Physics 217

Homework 12

1. (a) The probability of finding a particle between r_1 and r_2 is given by $P = \int_{r_1}^{r_2} |R(r)|^2 r^2 dr$.

   (b) $R_{10} = 2\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$. When the electron approaches the nucleus you can approximate the radial wave function as $R \sim r^l$. So for $r \leq r_p$ you get $R \approx 1$. Using this you get the probability of finding the electron inside the proton of being about 9×10^{-15}.

   (c) $R_{21} = \frac{1}{\sqrt{24}} a_0^{-3/2} \frac{r}{a} e^{-r/2a}$. Going through the same process you arrive at a probability of $\approx 2 \times 10^{-}26$ of finding the electron inside the nucleus. You see that there is a much smaller chance of finding the electron in $n = 2$, $l = 1$ state inside of the nucleus, which makes since because it has a greater energy.

2. (a) There are 3 allowed values of l for $n = 3$ which are $l = 2, 1, 0$. There are 2 allowed values of l for $n = 2$ which are $l = 1, 0$.

   (b) The $n = 3, l = 2$ level splits into 5 levels while the $n = 2, l = 1$ level splits into 3 levels.

   (c) There are 15 possible transitions.

   (d) There are 9 possible transitions, however there are only 3 different light frequencies.

   (e) The energy of each level is given by $E = E_n + \mu_B B_z m_l$ so the frequencies are dependent on changes of n and changes of m_l. By setting the energy differences of the levels equal to $h\nu$ you obtain the three desired frequencies which are $4.5694 \times 10^{14}, 4.56954 \times 10^{14}, 4.56968 \times 10^{14}$Hz.

3. $\vec{\mu} = -\frac{\mu_B}{\hbar}(\vec{L} + 2\vec{S})$. By choosing the Stern-Gerlach apparatus to be in the z direction the quantity in the brackets reduces to $(L + 2S_z)$. Manipulating this you arrive at a value of $l = 0$. The force acting on an electron going through a Stern-Gerlach apparatus is given by $F_z = -\frac{\partial B_z}{\partial z}\mu_B g_s m_s$. In order for this force to be equal to 100

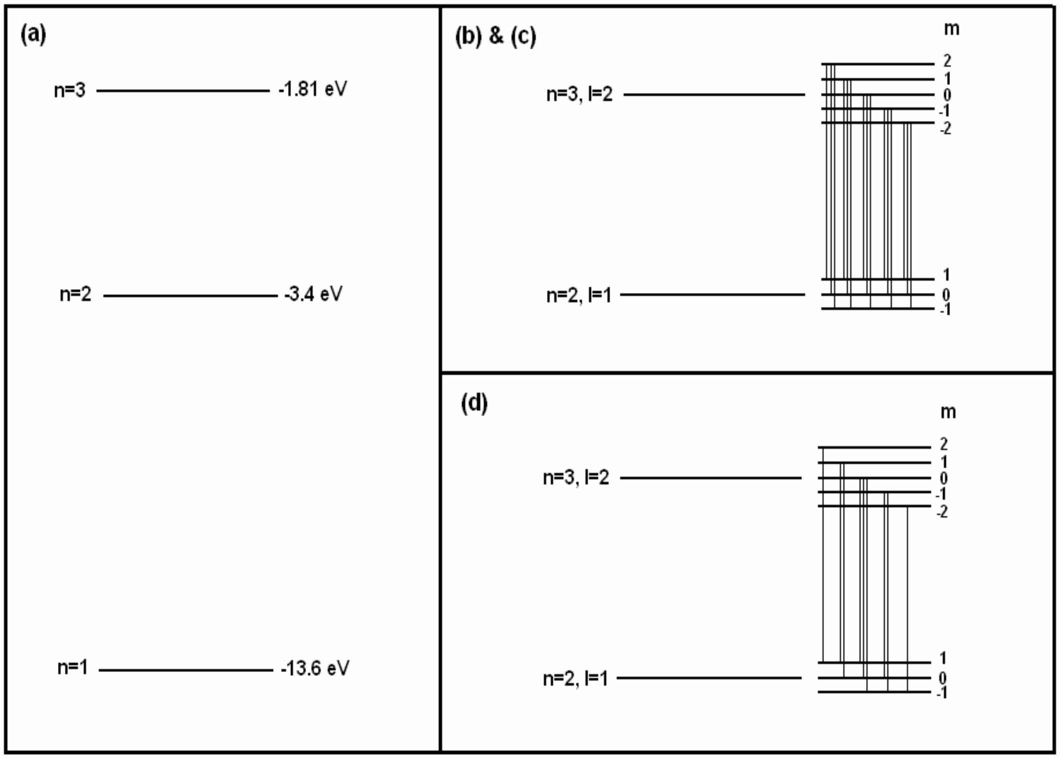

Figure 1: Energy level diagrams for question 2.

times the weight of the electron the value of the field gradient must be $\frac{\partial B_z}{\partial z} = 0.18\text{T/m}$. The deflection of an atom going through this apparatus is given by $Z = \frac{1}{2}a_z t^2$ where $a_z = F_z/m$ and t the amount of time that the atom spends in the magnetic field. The minimum resolution dictates the energy necessary so based upon what is chosen as the minimum resolution you get different values of K. The exact number is obtained from the equation $K = \frac{\partial B_z/\partial z \cdot \mu_B \cdot x^2}{2Z}$ where x is the length of the apparatus.