Physics 217
Problem Set 4
Due: Friday, September 28th, 2018

In this homework you will use Mathematica to do some visualization and numerical integrations.

1. (20 points) Consider a particle in one dimension. Its wavefunction at $t = 0$ is

$$\psi(x) = \frac{C}{1 + (x/a)^2}$$

The constant a (which is a length) gives the width of the wavefunction, i.e. it tells us how tightly localized the particle is around $x = 0$. The normalization constant C is a function of a.

(a) Calculate $C(a)$ by normalizing the wavefunction. What is the dimension of C? Do the calculation by hand.
You may use the fact that

$$\int_{-\infty}^{\infty} \frac{1}{(1+u^2)^2}\, du = \frac{\pi}{2} \ .$$

(b) Create a Mathematica function called "`psi`" that gives the value of the normalized wavefunction as a function of position:

```
psi(x,a) = C/(1+(x/a)^2);
```

where "`C`" is to be replaced with the expression that you found previously. Make sure you define the function using Wolfram language instead of simply typing the above expression into Mathematica. If you type `psi(1,2.5)` into Mathematica it will return the value at $x = 1$ of the wavefunction with width $a = 2.5$. *What is this value?*

(c) Use Mathematica to check that our function `psi(x,a)` is properly normalized (i.e. that we got the expression for `C` right). We will first check it for $a = 2.5$. We numerically integrate the probability density function from the wavefunction over all x (there is a built in function named `Infinity` which you can take advantage of, and you should find out the function that gives the complex conjugate of a number by yourself (even though here we are solely dealing with real numbers, we should always strictly stick to the definitions)):

$$\int_{-\infty}^{\infty} \psi^*(x)\psi(x)\, dx \ .$$

What should the answer be? Is that what you got? (If not, you either made a mistake in part (a), or in your Mathematica function: find the error and correct it.) Do the numerical integration again for several different values of a, and make sure that you always get the expected answer.

(d) Make plots of the normalized wave function for $a = 1, 3, 5$. Do several plots on the same set of axes and declare the legend of your plots. *How does the value of "a" affect the shape of the wavefunction?*

2. (20 points)

(a) Calculate the expectation value of position in the correctly-normalized wavefunction of question 1, for arbitrary width a:

$$\langle \hat{x} \rangle = \int \psi^*(x) x \psi(x) \, dx \ .$$

Now check your answer using Mathematica. Try different widths. Do the answers agree with your analytic calculation?

(b) Calculate the expectation value of the position-squared in the wavefunction of question 1, for arbitrary width a. You may use the fact that

$$\int_{-\infty}^{\infty} \frac{u^2}{(1+u^2)^2} \, du = \frac{\pi}{2} \ .$$

Now check your answer using Mathematica, for various widths a.

3. (20 points) Consider a free particle in one dimension. At $t = 0$ its wavefunction is

$$\psi(x) = \cos(kx)$$

(a) Is ψ an eigenfunction of the position operator? If so, what is its eigenvalue?

(b) Is ψ an eigenfunction of the momentum operator? If so, what is its eigenvalue?

(c) Is ψ an eigenfunction of the Hamiltonian? If so, what is its eigenvalue? How will it evolve over time?