Physics 217

Homework 4

1. (a) To normalize the wavefunction set $1 = \int_{-\infty}^{\infty} \psi^\star \cdot \psi dx$. Making use of integral given you get $C = \sqrt{\frac{2}{a\pi}}$.

   (b) bump(1,2.5) = 0.4350

   (c) When you integrate a properly normalized wavefunction over all space you should get the value of 1. Using the MatLab formula given you should get exactly this.

   (d) The plot on the next page is for the 3 different values of a requested. As the value of a increases the wavefunction spreads out.

2. (a) $\langle \hat{x} \rangle = \int_{-\infty}^{\infty} \psi^\star \cdot x \cdot \psi dx$. This is the integral of an odd function over even limits thus it is equal to zero $\langle \hat{x} \rangle = 0$.

   (b) $\langle x^2 \rangle = \int_{-\infty}^{\infty} \psi^\star x^2 \psi dx$. Making use of the integral given you get $\langle x^2 \rangle = a^2$. When I plugged the given formula into MatLab I got $\langle x^2 \rangle = 15.9999$ for a value of $a = 4$ which checks out.

3. (a) $\hat{x}\psi(x) = \hat{x}\cos(kx) = x\cos(kx)$, this operator produces a new function $(x\cos(kx))$ when it acts on $\psi(x)$ so $\psi(x)$ is not an eigenfunction of the position operator.

   (b) $\hat{p}\psi(x) = \hat{p}\cos(kx) = \hat{p}(2)[e^{ikx} + e^{-ikx}]$. $\psi(x) = \cos(kx)$ is a linear combination of two eigenstates of momentum, one with eignevalue k and the other with eigenvalue $-k$. Thus when $\hat{H}$ acts on $\psi(x)$ both states yield the same energy proving that $\psi(x)$ is an eigenfunction.

   (c) $\hat{H}\psi(x) = \left[\frac{\hbar^2 k^2}{2m}\right]\psi(x)$. $\psi(x)$ is an eigenvalue of the Hamiltonian with eigenvalue of $\frac{\hbar^2 k^2}{2m}$. The wavefunction will evolve in time according to the equation $\Psi(x,t) = \cos(kx)e^{-iEt/\hbar}$.

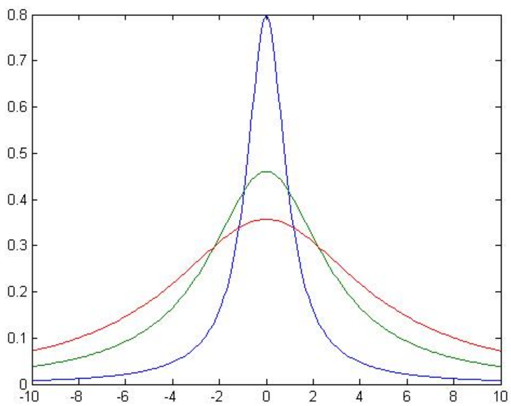

Figure 1: As a increase the width of the wavefunction increases.