QUANTUM MECHANICS II (524)
PROBLEM SET 4 (hand in February 13)

10) (10 pts) Consider a free particle in a state with definite momentum $\boldsymbol{p}$.

a) Write down the corresponding wave function $\psi(\boldsymbol{r}, t)$ and show that $\psi^*(\boldsymbol{r}, -t)$ is the wave function for the state with the momentum direction reversed.

b) Consider the above wave function at $t = 0$. Note that it is a complex wave function and explain why this doesn't violate time-reversal invariance.

11) (10 pts) Let $\psi(\boldsymbol{p})$ be the momentum-space wave function for the state $|\psi\rangle$. Construct the momentum-space wave function for the time-reversed state $\mathcal{I}_t\,|\psi\rangle$ in two different ways:

a) By using the decomposition of $|\psi\rangle$ in momentum-space eigenstates.

b) By Fourier-transforming the corresponding wave function of the time-reversed state in coordinate space.

Make sure the results in 10) and 11) agree.

12) (10 pts) A system with spin 1 has a Hamiltonian given by

$$H = aS_z^2 + b(S_x^2 - S_y^2),$$

with the constants a and b real.

a) Solve this problem exactly for the eigenvalues and corresponding normalized eigenkets.

b) Show that this Hamiltonian either is invariant under time reversal or that it is not.

13) (10 pts) We have considered two versions of the rotation matrix for a spin $\frac{1}{2}$ system: the one using Euler angles (α, β, γ) and the one with a unit vector and an angle $(\hat{n}, \phi)$. Use these two expressions to derive formulas for the connection between these two types of angles. Do it both ways: *i.e.* write Euler angle $= f(\hat{n}, \phi)$, *etc.* and also the other way around. As an example, represent a rotation about the x-axis by 45 degrees in terms of Euler angles.