# Physics 217

# Homework 10

1. $E = \frac{p^2}{2m} = \frac{\hbar^2 k_1^2}{2m}$ or $k_1 = \frac{n\pi}{a} \rightarrow n^2 = \frac{a^2 k_1^2}{\pi^2} \rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{\hbar^2 k_1^2}{2m}$

2. By combining these two equations we get $k_1 \tan(k_1 a/2) = \sqrt{\frac{2mV_0}{\hbar^2} - k_1^2}$. Next make the substitution $\theta = \frac{k_1 a}{2}$. From this we arrive at $\theta \tan(\theta) = \sqrt{\frac{ma^2V_0}{2\hbar^2} - \theta^2}$ meaning that $\Upsilon = \sqrt{\frac{ma^2V_0}{2\hbar^2}}$. From the graphs below you can see that there are 2 bound states for $\Upsilon = 6$ and one each for $\Upsilon = 1, 2$.

3. By following the same process as in class you arrive $k_2 = -k_1 \cot(k_1 a/2)$ and $k_2^2 = \frac{2mV_0}{\hbar^2} - k_1^2$ which leads to $\theta \cot\theta = -\sqrt{\Upsilon^2 - \theta^2}$. From the graphs below you see that for $\Upsilon = 1$ you get no bound states. For $\Upsilon = 2$ you get one bound state and for $\Upsilon = 6$ you get 2 bound states.

4. $\Upsilon = \sqrt{\frac{ma^2V_0}{2\hbar^2}}$. In the limit that $\Upsilon \rightarrow 0$ the depth of the potential well is going to 0. As $\Upsilon$ decreases there are fewer and fewer bound states which makes sense because the depth of the well is decreasing. It is interesting to note that below $\Upsilon = \pi/2$ there is only one bound state, but there will always be at least one bound state.

5. For even states $\theta \tan\theta = \sqrt{\Upsilon^2 - \theta^2}$ which approaches $\theta \tan\theta = \Upsilon$ for large values of $\Upsilon$. Rearranging this we get $\frac{1}{\Upsilon}\theta = \cot\theta$. This again is a transcendental equation which is solved graphically. The $\cot\theta$ function crosses the $\theta$ axis at $\pi/2, 3\pi/2, ... = (j + 1/2)\pi$ where $j = 0, 1, 2, ....$ The left hand side of the equation is that of a straight line with a very small slope ($\Upsilon$ is very large) meaning that it is a line which runs almost parallel to the $\theta$ axis. Thus the solutions to this equation occur at $\theta \approx (j + 1/2)\pi$. From before we know that $\theta = k_1 a/2$ thus when we plug $k_1$ into the energy equation we get $E = \frac{\hbar^2\pi^2}{2ma^2}(2j + 1)^2$ where $2j + 1 = n$ for odd values of $n$ and the odd values of $n$ correspond to the even eigenstates of the infinite square well.

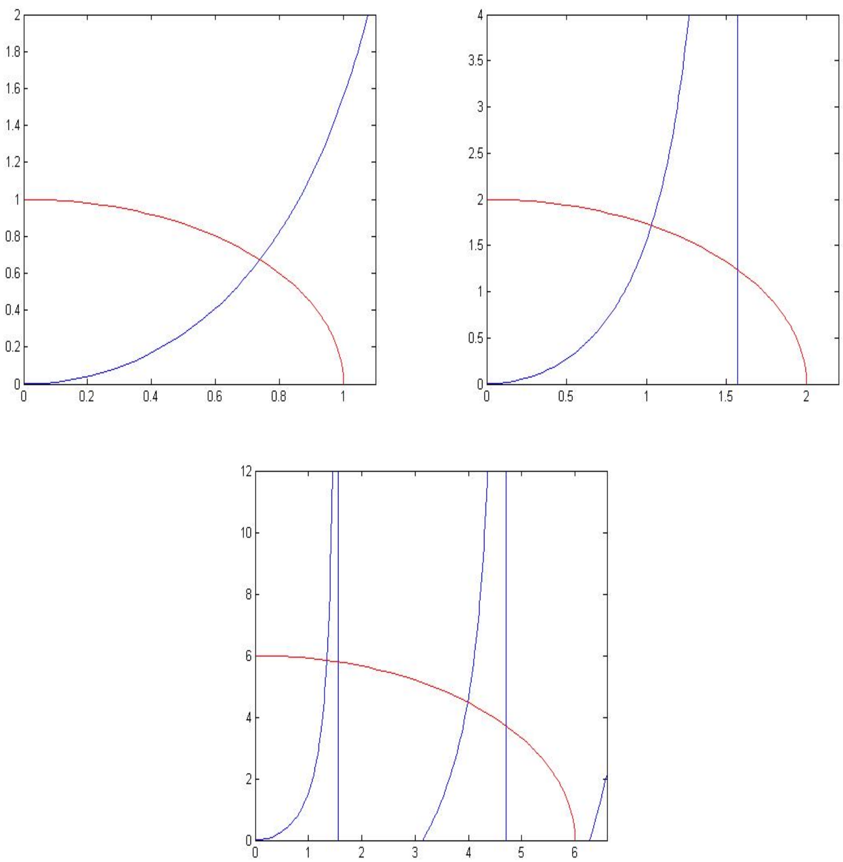


Figure 1: Graphs for problem 2: $\theta \tan\theta = \sqrt{\Upsilon^2 - \theta^2}$ for $\Upsilon = 1, 2, 6$.

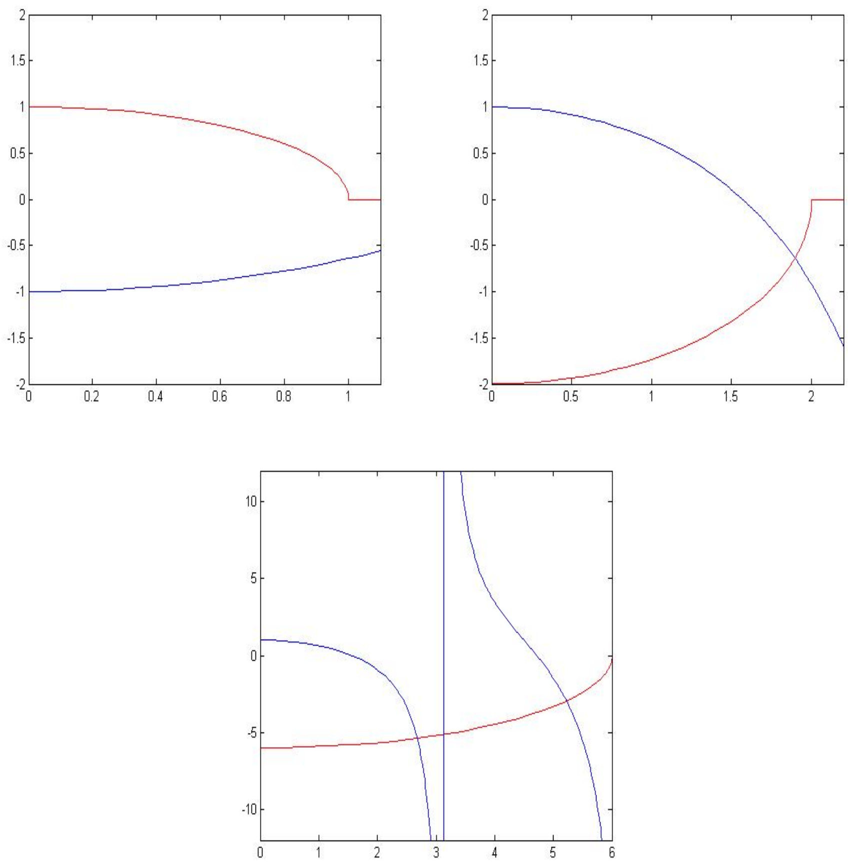


Figure 2: Graphs for problem 3: $\theta \cot \theta = \sqrt{\Upsilon^2 - \theta^2}$ for $\Upsilon = 1, 2, 6$.