QUANTUM MECHANICS II (524)
NUMERICAL PROBLEM SET 2 (hand in within a reasonable amount of time)

B) Write a computer program that solves the differential equation for the radial wave function at positive energy for a spherical potential using the Numerov method. This means that you should not ask a Math package to do everything for you. Include in the program the determination of the phase shift δ_ℓ (outside the range of the potential).

Plot these phase shifts as a function of energy in a suitable energy range (see below). Calculate the differential cross section (and plot it) for a couple of representative energies. Also, determine the total cross section and plot it as a function of energy. Use your code for the following three potentials:

a) (40 points)

$$V(r) = \frac{V_0}{1 + e^{(r-R_0)/a}},$$

where $V_0 = -51$ MeV, $R_0 = r_0 A^{1/3}$, $r_0 = 1.27$fm, $a = 0.67$fm and $A = 16$. This potential is adequate for the scattering of a nucleon from the nucleus ^{16}O. So the relevant mass is $mc^2 = 939$ MeV and the relevant energy scale is in MeV. Note that you should plot your phase shifts as a continuous function of the energy. The energy range can be taken from 0 to 25 MeV. Check that the convergence in the number of partial waves has occurred (this maximum ℓ-value increases with energy). Also, determine the total elastic cross section and plot it as a function of energy.

b) (30 points)

$$V(r) = 4\epsilon \left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6} \right],$$

where $\sigma = 2.556\mathring{A}$, $\epsilon = 10.22$ K (Kelvin), and the interaction is a reasonable description of the ^{4}He- ^{4}He atom-atom interaction. So the mass is the mass of the ^{4}He atom. Energy scale is in K. Calculate only the phase shifts for $\ell = 0, 1$ and 2 and plot these as a function of the energy up to the value that corresponds to $k = 2\mathring{A}^{-1}$.

c) (30 points)

$$V(r) = -h\frac{e^{-x}}{x} - 1650.6\frac{e^{-4x}}{x} + 6484.2\frac{e^{-7x}}{x},$$

where $h = 10.463$ MeV (also the unit for the other terms), $x = \mu r$, $\mu = 0.7\text{fm}^{-1}$, and $\hbar^2/m = 41.47$ MeV fm^2. This interaction describes certain aspects of strong part of the proton-proton interaction. Calculate only the $\ell = 0$ phase shift. You will find that it starts positive and rises rapidly before becoming smaller again, eventually crossing zero at rather large energy. Make sure you go high enough in energy to see this crossing. As it stands, the potential cannot support a bound state for $\ell = 0$ but it almost does. Change the -1650.6 constant (try initially steps of 5%) to generate a bound state at about -2 MeV using your bound-state program (but don't include Coulomb). Plot the corresponding $\ell = 0$ phase shift for this new potential and compare with the result for the original one.

Note that for the last two interactions you have to deal with the relevant reduced mass of the problem.