1/26/18

MECHANICS (411)
PROBLEM SET 2 (hand in February 2 at the beginning of class)

5) (10 points) A projectile is fired from the origin of a coordinate system, in the $x-y$ plane (x is the horizontal displacement; y, the vertical) with initial velocity $\boldsymbol{v}_0 = (v_{0x}, v_{0y}, 0)$. Consider only the force of gravity on this projectile.

(a) Find the components of the velocity (v_x and v_y) and the components of the displacement (x and y) as functions of time.

(b) Find, as a function of time, the torque $\boldsymbol{N}$ about the origin which is exerted by gravity on the projectile. Give magnitude and direction.

(c) Find, as a function of time, the angular momentum $\boldsymbol{L}$ of the particle about the origin. Give magnitude and direction.

(d) Check that $\boldsymbol{N} = \frac{d\boldsymbol{L}}{dt}$.

6) (10 points) A particle is projected vertically upward in a constant gravitational field with an initial speed v_0. There is a drag force proportional to the square of the instantaneous speed.

a) Write Newton's second law for the upward motion and integrate it to find the velocity and the height, $v(t)$, $y(t)$. Show that the particle reaches an altitude

$$y_{max} = \frac{v_{ter}^2}{g} \log\left(\sqrt{1 + v_0^2/v_{ter}^2}\right),$$

where $v_{ter} = \sqrt{mg/c}$ is the terminal velocity for the *downward* motion.

b) Show that when the particle returns to the initial position, its speed is $v_0 v_{ter}/\sqrt{v_0^2 + v_{ter}^2}$.

7) (10 points) Consider a rocket taking off vertically from rest in a gravitational field g. Assume that the rocket ejects fuel at a constant rate, $\dot{m} = -\mu$ (where μ is a positive constant), so that $m = m_0 - \mu t$. The exhaust speed of the fuel with respect to the rocket is also constant, v_{ex}.

a) Write the equation of motion, and solve for v as a function of t, using separation of variables.

b) Describe what would happen if the thrust were smaller than the weight of the rocket.

c) Integrate $v(t)$ and show that the rocket's height as a function of t is

$$y(t) = v_{ex}t - \frac{1}{2}gt^2 - \frac{mv_{ex}}{\mu}\log\left(\frac{m_0}{m}\right).$$

8) (10 points) A particle of mass m can slide on a smooth horizontal table. The particle is connected to a light inextensible string which passes through a small smooth hole O in the table, so that the lower end of the string hangs vertically below the table. Initially the string is held fixed with the particle moving with speed v_0 on a circle of radius a.

a) Calculate the tension in the string, and the angular momentum with respect to O.

b) The string is now pulled down from below in such a way that the length between O and the particle is $r(t)$. What is the value of the angular momentum with respect to O as a function of time?

c) Use Kepler's Second Law to estimate qualitatively the velocity of the particle as it gets closer to the hole, O. Why are you allowed to use Kepler's Law? Do you think that it would be possible to pull the particle through the hole?