

Physics 217

Homework 8

1. We know $\Delta y = 1 \times 10^{-5} \text{m}$ when combined with the uncertainty relation you get $\Delta p_y = 5.25 \times 10^{-30} \text{kg} \cdot \text{m/s}$. The uncertainty in the y-momentum is related to the spread by the equation $\Delta p_y \approx p_y = p \sin \theta$, thus we obtain $\sin \theta = \frac{\Delta p_y}{p} = \frac{\Delta p_y}{mv}$. We also know that the spread of the image is given by $\tan \theta = \frac{\text{spread}}{L}$ (L is the distance to the screen) thus we get $\text{spread} = L \tan \theta$. For small angles $\tan \theta \approx \sin \theta \approx \theta$ so we get $\text{spread} \approx L\theta \approx \frac{L\Delta p_y}{mv}$.
2. The point of this problem is to illustrate that if a particle's position is well known its momentum is not. Initially we know the particle is located inside the nucleus meaning $\Delta r = 7.8 \times 10^{-15} \text{m}$. Using the uncertainty relation we get $\Delta p \geq \frac{\hbar}{2\Delta r}$. When you put the numbers in you see that Δp is a large number. This may be better illustrated by looking at the uncertainty in the velocity ($\Delta v = \frac{\Delta p}{m}$). You notice that $\Delta v = 7.4 \times 10^9$; the uncertainty in the velocity is larger than the highest possible value of the velocity. This illustrates the fact that we don't know anything about the momentum. Anyway at this point you need to find the possible energy of the electron which is given by $\Delta E = \frac{(\Delta p)^2}{2m} = 2.49 \times 10^{-11} \text{J} = 1.56 \times 10^8 \text{eV} = 156 \text{MeV}$. Here you notice that this energy value is much greater than the rest mass of the electron ($m_e = .511 \text{MeV}$) so if we were doing this problem entirely correctly we would have used the relativistic energy relation $E^2 = p^2 c^2 + m_e^2 c^4$.
3. (a) We take the best case scenario of the uncertainty principle $\Delta x \Delta p_x = \hbar/2$. $E = \frac{1}{2m} \left(\frac{\hbar}{2x} \right)^2 + \frac{Cx^2}{2} = \frac{\hbar^2}{32\pi^2 m x^2} + \frac{Cx^2}{2}$.
(b) We now want to minimize the energy with respect to x so we use $\frac{dE}{dx} = 0$ to find

$x_{min} = \left[\frac{h^2}{16\pi^2 mc} \right]^{1/4}$. Putting this value into the energy equation we obtain the desired result $E_{min} = \frac{h\sqrt{c}}{4\pi\sqrt{m}} = \frac{h\nu}{2}$ where $\nu = \frac{1}{2\pi} \sqrt{C/m}$.

4. In order to know which slit the electron goes through we set the condition $\Delta y \leq a/2$. In order for diffraction to occur we impose the condition that $\Delta\theta < (n + \frac{1}{2})\frac{\lambda}{a}$, that is we don't want the peaks to overlap. Now let us deal with the situation where $n = 1$ and relate the uncertainty in angle to uncertainty in momentum. $\Delta p \approx p \sin\theta$ and $\sin\theta \approx \theta$. So we get $\Delta\theta \approx \frac{\Delta p}{p} < \frac{\lambda}{2a}$ which reduces to $\Delta p < \frac{h}{2a}$. Now multiplying our two results we get $\Delta p \Delta y < \frac{h}{4}$ which violates the uncertainty relation. So you see we can not know which slit the electron when through and still have a diffraction pattern.