Physics 217

Homework 6

1. See Figure 1. $\langle\psi_1|\hat{x}|\psi_1\rangle = \frac{2}{a}\int_{-a/2}^{a/2} x\cos^2(\frac{\pi x}{a})dx = 0$ because you are integrating an odd function over even limits, likewise $\langle\psi_2|\hat{x}|\psi_2\rangle = 0$.

2. (a) From Figure 2 you can see that you are most likely to find the particle in the x interval [0,50].

   (b) Doing the relevant integration you see that the wavefunction is normalized. Figure 3 shows the expectation value of x. This makes sense because the largest amount of the wavefunction was located in this interval.

3. (a) $\omega_1 = \frac{E_1}{\hbar} = \frac{\pi^2\hbar}{2ma^2}$ and likewise $\omega_2 = \frac{2\pi^2\hbar}{ma^2}$. ψ_1 evolves as $exp[-\frac{iE_1 t}{\hbar}] = exp[-i\omega_1 t]$ and ψ_2 evolves as $exp[-i\omega_2 t]$.

   (b) Figure 4 you can see that the $wavefn(x,t)$ returns the same thing as $wavefn\_t0(x)$.

   (c) In the animated plot the probability density oscillates between the positive and negative x regions with a period of ~ 42.

   (d) Figure 5 shows the plot of $expected\_x(t)$.

4. $\langle\Psi|x|\Psi\rangle = \int_{-a/2}^{a/2}\Psi^* x\Psi dx$. Using the result of question one we know that the terms which look like $\psi_1^* x\psi_1$ and $\psi_2^* x\psi_2$ don't contribute anything. Thus we are left with $\frac{1}{a}\left(e^{\frac{i\Delta Et}{\hbar}} + e^{-\frac{i\Delta Et}{\hbar}}\right)\int_{-a/2}^{a/2} x\cos(\frac{\pi x}{a})\sin(\frac{2\pi x}{a})dx$. Upon substitution and making use of the given integral and Euler's equation we get $\langle x\rangle = \frac{16a}{9\pi^2}\cos(\frac{\Delta Et}{\hbar})$. Plugging the appropriate values we see that the amplitude and period match the earlier value ($Amp \sim 18.01$ and $T \sim 42$).

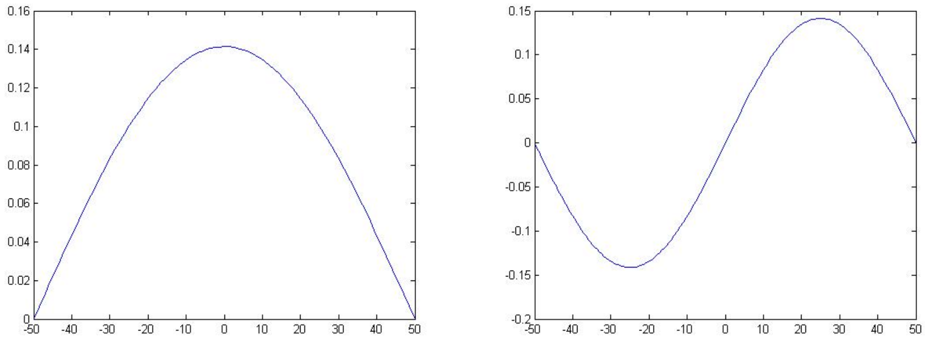

Figure 1: ψ_1 and ψ_2

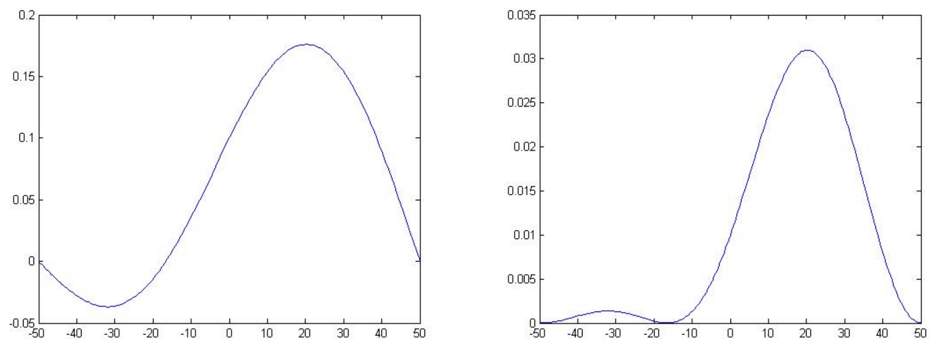

Figure 2: $\psi(x)$ and $|\psi(x)|^2$

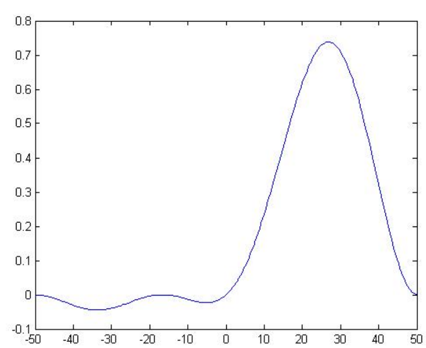

Figure 3: The expectation value of x.

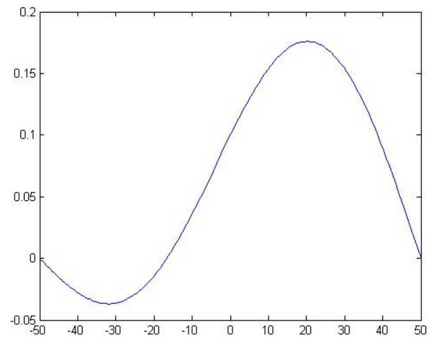

Figure 4: Initial wavefunction using new .m file.

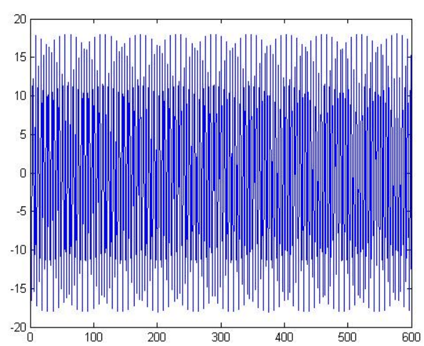

Figure 5: plot of $expected\_x(t)$