Physics 217

Homework 11

1. $\vec{L_1} = \vec{r_1} \times \vec{p_1} = 1.2 \times 10^{-5} kg \cdot m^2/s$. This is the angular momentum due to one ball, the total angular momentum is 2 times this number. We also know that $L = \sqrt[l(l+1)]{\hbar}$. Equating these two we get $l = 2.37 \times 10^{29}$. There are $2l + 1 = 4.74 \times 10^{29}$ possible values of m_l.

2. $Y_{11} = -\frac{1}{2}\sqrt{\frac{3}{2\pi}} \sin\theta e^{i\phi}$. Plugging this into the two given equations and having fun with some algebra you see that the equations are satisfied.

3. (a) The wave function of the ground state is given by $\psi = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$ and the potential is given by $V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$. Using these two quantities you find that the expectation value of the potential energy is $\bar{V} = -27.2$eV.
   (b) $E_1 = -\frac{13.6}{1^2} = -13.6\text{eV} = \frac{\bar{V}}{2}$.
   (c) $\bar{K} = E - \bar{V} = 13.6\text{eV} = -\frac{\bar{V}}{2}$.

4. The desired quantity looks like $\frac{1}{9}(\psi^*_{300}\psi_{300} + \psi^*_{31-1}\psi_{31-1} + \psi^*_{311}\psi_{311} + \psi^*_{310}\psi_{310} + \psi^*_{32-2}\psi_{32-2} + \psi^*_{32-1}\psi_{32-1} + \psi^*_{320}\psi_{320} + \psi^*_{321}\psi_{321} + \psi^*_{322}\psi_{322})$. After a lot more exciting algebra you arrive at something like $\frac{1}{9}\left(\frac{1}{81\pi^2}\right)\left(\frac{z}{a_0}\right)^3 e^{-2Zr/3a_0}\left(\frac{1}{2}(27 - 18Zr/a_0 + 2Z^2r^2/a_0)^2 + 2(6 - Zr/a_0)^2(Zr/a_0)^2 + \frac{2}{3}(Z^2r^2/a_0)\right)$ which you notice is independent of any angles implying spherical symmetry.