QUANTUM MECHANICS I (523)
PROBLEM SET 6 (hand in October 24)

22) Consider a free particle in one dimension in part a) and a particle with a Hamiltonian $H = \boldsymbol{p}^2/2m + V(\boldsymbol{x})$ in part b).

a) For the case of the one-dimensional problem, consider the position operator in the Heisenberg picture $x_H(t)$. Evaluate

$$[x_H(t), x_H(t=0)]\,.$$

b) Now working in three dimensions, calculate

$$[\boldsymbol{x}_H \cdot \boldsymbol{p}_H, H_H]$$

to obtain

$$\frac{d}{dt}\langle \boldsymbol{x} \cdot \boldsymbol{p} \rangle = \left\langle \frac{\boldsymbol{p}^2}{m} \right\rangle - \langle \boldsymbol{x} \cdot \nabla V \rangle.$$

In order to identify this results as the quantum analog of the virial theorem, the left-hand side should vanish. Under what condition does this happen?

23) Consider the spin precession problem with the Hamiltonian

$$H = \omega S_z.$$

The system is represented at time $t = 0$ by the ket

$$|\psi; t=0\rangle = \frac{1}{2}\,|S_z; +\rangle + \frac{i\sqrt{3}}{2}\,|S_z; -\rangle\,.$$

a) Calculate the energy dispersion for this state.

b) Determine the state at time t and calculate the probability that a measurement of S_y yields $\hbar/2$.

c) Evaluate τ_{S_x} which represents the characteristic time of the evolution of the statistical distribution of S_x for the ket $|\psi; t=0\rangle$

$$\tau_{S_x} = \frac{\langle (\Delta S_x)^2 \rangle^{1/2}}{\left| \frac{d\langle S_x \rangle}{dt} \right|}.$$

by first evaluating the time dependence of $\langle S_x \rangle$ and $\langle (S_x)^2 \rangle$. Be sure to check the time-energy uncertainty relation.

24) Consider the one-dimensional harmonic oscillator. Do the following without using wave functions.

a) Construct a linear combination of $|0\rangle$ and $|1\rangle$ such that $\langle q \rangle$ is as large as possible.

b) Assume that at $t = 0$ the system is in this state. Determine the time-evolved state at t in the Schrödinger picture and evaluate the expectation value $\langle q \rangle$ as a function of time in both the Schrödinger **and** Heisenberg picture.

c) Evaluate $\langle (\Delta q)^2 \rangle$ using either picture.

25) Consider a particle with mass m in a one-dimensional potential of the following form:

$$V = \begin{cases} \frac{1}{2}kq^2 & \text{for } q > 0 \\ \infty & \text{for } q < 0. \end{cases}$$

a) Determine the ground state energy by "thinking outside the box."

b) Determine the expectation value $\langle q^2 \rangle$ for the ground state.