2/13/15

QUANTUM MECHANICS II (524)
PROBLEM SET 5 (hand in February 20)

14) (30 pts) Consider a system with $j = 1$.

a) Write

$$\langle j = 1, m' | \, J_y \, | j = 1, m \rangle$$

in 3×3 matrix form.

b) Show that it is legitimate to replace $e^{-iJ_y\beta/\hbar}$ by

$$1 - i\left(\frac{J_y}{\hbar}\right)\sin\beta - \left(\frac{J_y}{\hbar}\right)^2(1-\cos\beta).$$

c) Use *b*) to determine the matrix elements of $\mathcal{D}^{j=1}_{m,m'}(\alpha,\beta,\gamma)$.

d) Use the Kronecker product involving two rotation matrices $[\mathcal{D}^{j=1/2}_{m_s,m'_s}(\alpha,\beta,\gamma)]_i$ $(i = 1, 2)$ representing rotation matrix elements for two spin-1/2 objects to arrive at the same result as in c).

15) (10 pts) Construct a spherical tensor of rank 1 out of two different vector operators $\boldsymbol{F} = (F_x, F_y, F_z)$ and $\boldsymbol{G} = (G_x, G_y, G_z)$. Write the components of the resulting tensor $T^{(1)}_\kappa$ in terms of the x, y, and z-components of $\boldsymbol{F}$ and $\boldsymbol{G}$.