08/29/14

QUANTUM MECHANICS I (523)
PROBLEM SET 1 (hand in September 12)

1) (10 pts) A gyroscope has a magnetic moment $\boldsymbol{\mu} = \gamma \boldsymbol{L}$. Show that the gyroscope carries out a precessional motion of circular frequency $\omega_L = \gamma B$ (Larmor frequency) in a constant magnetic field $\boldsymbol{B} = (0, 0, B)$.

2) (10 pts) A beam of hydrogen atoms, emitted from an oven running at a temperature of $T = 400K$, is sent through a Stern-Gerlach magnet of length $\ell = 1$ m. The atoms experience a magnetic field with a gradient of 10 tesla/m. Calculate the transverse deflection of a typical atom in the beam, due to the force exerted on its spin magnetic dipole moment, at the point where the beam leaves the magnet. Assume that the atoms emitted from the oven have a kinetic energy of $2k_BT$, where k_B is Boltzmann's constant (give a qualitative argument why this is plausible and larger than the typical kinetic energy of $3/2k_BT$).

3) (10 pts) Use the rules of bra-ket algebra to show the following

   a) $tr(AB) = tr(BA)$, where A and B are operators;
   b) $(AB)^\dagger = B^\dagger A^\dagger$, where A and B are operators;

4) (10 pts) Consider another set of sequential Stern-Gerlach experiments. Based on the results discussed in class describe the fraction of the atoms of the original beam that emerge either up or down from the last Stern-Gerlach magnet.

   a) Oven - SG$\hat{z}$ (block down spins) - SG$\hat{y}$ (block down spins) - SG$\hat{z}$
   b) Oven - SG$\hat{z}$ (block down spins) - SG$\hat{y}$ (block up spins) - SG$\hat{z}$
   c) Oven - SG$\hat{z}$ (block down spins) - SG$\hat{y}$ (both spins let through and beams recombined) - SG$\hat{z}$

5) (20 pts) Suppose we prepare a beam with angular momentum $\hbar$ (spin 1) which can yield a z-component of the angular momentum of $1 \times \hbar, 0 \times \hbar$, or $-1 \times \hbar$. What are your predictions for the intensities of the corresponding splitting of the beam in a Stern-Gerlach (z-axis) arrangement. Use the results of a single spin-$\frac{1}{2}$ atom to plausibly argue the outcome (argue quantitatively). If we select one type of atom from the SG$\hat{z}$ magnet and then subject the atom to a SG$\hat{x}$ arrangement, what will happen (answer qualitatively)?