3/20/15

QUANTUM MECHANICS II (524)
PROBLEM SET 3 (hand in March 27)

18) (20 pts)

a) Complete the steps discussed in class for a wave packet with a weight function

$$g(\boldsymbol{k}) \propto \exp[-\Delta_0^2(\boldsymbol{k}-\boldsymbol{k}_0)^2 - i\hbar\frac{\boldsymbol{k}\cdot\boldsymbol{k}_0}{\mu}t_0 + i\hbar\frac{\boldsymbol{k}^2}{2\mu}t_0]$$

and construct the probability density $|\langle\phi_{\boldsymbol{r}}|\psi_g(t)\rangle|^2$ for large negative times.

b) Apply the stationary phase method to perform the k_z integration to obtain the asymptotic form of $\Psi_g(\boldsymbol{r},t)$ for $t\to\infty$.

c) Evaluate

$$\rho_\perp = \int_{-\infty}^{\infty} dz \;\; |\Psi_g(0,z,t)|^2$$

for $t\to-\infty$ and obtain the result discussed in class.

19) (20 pts) Determine the matrix elements in coordinate space of the unperturbed propagator which is given by

$$\mathcal{G}_0(E-i\epsilon) = \frac{1}{E-H_0-i\epsilon}$$

using the same steps as in class for the propagator $\mathcal{G}_0(E+i\epsilon)$ with $i\epsilon$. Show that

$$\left|\psi_{\boldsymbol{k}}^{(-)}\right\rangle = |\phi_{\boldsymbol{k}}\rangle + \frac{1}{E-H_0-i\epsilon}V\left|\psi_{\boldsymbol{k}}^{(-)}\right\rangle$$

also solves $H\left|\psi_{\boldsymbol{k}}^{(-)}\right\rangle = E(k)\left|\psi_{\boldsymbol{k}}^{(-)}\right\rangle$. Follow the asymptotics analysis for this Lippmann-Schwinger equation and show that the second term now corresponds to an incoming spherical wave. Note that this corresponds to a boundary condition in **three** dimensions that cannot be realized physically.

20) (20 pts) The Lippmann-Schwinger equation can also be applied to one-dimensional problems.

a) Determine the unperturbed propagators

$$\mathcal{G}_0(x, x'; E \pm i\epsilon) = \langle x| \frac{1}{E - H_0 \pm i\epsilon} |x'\rangle ,$$

with $H_0 = \hbar^2 k^2/2m$.

b) Derive the corresponding Lippmann-Schwinger equations for the wave functions $\Psi_k^{\pm}(x)$ in the case of a local potential of finite range ($V(x) \neq 0$ only for $0 < |x| < a$).

c) Suppose an incident wave comes from the left described by a corresponding wave function

$$\langle x|k\rangle = \frac{1}{\sqrt{2\pi}} e^{ikx}.$$

We expect a transmitted wave only for $x > a$ and a reflected wave and the original wave for $x < -a$. Using your wisdom from the previous problem to identify (and argue why) which of the two Lippmann-Schwinger equations from b) applies in each case.