QUANTUM MECHANICS (471)
PROBLEM SET 9 (hand in November 11)

26) (10 points) Consider the 3D harmonic oscillator.

a) Construct the first excited state of the 3D harmonic oscillator as outlined in class by applying the D_0^+ operator to the radial wave function of the ground state.

b) Find the linear combinations of the three $|N = 1 \ell = 1 m_\ell\rangle$ states ($m_\ell = -1, 0, 1$) in terms of the three cartesian solutions of the 3D harmonic oscillator at the same energy
$|n_x = 1, n_y = 0, n_z = 0\rangle\,, |n_x = 0, n_y = 1, n_z = 0\rangle\,, |n_x = 0, n_y = 0, n_z = 1\rangle$.

27) (10 points) Consider a simple one-dimensional harmonic oscillator subject to a perturbation $V = bx$, where b is a real constant.

a) Calculate the energy shift of the ground state to lowest nonvanishing order.

b) Solve the problem exactly and compare with your result obtained in part a).

28) (10 points) Consider the first-order correction to the ground-state energy of the Helium atom discussed in class. Carry out the integration necessary for the evaluation of this correction, *i.e.* determine

$$\epsilon_1 = \frac{e^2}{\pi^2 a^6} \int d\boldsymbol{r}_1 \int d\boldsymbol{r}_2 \, \frac{e^{-2(r_1+r_2)/a}}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|},$$

using the expansion

$$\frac{1}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|} = \sum_{\ell=0}^{\infty} \frac{r_<^\ell}{r_>^{\ell+1}} P_\ell(\cos\theta),$$

where $r_<$ is the smaller of r_1 and r_2 and $r_>$ the larger. Also use the addition theorem of the spherical harmonics in terms of the Legendre polynomial

$$\frac{2\ell+1}{4\pi} P_\ell(\cos\theta) = \sum_{m_\ell=-\ell}^{\ell} Y_{\ell m_\ell}^*(\hat{\mathbf{1}}) Y_{\ell m_\ell}(\hat{\mathbf{2}}).$$

29) (10) Consider a one-electron atom with a nondegenerate ground state placed in an electric field which points in the z-direction. What is the corresponding potential that the electron experiences? Construct the ground state to first-order in this perturbing potential and determine the expectation value of the operator ez with respect to this state. Obtain the energy shift $\Delta = \alpha|\boldsymbol{E}|^2/2$ of the ground state computed to second order.