3/25/11

QUANTUM MECHANICS II (524)
PROBLEM SET 8 (hand in as soon as possible)

26) (40 points)

Write a computer program that solves the differential equation for the radial wave function at positive energy for a spherical potential. Include in the program the determination of the phase shift δ_ℓ (outside the range of this potential) for both $\ell = 0$ and 1. Plot both phase shifts as a function of energy in a suitable energy range. Calculate the differential cross section (and plot it) for a couple of energies which are low enough so that the $\ell = 0$ and 1 contributions dominate the cross section (potential in a) also has contributions from $\ell = 2$ at low energy but forget about these). Also, determine the total cross section and plot it as a function of energy. Choose one of the following three potentials:

a)

$$V(r) = \frac{V_0}{1 + e^{(r-R_0)/a}},$$

where $V_0 = -51$ MeV, $R_0 = r_0 A^{1/3}$, $r_0 = 1.27$fm, $a = 0.67$fm and $A = 16$. This potential is adequate for the scattering of a nucleon from the nucleus ^{16}O. So the relevant mass is $mc^2 = 939$ MeV and the relevant energy scale is in MeV.

b)

$$V(r) = 4\epsilon \left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6} \right],$$

where $\sigma = 2.556\mathring{A}$, $\epsilon = 10.22$ K (Kelvin), and the interaction is a reasonable description of the ^{4}He- ^{4}He atom-atom interaction. So the mass is the mass of the ^{4}He atom. Energy scale is in K.

c)

$$V(r) = -h\frac{e^{-x}}{x} - 1650.6\frac{e^{-4x}}{x} + 6484.2\frac{e^{-7x}}{x},$$

where $h = 10.463$ MeV (also the unit for the other terms), $x = \mu r$, $\mu = 0.7\text{fm}^{-1}$, and $\hbar^2/m = 41.47$ MeV fm^2. This interaction describes the strong part of the proton-proton interaction.

Note that for the last two interactions you have to deal with the relevant reduced mass of the problem.