2/18/11

QUANTUM MECHANICS II (524)
PROBLEM SET 5 (hand in February 25)

17) Construct a spherical tensor of rank 1 out of two different vectors $\boldsymbol{F} = (F_x, F_y, F_z)$ and $\boldsymbol{G} = (G_x, G_y, G_z)$. Write the components of the resulting tensor $T_\kappa^{(1)}$ in terms of the x, y, and z-components of $\boldsymbol{F}$ and $\boldsymbol{G}$.

18) (15 points) Consider the following matrix elements

$$\langle n\ell m| \mp \frac{1}{\sqrt{2}}(x \pm iy)\,|n'\ell' m'\rangle$$

and

$$\langle n\ell m|\, z\,|n'\ell' m'\rangle\,.$$

Relate these matrix elements as much as possible by using *only* the Wigner-Eckart theorem. State under what conditions these matrix elements are nonvanishing.

19) (15 points) This problem involves spherical tensors of rank 2.

a) Write xy, xz, and $(x^2 - y^2)$ as components of a spherical tensor of rank 2.

b) The expectation value

$$Q \equiv e\,\langle \alpha j m = j|\,(3z^2 - r^2)\,|\alpha j m = j\rangle$$

is known as the quadrupole moment. Determine

$$e\,\langle \alpha j m'|\,(x^2 - y^2)\,|\alpha j m = j\rangle\,,$$

(where $m' = j, j-1, \ldots$) in terms of Q and appropriate Clebsch-Gordan coefficients.

20) (20 bonus points total) As discussed in the previous problem the expectation value

$$Q \equiv e\,\langle \alpha j m = j|\,(3z^2 - r^2)\,|\alpha j m = j\rangle$$

is known as the quadrupole moment. The angular momentum quantum numbers j and m must be interpreted as the total angular momentum of a nucleon obtained by adding its orbital angular momentum to its spin.

a) (5 points) Show that Q can be written in terms of a radial integral and several Clebsch-Gordan coefficients.

b) (15 points) Proof that the result in part a) can be simplified to

$$Q = -e\frac{2j-1}{2j+1}\langle r^2\rangle_j,$$

with a self-evident notation for the radial integral. You will need to employ useful relations between Clebsch-Gordan coefficients that can be found in books on angular momentum. Some examples of these books are: "Angular Momentum" by Brink and Satchler, "Angular Momentum in Quantum Mechanics" by Edmonds. Equation (260) in Chapter 7 of our book (note typos) is one of these relations but you need others.