3/11/11

QUANTUM MECHANICS II (524)
PROBLEM SET 7 (hand in March 25)

23) Determine the matrix elements in coordinate space of the unperturbed propagator which is given by

$$\mathcal{G}_0(E - i\epsilon) = \frac{1}{E - H_0 - i\epsilon}$$

using the same steps as in class for the propagator $\mathcal{G}_0(E + i\epsilon)$ with $i\epsilon$. Show that

$$\left|\boldsymbol{k}^{(-)}\right\rangle = |\boldsymbol{k}\rangle + \frac{1}{E - H_0 - i\epsilon} V \left|\boldsymbol{k}^{(-)}\right\rangle$$

also solves $H\left|\boldsymbol{k}^{(-)}\right\rangle = E\left|\boldsymbol{k}^{(-)}\right\rangle$. Follow the asymptotics analysis for this Lippmann-Schwinger equation and show that the second term now corresponds to an incoming spherical wave. Note that this corresponds to a boundary condition in **three** dimensions that cannot be realized physically.

24) The Lippmann-Schwinger equation can also be applied to one-dimensional problems.

a) Determine the unperturbed propagators

$$\mathcal{G}_0(x, x'; E \pm i\epsilon) = \langle x| \frac{1}{E - H_0 \pm i\epsilon} |x'\rangle\,,$$

with $H_0 = k^2/2m$.

b) Derive the corresponding Lippmann-Schwinger equations for the wave functions $\Psi_k^{\pm}(x)$ in the case of a local potential of finite range ($V(x) \neq 0$ only for $0 < |x| < a$).

c) Suppose an incident wave comes from the left described by a corresponding wave function

$$\langle x|k\rangle = \frac{1}{\sqrt{2\pi}} e^{ikx}.$$

We expect a transmitted wave only for $x > a$ and a reflected wave and the original wave for $x < -a$. Using your wisdom from the previous problem to identify (and argue why) which of the two Lippmann-Schwinger equations from b) applies in each case.

25) (20 points) Consider a potential

$$V = 0 \quad \text{for } r > R, \quad V = V_0 = constant \quad \text{for} \quad r < R,$$

where V_0 may be positive or negative. Using the differential equation for the radial wave function (using partial waves), show that for $|V_0| \ll E = p^2/2m$ and $pR/\hbar \ll 1$ the differential cross section is isotropic and that the total (elastic) cross section is given by

$$\sigma_{tot} = \frac{16\pi}{9} \frac{m^2 V_0^2 R^6}{\hbar^4}.$$

It may be helpful to use (and look up) one of the recursion relations for spherical Bessel functions. Suppose that the energy is now raised slightly. Show that the angular distribution can then be written as

$$\frac{d\sigma}{d\Omega} = A + B\cos\theta.$$

Obtain an expression for A/B.