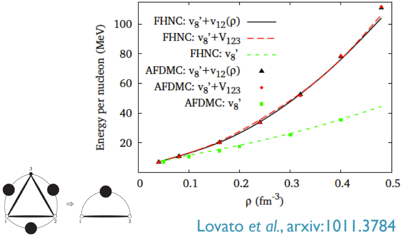
BHF

$$\overline{V}_{ij}(r) = \rho \int d^3 r_k \sum_{\sigma_k,\tau_k} g(r_{ik})^2 g(r_{jk})^2 V_{ijk}$$



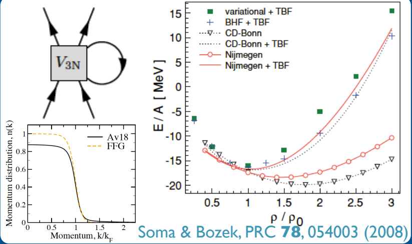
Li, Lombardo, Shulze & Zuo, PRC **77**, 034316 (2008)

FHNC



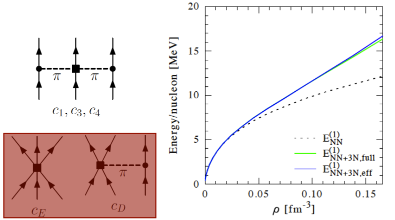
Lovato *et al.*, arxiv:1011.3784

SCGF



Soma & Bozek, PRC **78**, 054003 (2008)

RG



Hebeler *et al.*, PRC **82**, 014314 (2010)

Outline

UNIVERSITY OF SURREY

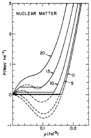
Hot nuclear matter

T=0

FHNC	Monte Carlo	BHF	SCGF	RG
✓	✓	✓	? Soma & Bozek, PRC **78**, 054003 (2008)	✓

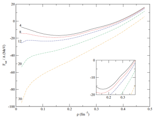
Finite Temperature

Free-energy minimization



Schmidt & Pandharipande, PLB **87**, 11 (1979)
Friedman & Panhdaripande, NPA **361** (1981)

Microcanonical ensemble

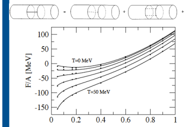


Mukherjee, PRC **75**, 035802 (2007) & **79** 045811 (2009)

?

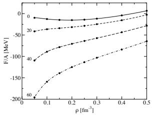
Hybrid approach

Bloch-de Dominicis



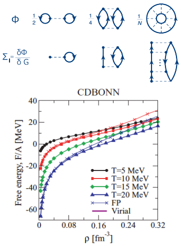
Baldo & Ferreira, PRC **59**, 682 (1999)
Nicotra *et al.*, A&A **451**, 213 (2006)

FT-BHF



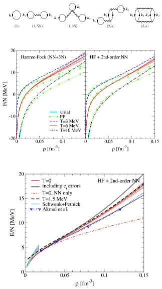
Rios *et al.*, PRC **72**, 024316 (2005)
Burgio *et al.*, PRC **83**, 025804 (2011)

Luttinger-Ward



Rios *et al.*, PRC **74**, 054317 (2006)
Soma & Bozek, PRC **74**, 045809 (2006)
Rios *et al.*, PRC **79**, 025802 (2009)

Finite temperature perturbation theory



Tolos *et al.*, NPA **74**, 054317 (2006)

Hypernuclear matter

FHNC

?

LOCV

Pandharipande, NPA **178**, 123 (1971)

Monte Carlo

✓

SCGF

✓

Impurity

Robertson, PRC **70**, 044301 (2004)

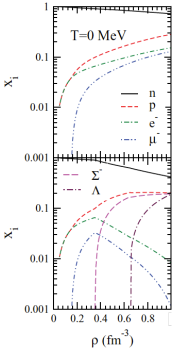
RG

✓

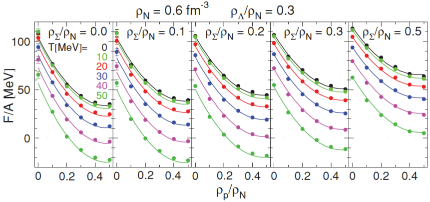
YN interactions

Dapo *et al*, PRC **81**, 035803 (2010)

Brueckner-Hartree-Fock



10^3 BHF data & fits for hot hyperonic matter EoS



Burgio, Schulze & Li, PRC **83**, 025804 (2011)

Schulze *et al.*, PLB **355**, 21 (1995)
Baldo, Burgio & Schulze, PRC **61**, 055801 (1999)
Vidaña *et al.*, PRC **61**, 025802 (2000)
Vidaña *et al.*, PRC **62**, 035801 (2000)
Schulze, Polls, Ramos, Vidaña, PRC **73**, 058801 (2006)
Burgio & Schulze, A&A518, 17 (2010)

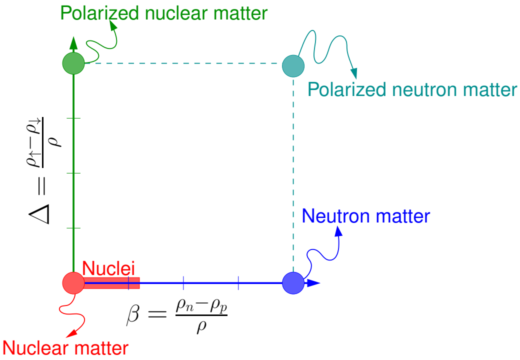
Q=0, S=-2 channels

$$\begin{pmatrix} \overline{G}_{\Lambda\Lambda\to\Lambda\Lambda} & \overline{G}_{\Lambda\Lambda\to\Xi^0 n} & \overline{G}_{\Lambda\Lambda\to\Xi^- p} & \overline{G}_{\Lambda\Lambda\to\Sigma^0\Lambda} & \overline{G}_{\Lambda\Lambda\to\Sigma^0\Sigma^0} & \overline{G}_{\Lambda\Lambda\to\Sigma^-\Sigma^+} \\ \overline{G}_{\Xi^0 n\to\Lambda\Lambda} & \overline{G}_{\Xi^0 n\to\Xi^0 n} & \overline{G}_{\Xi^0 n\to\Xi^- p} & \overline{G}_{\Xi^0 n\to\Sigma^0\Lambda} & \overline{G}_{\Xi^0 n\to\Sigma^0\Sigma^0} & \overline{G}_{\Xi^0 n\to\Sigma^-\Sigma^+} \\ \overline{G}_{\Xi^- p\to\Lambda\Lambda} & \overline{G}_{\Xi^- p\to\Xi^0 n} & \overline{G}_{\Xi^- p\to\Xi^- p} & \overline{G}_{\Xi^- p\to\Sigma^0\Lambda} & \overline{G}_{\Xi^- p\to\Sigma^0\Sigma^0} & \overline{G}_{\Xi^- p\to\Sigma^-\Sigma^+} \\ \overline{G}_{\Sigma^0 p\to\Lambda\Lambda} & \overline{G}_{\Sigma^0 p\to\Xi^0 n} & \overline{G}_{\Sigma^0 p\to\Xi^- p} & \overline{G}_{\Sigma^0 p\to\Sigma^0\Lambda} & \overline{G}_{\Sigma^0 p\to\Sigma^0\Sigma^0} & \overline{G}_{\Sigma^0 p\to\Sigma^-\Sigma^+} \\ \overline{G}_{\Sigma^0\Sigma^0\to\Lambda\Lambda} & \overline{G}_{\Sigma^0\Sigma^0\to\Xi^0 n} & \overline{G}_{\Sigma^0\Sigma^0\to\Xi^- p} & \overline{G}_{\Sigma^0\Sigma^0\to\Sigma^0\Lambda} & \overline{G}_{\Sigma^0\Sigma^0\to\Sigma^0\Sigma^0} & \overline{G}_{\Sigma^0\Sigma^0\to\Sigma^-\Sigma^+} \\ \overline{G}_{\Sigma^-\Sigma^+\to\Lambda\Lambda} & \overline{G}_{\Sigma^-\Sigma^+\to\Xi^0 n} & \overline{G}_{\Sigma^-\Sigma^+\to\Xi^- p} & \overline{G}_{\Sigma^-\Sigma^+\to\Sigma^0\Lambda} & \overline{G}_{\Sigma^-\Sigma^+\to\Sigma^0\Sigma^0} & \overline{G}_{\Sigma^-\Sigma^+\to\Sigma^-\Sigma^+} \end{pmatrix}$$

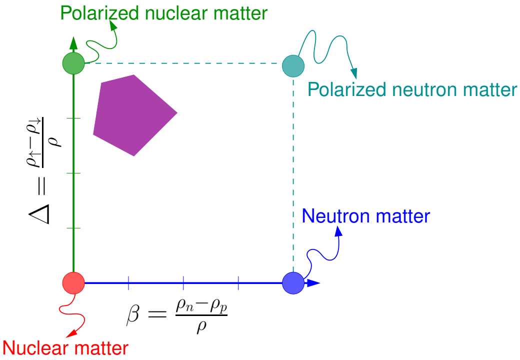
Issues

- Uncertainties in NY & YY interactions
- Hyperonic 3BF
- Softening of the EoS
- Other effects: response, transport, viscosities

Exotic phases of nuclear matter

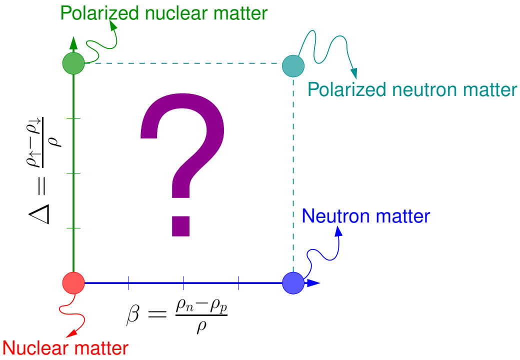


- Spin & isospin polarized nuclear matter
- Which phases are favored and why?
- Need of safe theoretical estimations!



- Spin & isospin polarized nuclear matter
- Which phases are favored and why?
- Need of safe theoretical estimations!

Exotic phases of nuclear matter

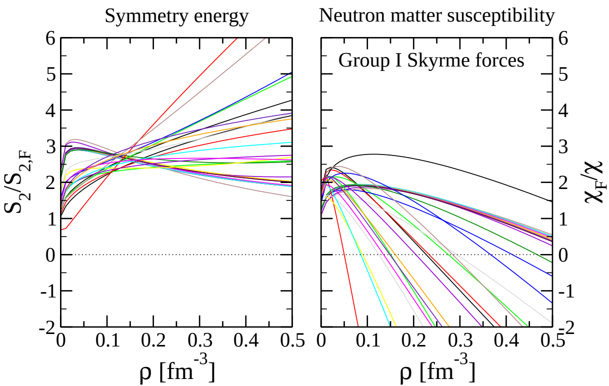


- Spin & isospin polarized nuclear matter
- Which phases are favored and why?
- Need of safe theoretical estimations!

Ferromagnetism?

Instabilities in phenomenological & microscopic models

$$S_2 = \frac{1}{2}\left(\frac{\partial^2 E/A}{\partial \beta^2}\right)_{\beta=0} \qquad \frac{1}{\chi} = \frac{1}{\mu^2 \rho}\left(\frac{\partial^2 E/A}{\partial \Delta^2}\right)_{\Delta=0}$$

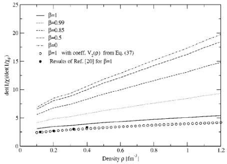


- Skyrme mean-field calculations predict instabilities
- Microscopic calculations do not predict transition

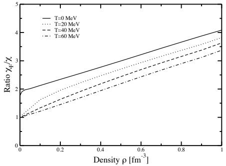
Ferromagnetism?

Instabilities in phenomenological & microscopic models

Brueckner-Hartree-Fock

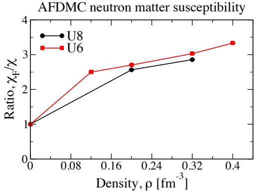


Vidaña & Bombaci, Phys. Rev. C **66**, 045801 (2001)
Vidaña et al., Phys. Rev. C **65**, 035804 (2002)

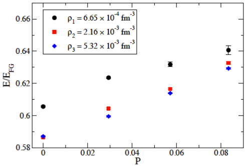


Bombaci *et al.*, Phys. Lett. B **632**, 638 (2006)

Monte-Carlo



Fantoni, Sarsa & Schmidt, PRL **87**, 181101 (2001)



Gezerlis, arxiv:1012.4464

FHNC

?

SCGF

?

RG

?

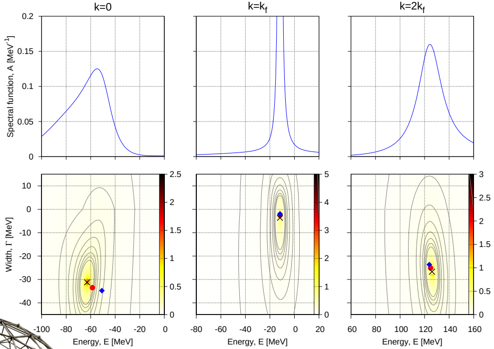
- Skyrme mean-field calculations predict instabilities
- Microscopic calculations do not predict transition

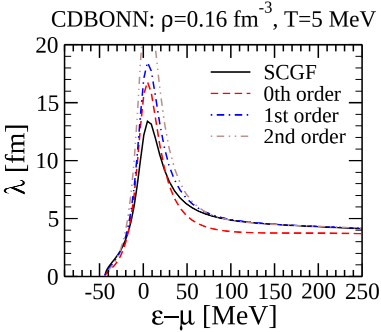
- Don’t stick to EoS only, aim at complete models
- Better if experimentally testable: mfp, viscosity, symmetry energy

See also Omar Benhar’s talk

Nucleon mean-free path within SCGF

$$\lambda = \frac{k}{m^* \Gamma}$$





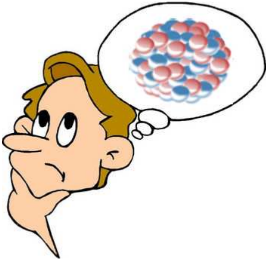
Rios & Somà, preliminary

Outline

Conclusions

- Nuclear physics is an exciting field
- but nuclear many-body problem is difficult!
- Combined with empirical knowledge, a powerful method
- Joint effort from Compstar nuclear theorists
- A certain degree of agreement... But also disagreement!
- Still work to do for exotic phases

Thank you!



MARIE CURIE ACTIONS

PEOPLE

Science & Technology Facilities Council