2/23/18

MECHANICS (411)
PROBLEM SET 6 (hand in March 2 at the beginning of class)

20) (10 points) A wedge of mass M moves on a horizontal surface. A block of mass m slides down the wedge (see Fig.7.8 of Taylor's book). Suppose that the wedge has a given motion, $x = \frac{1}{2}at^2$ (a is a fixed constant), **imposed** upon it.

   a) Set up the equations of motion using Newtonian mechanics and determine the constraint force $\boldsymbol{F_{cstr}}$ between the wedge and the block. Work in the given, inertial coordinate system.

   b) Set up the equations of motion using Lagrangian methods, with generalized coordinate q_1. Again, check that the the equation of motion for $\ddot{q}_1$ is the same as in part (a).

21) (10 points) A particle of mass m moves freely over the surface of the sphere with Lagrangian

$$\mathcal{L} = \frac{1}{2}m\left(\frac{\mathrm{d}\,s}{\mathrm{d}\,t}\right)^2 = \frac{1}{2}mR^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right). \qquad (1)$$

Show that both the Lagrangian and the quantity $p_\phi = mR^2\sin^2\theta\dot{\phi}$ are constants of the motion (they are conserved), and give a physical interpretation.

22) (20 points) Determine the degrees of freedom, the kinetic energy, the generalized forces and the equations of motion (you don't need to solve them!) for the following systems in a constant gravitational field:

   a) The double Atwood machine in Fig. 1.

   b) A mass m hanging from a spring with constant k.

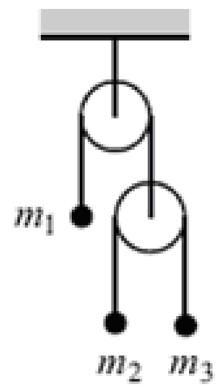

FIG. 1. Double Atwood machine. Neglect the mass of the pulleys and friction.