MECHANICS (411)
PROBLEM SET 4 (hand in February 16 at the beginning of class)

13) (10 points) At time $t = 0$, a particle moving in simple harmonic motion has $x = 2\sqrt{3}$, $\dot{x} = 6$, and $\ddot{x} = -18\sqrt{3}$.

   a) Write an expression for the motion of the form $x = Re(\mathcal{A}e^{i\omega t})$, where $\mathcal{A}$ is a complex number.
   b) Write an expression for the motion of the form $x = A\cos(\omega t - \phi)$, where A and ϕ are real.
   c) Represent $\mathcal{A}e^{i\omega t}$ as a rotating vector in the complex plane. Draw a diagram showing its position at $t = 0, t = \pi/18, t = 2\pi/9$, and $t = \pi/3$. What is x at each of these times?

14) (10 points) A particle of mass m undergoes damped oscillations with damping coefficient β and natural frequency ω_0 ($\omega_0 \gg \beta$). At $t = 0$, it starts at $x = A$ with $\dot{x} = 0$.

   a) Calculate the kinetic energy, potential energy, and total energy as functions of time.
   b) What is the average total energy (averaged over one cycle)? [Hint: Since $\omega_0 \gg \beta$, one may assume that $e^{-\beta t}$ stays relatively constant over one cycle. Then the average energy can be found by averaging only those terms that contain $\omega_1 t$. Answer: $E \cong \frac{1}{2}m\omega_0^2 A^2 e^{-2\beta t}$.]

15) (10 points) When a body is suspended from a fixed point by a certain linear spring (i.e. obeying Hooke's law), the natural frequency of its vertical oscillations is found to be ω_1. When a different linear spring is used, the oscillations have angular frequency ω_2.

   a) Find the angular frequency when the two springs are used together in parallel.
   b) Repeat the calculation when they are used in series.
   c) Show that the first of these frequencies is at least twice the second.

16) (10 points) The position of an overdamped harmonic oscillator is given by Eq.(5.40) in the text.

a) Find the constants C_1 and C_2 in terms of the initial position x_0 and velocity v_0.

b) Plot the resulting $x(t)$ for the two cases that $v_0 = 0$ and $x_0 = 0$.

c) Show that for $\beta \to 0$ your solution in a) approaches the solution for undamped motion.