MECHANICS (411)
PROBLEM SET 5 (hand in February 23 at the beginning of class)

17) (16 points) In many problems in the calculus of variations, one needs to determine the length ds of a short segment of a curve on a surface (see Eq.(6.1) in the book). Determine ds for the following situations:

   a) Curve given by $y = y(x)$ in a plane.
   b) As in a) but $x = x(y)$.
   c) As in a) but $r = r(\phi)$.
   d) As in a) but $\phi = \phi(r)$.
   e) Curve given by $\phi = \phi(z)$ on a cylinder of radius R.
   f) As in e) but $z = z(\phi)$.
   g) Curve given by $\theta = \theta(\phi)$ on a sphere of radius R.
   h) As in g) but $\phi = \phi(\theta)$.

18) (12 points) The shortest path between two points on a *curved surface*, such as the surface of a sphere is called a **geodesic**. To find a geodesic, one finds the curve that makes the path length stationary. In a plane, we saw that $ds = \sqrt{\mathrm{d}\,x^2 + \mathrm{d}\,y^2} = \sqrt{1 + y'^2}dx$.

   a) Use spherical polar coordinates (r, θ, ϕ) to show that the length of the path joining two points on a sphere of radius R is

$$L = R \int_{\theta_1}^{\theta_2} \sqrt{1 + \sin^2\theta \phi'(\theta)^2} \mathrm{d}\,\theta. \tag{1}$$

   b) Prove that the geodesic between two given points on a sphere is a great circle. Note that the integrand above is independent of ϕ, so the Euler-Lagrange equations reduce to $\partial f / \partial \phi' = c$, where c is a constant. You can always choose your z axis to pass through the point 1, which will allow you to show that $c = 0$ in this case. Describe the corresponding geodesics (Note that with this choice of axes, the point 1 is at the North pole.).

19) (12 points) A pendulum bob of mass m is suspended by a string of length ℓ from a point of support. The point of support moves back and forth along the

horizontal x-axis according to the equation $x = a\cos(\omega t)$, with a a constant. Assume that the pendulum's position remains in the $x - z$ plane (z is vertical), and describe the pendulums position by the angle θ that the string makes with the vertical.

a) Set up the Lagrangian, and from it find the equation of motion, using the single generalized coordinate θ.

b) Show that, for small values of θ, the equation reduces to that of a driven harmonic oscillator and find the corresponding steady state motion.

c) Comment on the use of such a device as a seismograph to sense horizontal oscillations of the Earths surface. Given the choice, would it be better to have the pendulums natural frequency much greater or much less than the typical vibrational frequencies of the Earth?