2/20/15

QUANTUM MECHANICS II (524)
PROBLEM SET 6 (hand in February 25)

16) (15 points) This problem involves spherical tensors of rank 2.

a) Write $xy, xz$, and $(x^2 - y^2)$ as components of a spherical tensor of rank 2.

b) The expectation value

$$Q \equiv e \langle \alpha jm = j | \left(3z^2 - r^2\right) | \alpha jm = j \rangle$$

is known as the quadrupole moment. Determine

$$e \langle \alpha jm' | \left(x^2 - y^2\right) | \alpha jm = j \rangle ,$$

(where $m' = j, j-1, ...$) in terms of $Q$ and appropriate Clebsch-Gordan coefficients.

Bonus problem

17) (20 bonus pts total) As discussed in the previous problem the expectation value

$$Q \equiv e \langle \alpha jm = j | \left(3z^2 - r^2\right) | \alpha jm = j \rangle$$

is known as the quadrupole moment. The angular momentum quantum numbers $j$ and $m$ must be interpreted as the total angular momentum of a nucleon obtained by adding its orbital angular momentum to its spin.

a) (5 points) Show that $Q$ can be written in terms of a radial integral and several Clebsch-Gordan coefficients.

b) (15 points) Proof that the result in part $a$) can be simplified to

$$Q = -e \frac{2j-1}{2j+1} \langle r^2 \rangle_j ,$$

with a self-evident notation for the radial integral. You may need to employ useful relations between Clebsch-Gordan coefficients that can be found in books on angular momentum. Some examples of these books are: "Angular Momentum" by Brink and Satchler, "Angular Momentum in Quantum Mechanics" by Edmonds. Equation (260) in Chapter 7 of our book (note typos) is one of these relations but you may need others.