

MECHANICS (411)

PROBLEM SET 1 (hand in January 26 at the beginning of class)

1) (10 points) Suppose it is known about a vector \mathbf{A} only that its projection on a given unit vector $\hat{\mathbf{e}}$ is $s = \hat{\mathbf{e}} \cdot \mathbf{A}$, and that $\hat{\mathbf{e}} \times \mathbf{A} = \mathbf{B}$. Show that this is sufficient to determine \mathbf{A} by expressing it in terms of $\hat{\mathbf{e}}$, s , and \mathbf{B} . [Hint: Resolve \mathbf{A} into its components along the three perpendicular axes given by $\hat{\mathbf{e}}$, $\mathbf{B}/|\mathbf{B}|$, and $\hat{\mathbf{e}} \times \mathbf{B}/|\mathbf{B}|$.]

2) (10 points) Prove the following identities

a)

$$\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$$

Note the cyclic order of the individual vectors.

b)

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}).$$

3) (10 points) David and Goliath are pushing a frictionless automobile of mass m which is initially at rest. Goliath, strong but out of shape, applies a **forward** push of magnitude $F(t) = F_0 e^{-t/\tau}$, where τ and F_0 are constants. David, weaker but with more endurance, applies a **backward** push of magnitude $F(t) = \frac{1}{2}F_0 e^{-t/2\tau}$. What net displacement do they produce as $t \rightarrow \infty$? [Ans: $x = F_0 \tau^2 / m$]

4) (10 points) A child stands on a scale in an elevator. In each of the following cases, explain whether the scale reads an amount which is less than, equal to, or greater than the child's weight. Discuss from the point of view of someone in an inertial frame, *i.e.*, a frame which is not accelerating.

(a) The elevator speeds up as it leaves floor 1 for floor 3.

(b) The elevator speeds up as it leaves floor 3 for floor 1.

(c) The elevator slows down as it approaches floor 1 from floor 3.

(d) The elevator moves at constant speed going from floor 1 to floor 3.