12/2/16

QUANTUM MECHANICS (471)
PROBLEM SET 11 (hand in December 9)

32) (10 points) Consider a system made up of two spin-$\frac{1}{2}$ particles in a spin-singlet state (meaning the total spin $S = 0$). Observer A measures spin components of particle 1 while B does the same for particle 2.

a) Determine the probability for A to obtain the spin up in the y-direction when B makes no measurement. Same for the $\hat{\boldsymbol{n}}$-direction, where this unit vector lies in the xz-plane and makes a 45 degree angle with the z-axis.

b) Observer B obtains the spin of particle 2 to be up in the $\hat{\boldsymbol{n}}$-direction. What can be concluded about the outcome of observer A's measurement if (i) A measures s_{1y}, and (ii) A measures s_{1x}?

33) (10 points) Demonstrate that the spin-correlation function $C_{QM}(\hat{\boldsymbol{n}}^{(1)}, \hat{\boldsymbol{n}}^{(2)})$ is indeed given by $-\cos\Phi$ as indicated in the Phys. Rev. Letter discussed in class. The angle Φ is illustrated in Fig. 3 of that paper.

34) (10 points) Consider a free particle in a state with definite momentum $\boldsymbol{p}$.

a) Write down the corresponding wave function $\psi(\boldsymbol{r}, t)$ and show that $\psi^*(\boldsymbol{r}, -t)$ is the wave function for the state with the momentum direction reversed.

b) Consider the above wave function at $t = 0$. Note that it is a complex wave function and explain why this doesn't violate time-reversal invariance.

35) (10 points) Let $\psi(\boldsymbol{p})$ be the momentum-space wave function for the state $|\psi\rangle$. Construct the momentum-space wave function for the time-reversed state $\mathcal{I}_t\,|\psi\rangle$ in two different ways:

a) By using the decomposition of $|\psi\rangle$ in momentum-space eigenstates.

b) By Fourier-transforming the corresponding wave function of the time-reversed state in coordinate space.

Make sure the results in 34) and 35) agree.