QUANTUM MECHANICS (471)
PROBLEM SET 5 (hand in October 7)

16) (20 points) Consider the spin precession problem with the Hamiltonian

$$H = \omega S_z.$$

The system is represented at time $t = 0$ by the ket

$$|\psi; t = 0\rangle = \frac{1}{2}|S_z; +\rangle + \frac{i\sqrt{3}}{2}|S_z; -\rangle .$$

a) Calculate the energy dispersion for this state.

b) Determine the state at time t and calculate the probability that a measurement of S_y yields $\hbar/2$.

c) Evaluate τ_{S_x} which represents the characteristic time of the evolution of the statistical distribution of S_x for the ket $|\psi; t = 0\rangle$

$$\tau_{S_x} = \frac{\langle(\Delta S_x)^2\rangle^{1/2}}{\left|\frac{d\langle S_x\rangle}{dt}\right|}.$$

by first evaluating the time dependence of $\langle S_x\rangle$ and $\langle(S_x)^2\rangle$. You can now write the product of the energy dispersion and the characteristic time to generate what is often considered the time-energy uncertainty relation.

17) (10 points) Consider the 1-D harmonic oscillator. Do the following without using wave functions. Review the 1-D harmonic oscillator before proceeding.

a) Construct a linear combination of $|0\rangle$ and $|1\rangle$ such that $\langle x\rangle$ is as large as possible.

b) Assume that at $t = 0$ the system is in this state. Determine the state at t and evaluate the expectation value $\langle x\rangle$ as a function of time.

c) Evaluate $\langle(\Delta x)^2\rangle$.

18) (10 points) Suppose a 2×2 matrix X (not necessarily Hermitian, nor unitary) is written as

$$X = a_0 + \boldsymbol{\sigma} \cdot \boldsymbol{a},$$

where all a_i are numbers.

a) How are a_0 and a_k $(k = 1, 2, 3)$ related to $tr(X)$ and $tr(\sigma_k X)$?

b) Determine a_0 and a_k in terms of the matrix elements X_{ij}.