Physics 217

Homework 9

1. We know that $T = \left[1 + \frac{\sin^2(k_2 a)}{4E/V_0(E/V_0-1)}\right]^{-1}$ and $R = 1 - T$. We must take care to use the correct value of a. In this formula a refers to the entire width of the well thus here $a = 2$nm. From the given energy we get a value of $k_2 = 7.24 \times 10^9 \text{m}^{-1}$. This leads to $R = 0.228$.

2. To do this problem you must solve the S.E. in 3 different regions and match the solutions at the boundaries. Also you can note that $R = 1-T = 1-\left[1+\frac{\sin^2(ka)}{4E/V_0(E/V_0-1)}\right]^{-1}$. Since T is always zero or a positive number R is a number which is always smaller than or equal to 1. We know that $R = |r|^2$ thus meaning that $|r| \leq 1$. If r were a number greater than 1 that would mean that particles were being created at the boundary of the potential which can't be true and we know that a value of r less than 0 has no physical meaning.

3. Here you must again solve the S.E. in different regions and match the solutions at the boundaries. In region I the solution is $\psi_1(x) = Ae^{ik_1 x} + Be^{-ik_1 x}$ where $k_1 = \frac{\sqrt{2mE}}{\hbar}$. In region II the solution is $\psi_2(x) = Ce^{k_2 x}$ (there is only one term because we have no waves traveling to the left in region 2). As in class we can set $A = 1, B = r, c = \tau$. Through use of the boundary conditions ($\psi_1(x = 0) = \psi_2(x = 0)$, $\frac{d\psi_1(x)}{dx}|_{x=0} = \frac{d\psi_2(x)}{dx}|_{x=0}$) we see that $r = \frac{(k_1+ik_2)^2}{k_1^2+k_2^2}$. This gives $|r|^2 = 1$ which makes since because we expect that all of the particles incident on this potential would get reflected.

4. This problem follows the same process of solving the S.E. in 3 regions and matching solutions at the boundary. When you do this you get to $T = \left[1 + \frac{\sin^2 k_2 a}{4E/V_0(E/V_0-1)}\right]^{-1}$. For perfect transition we must have $T = 1$ which happens when $\sin^2(k_2 a) = 0$. This gives the constraint on energy necessary for perfect transmission $E = \frac{n^2\pi^2\hbar^2}{2ma^2}$, precisely the energies of the infinite square well.