11/4/16

QUANTUM MECHANICS (471)
PROBLEM SET 8 (hand in November 11)

23) (10 points) Construct the normalized antisymmetric state for 4 particles in terms of product states using generic labels for the single-particle quantum numbers.

24) (10 points) Diagonalize $\boldsymbol{S}^2$ for a system of two spin-$\frac{1}{2}$ particles and generate the basis transformation to states with good total spin that way.

25) (20 points) The following operators are useful in dealing with the 3D harmonic oscillator:

$$C_\ell^\pm = p_r \pm \frac{i\hbar}{r}\left(\ell + \tfrac{1}{2} \pm \tfrac{1}{2}\right) \mp im\omega r.$$

We will also consider

$$D_\ell^\pm = p_r \pm \frac{i\hbar}{r}\left(\ell + \tfrac{1}{2} \pm \tfrac{1}{2}\right) \pm im\omega r.$$

a) Demonstrate that

$$H_{\ell-1} C_\ell^- \left|E\ell\right\rangle = \left(E + \hbar\omega\right) C_\ell^- \left|E\ell\right\rangle .$$

b) Show that the Hamiltonian can also be written as

$$H_\ell = \frac{1}{2m} D_{\ell\pm1}^\mp D_\ell^\pm - \hbar\omega\left(\ell + \tfrac{1}{2} \pm 1\right).$$

c) Show then that

$$D_\ell^\pm \left|E, \ell\right\rangle = d_{E\ell}^\pm \left|E \pm \hbar\omega, \ell \pm 1\right\rangle ,$$

with $d_{E\ell}^\pm$ appropriate normalization constants.