Physics 217

Homework 2

Before I start I would like to say that these solutions are written in the way which I solved the problems. There are certainly other ways of solving these problems (particularly the algebra) which may be quicker, thus more elegant. If upon reading these solutions you are still unclear on something please do not hesitate to ask.

1. The solution to this problem lies in conservation of momentum and trig identities. The initial momentum is $\vec{p} = \frac{h}{\lambda}\hat{x}$ and the momentum after scattering is $\vec{p} = \left[\frac{h}{\lambda'}\cos\theta + m_0 v\cos\phi\right]\hat{x} + \left[\frac{h}{\lambda'}\sin\theta - m_0 v\sin\phi\right]\hat{y}$ where v is the velocity of the electron after scattering. The v dependance can be eliminated by equating the initial and final momentum in the $\hat{y}$. The next step is to equate the $\hat{x}$ component of the initial and final momentum. After some algebraic manipulation you get to the equation $\tan\phi = \frac{\sin\theta}{\frac{\lambda'}{\lambda} - \cos\theta}$. At this point make use of the Compton Eq. in order to eliminate the λ' dependence. Next you arrive at $\left(1 + \frac{h\nu}{m_0 c^2}\right)\tan\phi = \frac{\sin\theta}{1-\cos\theta}$. Now using two trig identities gives the final answer. The identities are $\sin 2x = 2\sin x \cdot \cos x$ and $\sin^2(x) = \frac{1-\cos(2x)}{2}$.

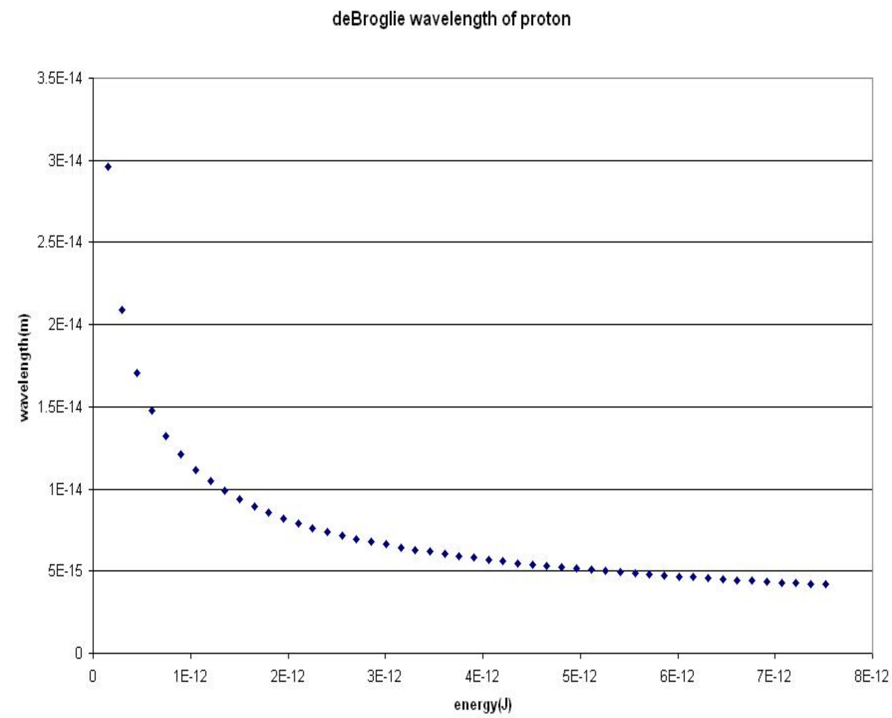
2. In this problem the photon scatters at an angle of $\theta = \pi/2$ meaning that after the collision all of the $\hat{x}$ momentum is possessed by the electron. Like in the previous problem you can eliminate the velocity variable by conserving momentum in the $\hat{x}$ direction (you get $v = \frac{h}{m_0 \lambda_0 \cos\phi}$). Next you conserve momentum in the $\hat{y}$ direction and use the Compton Eq. which will produce $\tan\phi = \frac{\lambda_0}{\lambda_0 + \frac{h}{m_0 c}}$. Now you must find the components of the electron's momentum after the collision. The $\hat{x}$ component is just equal to the initial momentum of the photon $p_{x,electron} = \frac{h}{\lambda_0} = 5.344 \times 10^{-23}\frac{kg \cdot m}{s}$. The

$\hat{y}$ component of the electron's momentum is $p_{y,electron} = -m_0 v \sin\phi = -\frac{h}{\lambda_0 + \frac{h}{m_0 c}} = -4.48 \times 10^{-23} \frac{kg \cdot m}{s}$. The angle of the recoil electron is $\phi \approx 40°$.

3. The Compton Eq. reads $\Delta\lambda = \frac{h}{m_p c}(1 - \cos\theta)$. This equation is maximized when $\cos\ \theta = -1$ which happens at $\theta = \pi\ rad = 180°$. Thus $\Delta\lambda_{max} = \frac{2h}{m_p c} = 2.645 \times 10^{-15} m$.

4. The de Broglie wavelength of a particle is given by the equation $\lambda = \frac{h}{p} = \frac{h}{(2mK)^{1/2}}$. On the next page are computer generated plots of the de Broglie wavelengths (you are not required to produce a computer generated image).

As a general comment I would like to say that many derivations can be done following the process I used in problem 1. Start with something you know is true, conservation of momentum in this case. Next you identify which variables you want to keep (in this case ν or equivalently λ) and which variables you want to eliminate (in this case λ'). Then you find equations which can eliminate the desired variables (Compton Eq.). Then just do a little algebra (trig identities) and you are there. Admittedly many derivations are tougher than this one, but this process can be useful even in those situations.



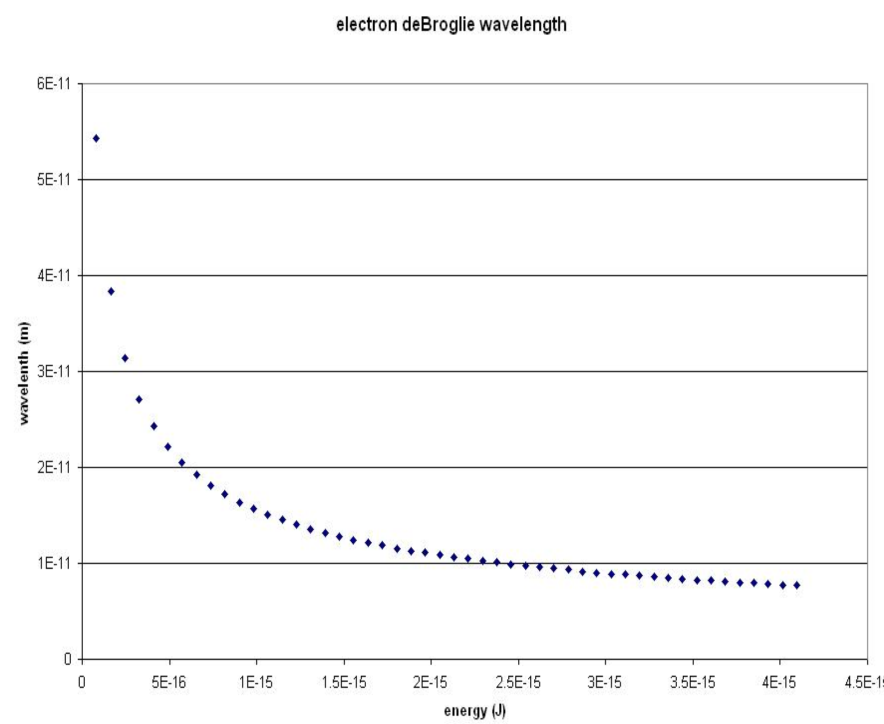


Figure 1: Plots of the deBroglie wavelenth as a function of kinetic energy.