4/3/15

QUANTUM MECHANICS II (524)
PROBLEM SET 8 (hand in #21 in on April 10 and the numerical work later)

21) (20 points) Consider a potential

$$V = 0 \quad \text{for } r > R, \quad V = V_0 = constant \quad \text{for} \quad r < R,$$

where V_0 may be positive or negative. Using the differential equation for the radial wave function (using partial waves), show that for $|V_0| \ll E = \hbar^2 k^2/2\mu$ and $kR \ll 1$ the differential cross section is isotropic and that the total (elastic) cross section is given by

$$\sigma_{tot} = \frac{16\pi}{9} \frac{\mu^2 V_0^2 R^6}{\hbar^4}.$$

Suppose that the energy is now raised slightly. Show that the angular distribution can then be written as

$$\frac{d\sigma}{d\Omega} = A + B \cos\theta.$$

Obtain an expression for A/B.

C) (30 points) This problem is an extension of the numerical work of the previous numerical assignment so do it when you have finished that one. The due date is somewhat flexible again. Consider the attractive central potential that you have used in part a) of that assignment. Generate the phase shift for $\ell = 3$. Since there are no $\ell = 3$ bound states, the phase shift starts at 0 for $E \to 0$. Plot the phase shift as a function of energy. Since the potential is attractive, the phase shift will initially rise and then at sufficiently high energy it will decrease and ultimately go to zero. Now increase the attraction of this potential by a suitable amount and study the behavior of the phase shift in this partial wave. Ultimately by increasing the attraction you should see a very fast rise of the phase through $\pi/2$ after which it will decrease again. Plot the phase shifts for relevant values of the depth of the potential and plot the potential as well. Also plot the partial cross section from this wave as a function of energy for each case. While you are looking into this, read corresponding material (in Sakurai for example) about resonance scattering. What will happen when you continue to increase the attraction of the potential?