Physics 217 Problem Set 12 Due: Friday, Dec 7th, 2018

- 1. (10 points) Problem 28 from Chapter 8 of the Harris book.
- 2. (10 points) Using the completeness relation

$$1 = \sum_{i} |a_i\rangle \langle a_i| \tag{1}$$

demonstrate the operator identity for the operators A and B

$$(AB)^{\dagger} = B^{\dagger}A^{\dagger}. \tag{2}$$

3. (20 points) Use the expressions for the $|S_x; +\rangle$ and $|S_x; -\rangle$ as well as the $|S_y; +\rangle$ and $|S_y; -\rangle$ in terms of the spin states $|S_z; +\rangle$ and $|S_z; -\rangle$ to demonstrate that Eq. (1) is also fulfilled for the spin-x and spin-y states. Use these same expression to show that

•
$$S_x = \frac{\hbar}{2} \left(\left| + \right\rangle \left\langle - \right| + \left| - \right\rangle \left\langle + \right| \right)$$

• $S_y = \frac{i\hbar}{2} \left(- \left| + \right\rangle \left\langle - \right| + \left| - \right\rangle \left\langle + \right| \right)$.

Note that we used the notation $|S_z; +\rangle = |+\rangle$ and $|S_z; -\rangle = |-\rangle$ here and also below.

4. (10 points) Using the orthonormality of $|+\rangle$ and $|-\rangle$ and the expression in the previous problem to prove

$$[S_x, S_y] = i\hbar S_z. \tag{3}$$