10/31/14

QUANTUM MECHANICS I (523)
PROBLEM SET 8 (hand in November 7)

30) Consider a system with angular momentum $j = 1$, whose ket space is spanned by the basis $|+1\rangle, |0\rangle, |-1\rangle$ of three eigenkets common to $\boldsymbol{J}^2$ (eigenvalue $2\hbar^2$) and J_z (respective eigenvalues $+\hbar, 0$ and $-\hbar$). The state of the system is given by

$$|\psi\rangle = \alpha\,|+1\rangle + \beta\,|0\rangle + \gamma\,|-1\rangle\,,$$

where α, β and γ are three complex parameters.

a) Calculate the mean value $\langle \boldsymbol{J} \rangle$ of the angular momentum in terms of α, β and γ.

b) Give the expression for the three mean values $\langle J_x^2 \rangle$, $\langle J_y^2 \rangle$, and $\langle J_z^2 \rangle$ in terms of the same quantities.

31) Consider a physical system whose four-dimensional vector space is spanned by a basis of four eigenkets $|jm\rangle$ common to $\boldsymbol{J}^2$ and J_z with $j = 0$ or 1 and $-j \leq m \leq j$.

a) Determine the eigenkets common to $\boldsymbol{J}^2$ and J_x in terms of these basis states.

b) Consider the normalized state

$$|\psi\rangle = \alpha\,|j=1, m=1\rangle + \beta\,|j=1, m=0\rangle + \gamma\,|j=1, m=-1\rangle + \delta\,|j=0, m=0\rangle\,.$$

Determine the probability of finding $2\hbar^2$ and $\hbar$ if $\boldsymbol{J}^2$ and J_x are measured simultaneously.

c) Calculate the mean value of J_z when the system is in the state $|\psi\rangle$. Do the same for the operators $\boldsymbol{J}^2$ and J_x.

d) Now J_z^2 is measured. Determine the possible results, their probabilities and their mean value.

32) An angular momentum eigenstate $|j, m = j\rangle$ is rotated by an infinitesimal angle ϵ about the y-axis. Calculate to order ϵ^2 the probability that a measurement on the rotated state will yield the original state.

33) The wave function of a particle in a spherically symmetric potential $V(r)$ is given by

$$\psi(\boldsymbol{r}) = (x + y + 3z)f(r)$$

a) Is ψ an eigenfunction of $\boldsymbol{\ell}^2$? If yes, determine the ℓ-value. If no, what are the possible ℓ-values one may obtain upon measuring $\boldsymbol{\ell}^2$?

b) What are the probabilities for the particle to be found in various m_ℓ states?