QUANTUM MECHANICS II (524)
PROBLEM SET 3 (hand in February 6)

7) (10 pts) A quantum state represented by $|\Psi\rangle$ is known to be a simultaneous eigenket of the Hermitian operators A and B which *anticommute*,

$$AB + BA = 0.$$

What can you say about the eigenvalues of A and B for the state $|\Psi\rangle$? Illustrate your point using the parity operator Π and the momentum operator $\boldsymbol{p}$.

8) (20 pts) Consider a spin 1/2 particle bound in a spherically symmetric potential well (like for hydrogen or the 3-D oscillator). Define spin-angular functions in two-component form as follows

$$\mathcal{Y}_\ell^{j=\ell\pm1/2,m} = \pm\sqrt{\frac{l \pm m + 1/2}{2\ell+1}} Y_{\ell,m-1/2}(\theta,\phi)\chi_+ + \sqrt{\frac{l \mp m + 1/2}{2\ell+1}} Y_{\ell,m+1/2}(\theta,\phi)\chi_-,$$

where $\chi_\pm$ describe spinors with spin up and down, respectively. Note that the spin-angular functions are simultaneous eigenfunctions of $\boldsymbol{S}^2, \boldsymbol{L}^2, \boldsymbol{J}^2$ and J_z, since the linear combinations above involve the appropriate Clebsch-Gordan coefficients to couple orbital angular momentum and spin to total angular momentum (and its projection).

a) Write out (in as simple a form as possible) the spin-angular function $\mathcal{Y}_{\ell=0}^{j=1/2,m=1/2}$.
b) Express $(\boldsymbol{\sigma}\cdot\boldsymbol{r})\mathcal{Y}_{\ell=0}^{j=1/2,m=1/2}$ in terms of other $\mathcal{Y}_\ell^{j,m}$.
c) Show that your result in b) is understandable by considering the transformation properties of the operator $\boldsymbol{s}\cdot\boldsymbol{r}$ under rotations and parity.

9) (10 pts) Let $\chi(\hat{\boldsymbol{n}})$ be the two-component eigenspinor of $\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}$ with eigenvalue $+1$. Use the explicit form of $\chi(\hat{\boldsymbol{n}})$ (in terms of the angles that characterize $\hat{\boldsymbol{n}}$) to verify that $-i\sigma_y\chi^*(\hat{\boldsymbol{n}})$ is the two-component spinor with the spin direction reversed.