09/19/14

QUANTUM MECHANICS I (523)
PROBLEM SET 3 (hand in September 26)

10) (10 points) A beam of spin-3/2 particles is directed into a Stern-Gerlach analyzer (in the $\hat{\boldsymbol{z}}$-direction). Four output beams are observed with deflections consistent with magnetic moments arising from spin angular momentum components of $3/2\hbar$,$1/2\hbar$,-$1/2\hbar$, and $-3/2\hbar$. For such a physical system write down

a) The eigenvalue equations for the S_z operator.
b) The matrix representations of the S_z eigenstates.
c) The matrix representation of the S_z operator.
d) The eigenvalue equations for the $\boldsymbol{S}^2$ operator and its matrix representation.

11) (10 points) A certain observable O has a 3×3 matrix representation in some basis as follows:

$$O \Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}.$$

Find all the normalized eigenvectors and the corresponding eigenvalues.

12) (5 points) Let A and B be observables. Suppose the simultaneous eigenkets of A and B $\{|a_i b_j\rangle\}$ form a *complete* orthonormal set of basis kets. Can one always conclude that $[A, B] = 0$? If yes, prove it. If no, give a counterexample.

13) (15 points) Consider a three-dimensional ket space. In a certain orthonormal basis with kets $|1\rangle$, $|2\rangle$, and $|3\rangle$ the operators A and B are represented by

$$A \Rightarrow \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}$$

and

$$B \Rightarrow \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix},$$

with a and b both real.

a) Clearly A has a degenerate spectrum. Does B also have a degeneracy?

b) Show that A and B commute.

c) Find a new set of orthonormal kets which are simultaneous eigenkets of A and B. Specify the eigenvalues of A and B for each of these three eigenkets. Does this specification completely chracterize each eigenket or is there another commuting observable?