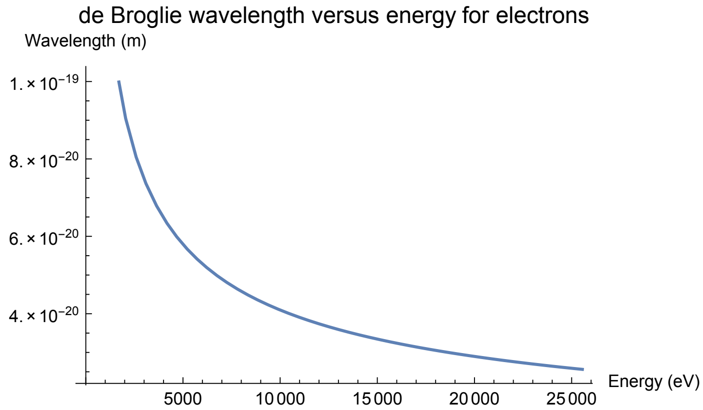
Problem set 3

1. For electrons and protons, kinetic energy $E = \frac{m v^2}{2} = \frac{p^2}{2 m}$,

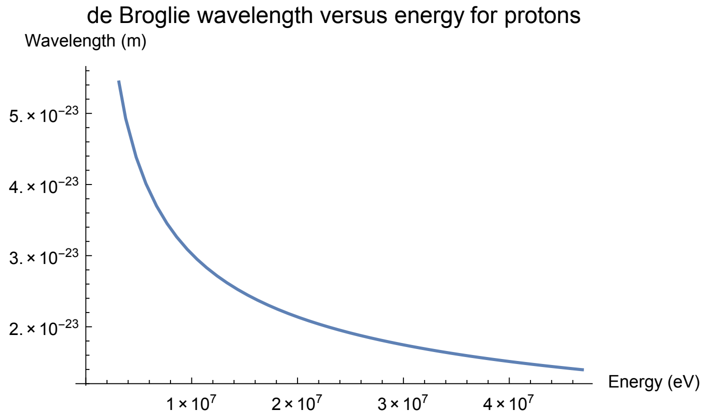
and de Broglie wavelength $\lambda = \frac{h}{p}$. Therefore we get $\lambda = \frac{h}{\sqrt{2 m E}}$.

In[1]:=
```
h = 6.626 × 10^-34 / (1.6 × 10^-19);      (* in units of eV.s *)
me = 0.511 × 10^6;
(* in units of eV/c^2. hence the rest energy of electrons is me*c^2=
 0.511*10^6 eV *)
mp = 938 × 10^6;       (* in units of eV/c^2. hence the
  rest energy of protons is mp*c^2 = 938*10^6 eV *)
lambda[e_, m_] := h / Sqrt[2 m e];
Show[Plot[lambda[e, me], {e, 0, me * 0.05}],
 AxesLabel → {"Energy (eV)", "Wavelength (m)"},
 PlotLabel → "de Broglie wavelength versus energy for electrons"]
Plot[lambda[e, mp], {e, 0, mp * 0.05},
 AxesLabel → {"Energy (eV)", "Wavelength (m)"},
 PlotLabel → "de Broglie wavelength versus energy for protons"]
```

Out[5]=



Out[6]=



2. (a) Convert the unit of Planck ' s constant into the SI base units.

$J\ s = (m / s^2)\ kg\ m\ s = m^2\ kg\ s^{-1}$.

Since angular momentum is $r \times p$, the SI base units for angular momentum is $m\ kg\ (m / s) = m^2\ kg\ s^{-1}$.

Hence it is obvious that Planck ' s constant has the dimension of angular momentum.

(b) The average radius of a ground state hydrogen atom (which is also the distance of the electron and the proton for the ground state hydrogen atom) is r = 0.0529 nm.

The gravitational attraction between the electron and the proton is :

In[7]:=
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r = 0.0529;
g = 6.674 × 10^-11;      (* gravitational constant, in units of m^3 kg^-1 s^-2 *)
me = 9.109 × 10^-31;     (* electron mass, in units of kg *)
mp = 1.673 × 10^-27;     (* proton mass, in units of kg *)
fgrav = g me mp / (r 10^-9)^2              (* gravitational force, in units of N *)
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Out[11]= 3.63447×10^{-47}

The Coulomb attraction between the electron and the proton is :

In[12]:=
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k = 8.99 × 10^9;      (* Coulomb constant, in units of N m^2 C^-2 *)
fcou = k (1.6 × 10^-19)^2 / (r 10^-9)^2         (* Coulomb force, in units of N *)
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Out[13]= 8.2241×10^{-8}

Since gravitational force is 10^{39} order smaller than Coulomb force, we can ignore the gravitational force.

3. (a) According to Bohr ' s model, the quantum number of the ground state of the hydrogen atom is n = 1.

(b) The orbital radius is $r = a_0 n^2 = 0.0529$ nm;

(c) The angular momentum is $L = n \hbar = 1.055 * 10^{-34}$ J s;

(d) The linear moment is $m v = L / r = 1.99 \times 10^{-24}$ kg m s^{-1};

(e) The angular velocity is $\omega = v / r = 4.14 \times 10^{16}\ s^{-1}$;

(f) The linear speed is $v = L / (m r) = 2.19 \times 10^6$ m s^{-1};

(g) The force on the electron is $m \frac{v^2}{r} = 8.26 \times 10^{-8}$ N (which is approximately the same with the result from 2 (b) as calculated from the Coulomb force);

(h) The acceleration of the electron is $a = \frac{v^2}{r} = 9.07 \times 10^{22}$ m s^{-2};

(i) The kinetic energy is $E_k = m \frac{v^2}{2} = 2.18 \times 10^{-18}$ J;

(j) The potential energy is $E_P = -\frac{1}{4 \pi \epsilon_0} \frac{e^2}{r} = -4.35 * 10^{-18}$ J;

(k) The total energy is $E_k + E_P = -2.17 * 10^{-18}$ J (the absolute value of potential energy is almost twice the kinetic energy. and this result is the same as $E = -13.6\ eV \frac{1}{n^2}$).

Since $r = a_0 n^2$ and $E = -13.6\ \text{eV}\ \frac{1}{n^2}$,
the orbital radius and the total energy will both increase with increasing quantum number (the total energy is negative, hence the smaller the absolute value, the larger the total energy).