QUANTUM MECHANICS (471)
PROBLEM SET 5 (hand in October 14)

19) (10 points) Find the eigenvalues of

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Suppose an electron is in the spin state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. If $S_y$ is measured, what is the probability of the result $\hbar/2$?

20) (10 points) Consider a particle with intrinsic spin 1 with eigenstates

$$\boldsymbol{S}^2 \left|S = 1M\right\rangle = \hbar^2 S(S+1) \left|S = 1M\right\rangle$$

and

$$S_z \left|S = 1M\right\rangle = \hbar M \left|S = 1M\right\rangle.$$

Evaluate the matrix elements of

$$S_z(S_z + \hbar)(S_z - \hbar)$$

and

$$S_x(S_x + \hbar)(S_x - \hbar)$$

in the above representation.

21) (10 points) Show in detail using the strategy discussed in class that

$$\langle r\theta\phi| \ell_x |\psi\rangle = -i\hbar \left( -\sin\phi \frac{\partial}{\partial\theta} - \cot\theta \cos\phi \frac{\partial}{\partial\phi} \right) \psi(r, \theta, \phi).$$

22) (10 points for part a) and 10 bonus points for part b)) Consider the matrix operators

$$\frac{1}{2}(1 + \boldsymbol{\sigma} \cdot \hat{\boldsymbol{n}}) \text{ and } \frac{1}{2}(1 - \boldsymbol{\sigma} \cdot \hat{\boldsymbol{n}}),$$

where $\hat{\boldsymbol{n}}$ is a unit vector characterized by the usual angles $\alpha$ and $\beta$.

a) Apply these operators to the spinor $\chi_-$ and normalize the resulting spinors. Compare these spinors with the eigenstates obtained in Problem 6.

b) Apply these operators to a general spinor $\chi$ and normalize the resulting spinors. Compare with a) and discuss the properties of these operators.