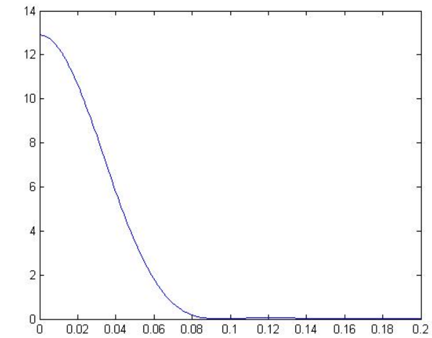
Physics 217

Homework 7

1. $\tilde{\psi}(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ikx}\psi(x,0)dx = \frac{1}{\sqrt{a\pi}}\int_{-a/2}^{a/2} e^{-ikx}\cos(\frac{\pi x}{a})dx$ after a lot of algebra you arrive at a nice answer of $\tilde{\psi}(k) = 2\sqrt{a\pi}\frac{\cos(ka/2)}{\pi^2-k^2a^2}$. Using this answer you get $|\tilde{\psi}(k)|^2 = 4a\pi\frac{\cos^2(ka/2)}{(\pi^2-k^2a^2)^2}$. Figure 1 shows a plot of $|\tilde{\psi}(k)|^2$. Notice that the first zero occurs at around 0.094 which will be relevant in question 2.

2. $|\tilde{\psi}(k)|^2 = 0$ when $\cos(ka/2) = 0$ or in other words when $ka/2 = (n + 1/2)\pi$. However for $n = 0$ the denominator of $|\tilde{\psi}(k)|^2$ blows up meaning that we must go to $n = 1$ to find the first zero of $|\tilde{\psi}(k)|^2$. Thus we see that the first zero of $|\tilde{\psi}(k)|^2$ occurs at $k = \frac{3\pi}{a}$. Here you notice that this number is consistent with the location of the first zero in the graph which you produced for question 1. You see that k is inversely proportional to a thus as a increases k_0 (the location of the first zero of $|\tilde{\psi}(k)|^2$) decreases. This means as the size of the well increases the wave function becomes more sharply spiked in k-space. This should make sense because as the size of the well increases you are losing information about the location which means you should be gaining information about the momentum and this is precisely what is happening. Now multiplying our value of Δk by Δx we get $\Delta x\Delta k = 3\pi > 1/2$ thus the uncertainty relation is satisfied.

3. Figures 2-5 show the plots which you were asked to produce. As was noticed before the wave numbers below k_0 contribute the most to the wave function thus the significance of k values near 0.05. If the lower wave numbers are left out the hump in the middle of the graph does not appear, the wave function looks more like a pure sine wave. If the higher wave numbers are left out the wave function has bumps outside of the desired hump in the middle.



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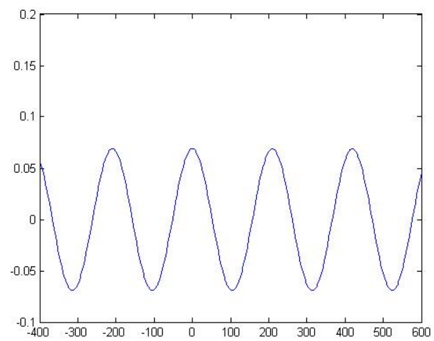

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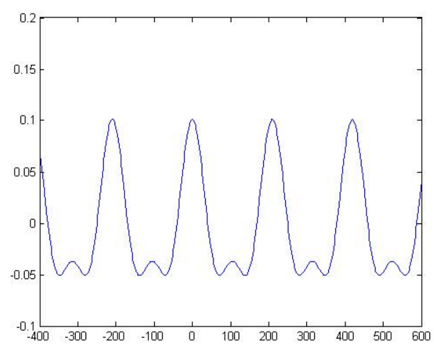

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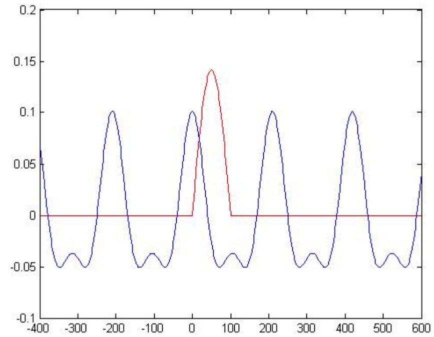
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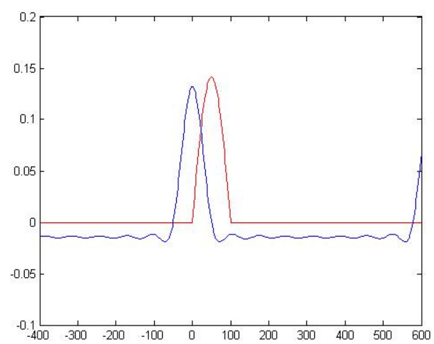
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