## Physics 217

Problem Set 12
Due: Friday, Dec 7th, 2018

1. (10 points) Problem 28 from Chapter 8 of the Harris book.
2. (10 points) Using the completeness relation

$$
\begin{equation*}
1=\sum_{i}\left|a_{i}\right\rangle\left\langle a_{i}\right| \tag{1}
\end{equation*}
$$

demonstrate the operator identity for the operators $A$ and $B$

$$
\begin{equation*}
(A B)^{\dagger}=B^{\dagger} A^{\dagger} \tag{2}
\end{equation*}
$$

3. (20 points) Use the expressions for the $\left|S_{x} ;+\right\rangle$ and $\left|S_{x} ;-\right\rangle$ as well as the $\left|S_{y} ;+\right\rangle$ and $\left|S_{y} ;-\right\rangle$ in terms of the spin states $\left|S_{z} ;+\right\rangle$ and $\left|S_{z} ;-\right\rangle$ to demonstrate that Eq. (1) is also fulfilled for the spin- $x$ and spin- $y$ states. Use these same expression to show that

- $S_{x}=\frac{\hbar}{2}(|+\rangle\langle-|+|-\rangle\langle+|)$
- $S_{y}=\frac{i \hbar}{2}(-|+\rangle\langle-|+|-\rangle\langle+|)$.

Note that we used the notation $\left|S_{z} ;+\right\rangle=|+\rangle$ and $\left|S_{z} ;-\right\rangle=|-\rangle$ here and also below.
4. (10 points) Using the orthonormality of $|+\rangle$ and $|-\rangle$ and the expression in the previous problem to prove

$$
\begin{equation*}
\left[S_{x}, S_{y}\right]=i \hbar S_{z} \tag{3}
\end{equation*}
$$

