11/16/12

QUANTUM MECHANICS I (523)
PROBLEM SET 12 (hand in December 7 or earlier)

43) (15 points) Consider a particle with orbital angular momentum $\ell = 0$ in the central potential

$$V(r) = \frac{-V_0}{\exp\{\kappa r\} - 1}$$

called Hulthen's potential. Find the lowest energy eigenvalue using the operator method discussed in class for the three-dimensional oscillator and the Hydrogen-like Hamiltonian. Try

$$G^+_{\ell=0} \approx p_r + ib_0 + \frac{ic_0}{\exp\{\kappa r\} - 1}$$

with b_0 and c_0 constants.

44) (25 points) Define the operator

$$\boldsymbol{M} = \frac{1}{2m}\left(\boldsymbol{p} \times \boldsymbol{\ell} - \boldsymbol{\ell} \times \boldsymbol{p}\right) - e^2\frac{\boldsymbol{r}}{r}.$$

a) Show that

$$[\ell_i, M_j] = i\hbar\epsilon_{ijk}M_k.$$

You should of course make use of the results of problem 37.

b) The Hamiltonian of the hydrogen atom

$$H = \frac{\boldsymbol{p}^2}{2m} - \frac{e^2}{r}$$

commutes with $\boldsymbol{M}$. Demonstrate this. This symmetry is responsible for the accidental degeneracy of the Hydrogen atom (see Gottfried Sec. 5.2 for many more details and also Sakurai p. 265 for more information).