Nuclear Physics

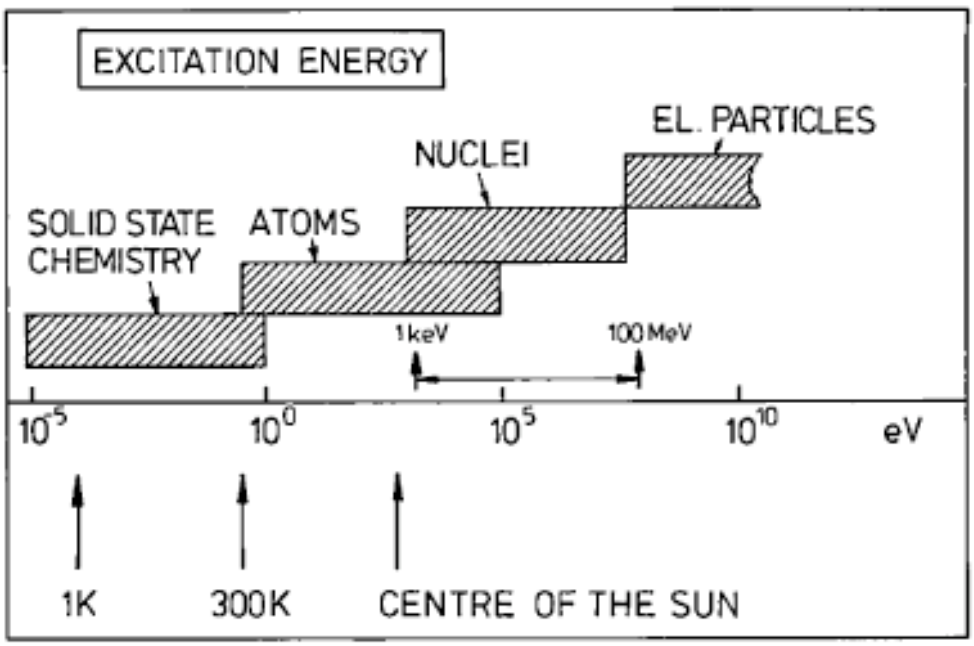
- Broad field with many applications
- Current funding at NSF for nuclear theory
  - Nuclear Structure (DOE -> FRIB)
  - Hadron Structure (DOE -> JLAB)
  - Up and Down quark physics (DOE -> RHIC) also considered particle physics by many
- Publications in
  - Physical Review C (D)
  - Physical Review Letters
  - European counterparts and astrophysics journals
- WU: 9 senior people plus grad students and postdocs (including people in Radiochemistry)

Energy scales

- Use Heisenberg uncertainty relation

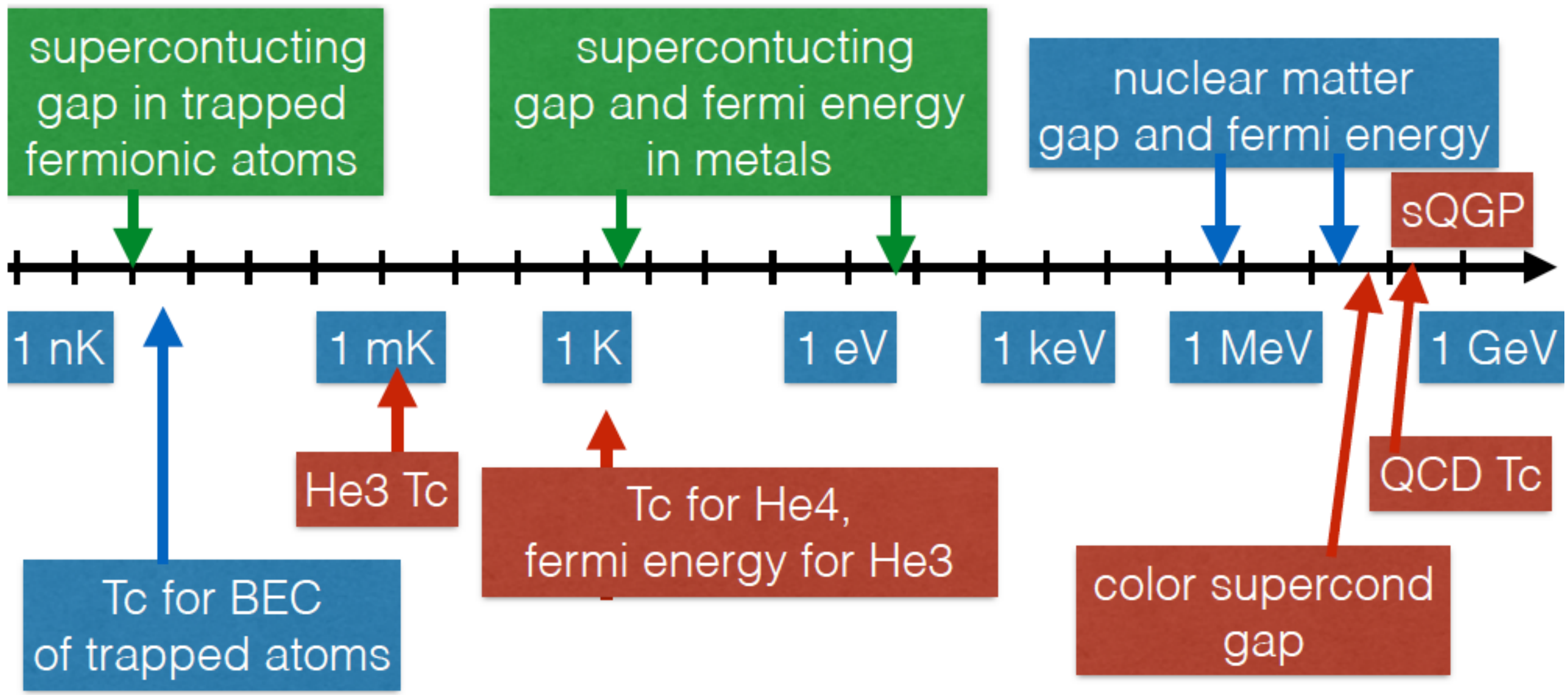
$$\Delta x \; \Delta p \geq \frac{\hbar}{2}$$

- for localization of about 10^{-15} m --> $\dfrac{(\Delta p)^2}{2m_N} \approx \text{few MeV}$



A wider temperature perspective

- From Shuryak: Quantum Many-Body Physics in a Nutshell



- 1 eV —> 1.160 452 21(67) x 10^4K (NIST value)

Many manifestations of quantum physics

- Ultimately linked to a strong interaction Hamiltonian describing nucleons

$$H_A = \sum_{i=1}^{A} \frac{\boldsymbol{p}^2}{2m_i} + \sum_{i<j=1}^{A} V(i,j) + \ldots$$

- A nucleons either p (proton) or n (neutron)
- Can possibly be approximately treated by

$$H_A = \sum_{i=1}^{A} \frac{\boldsymbol{p}^2}{2m_i} + \sum_{i<j=1}^{A} V(i,j) + \ldots = \sum_{i=1}^{A} \left\{ \frac{\boldsymbol{p}^2}{2m_i} + U(i) \right\} + H_A(residual) \rightarrow \sum_{i=1}^{A} \left\{ \frac{\boldsymbol{p}^2}{2m_i} + U(i) \right\}$$

- with a single-particle Hamiltonian and the fermion character of nucleons
- but ... nucleons are themselves composites ...

Bosons and Fermions

- Use experimental observations to conclude consequences of identical particles
- Two possibilities
  - antisymmetric states $\Rightarrow$ fermions half-integer spin
    - Pauli from properties of electrons in atoms
  - symmetric states $\Rightarrow$ bosons integer spin
    - Considerations related to electromagnetic radiation (photons)
- Can also consider quantization of "field" equations
  - e.g. quantize "free" Maxwell equations (Dirac)
- Protons and neutrons have intrinsic spin $\frac{1}{2}$ --> fermions

Global properties of nuclei

- Different levels of description
- Identify most relevant degrees of freedom to describe physics of interest
- Depends on probe --> wavelength of probing radiation and energy scales studied
- Mostly nucleons at low energy in this course --> quantum many-body problem and one of the most difficult ones

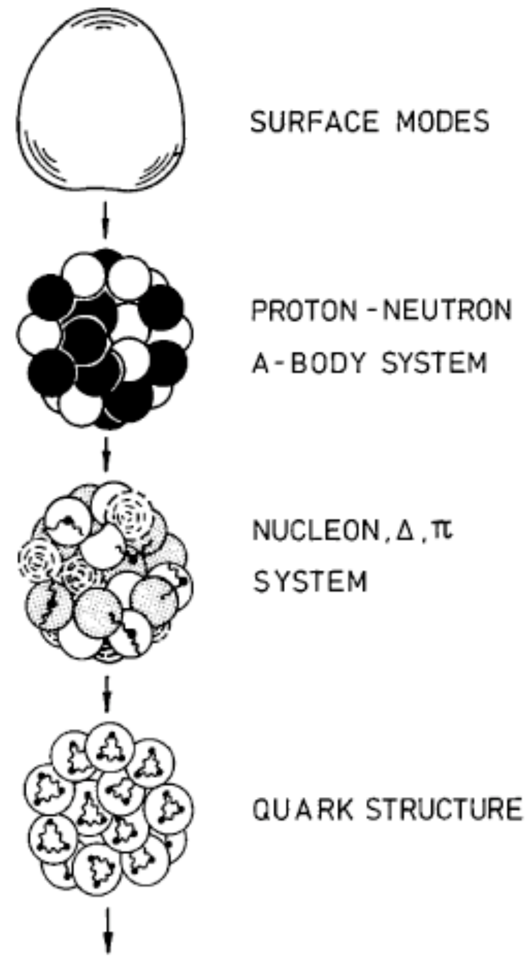


Chart of nuclides

- Brookhaven data base http://www.nndc.bnl.gov/chart/

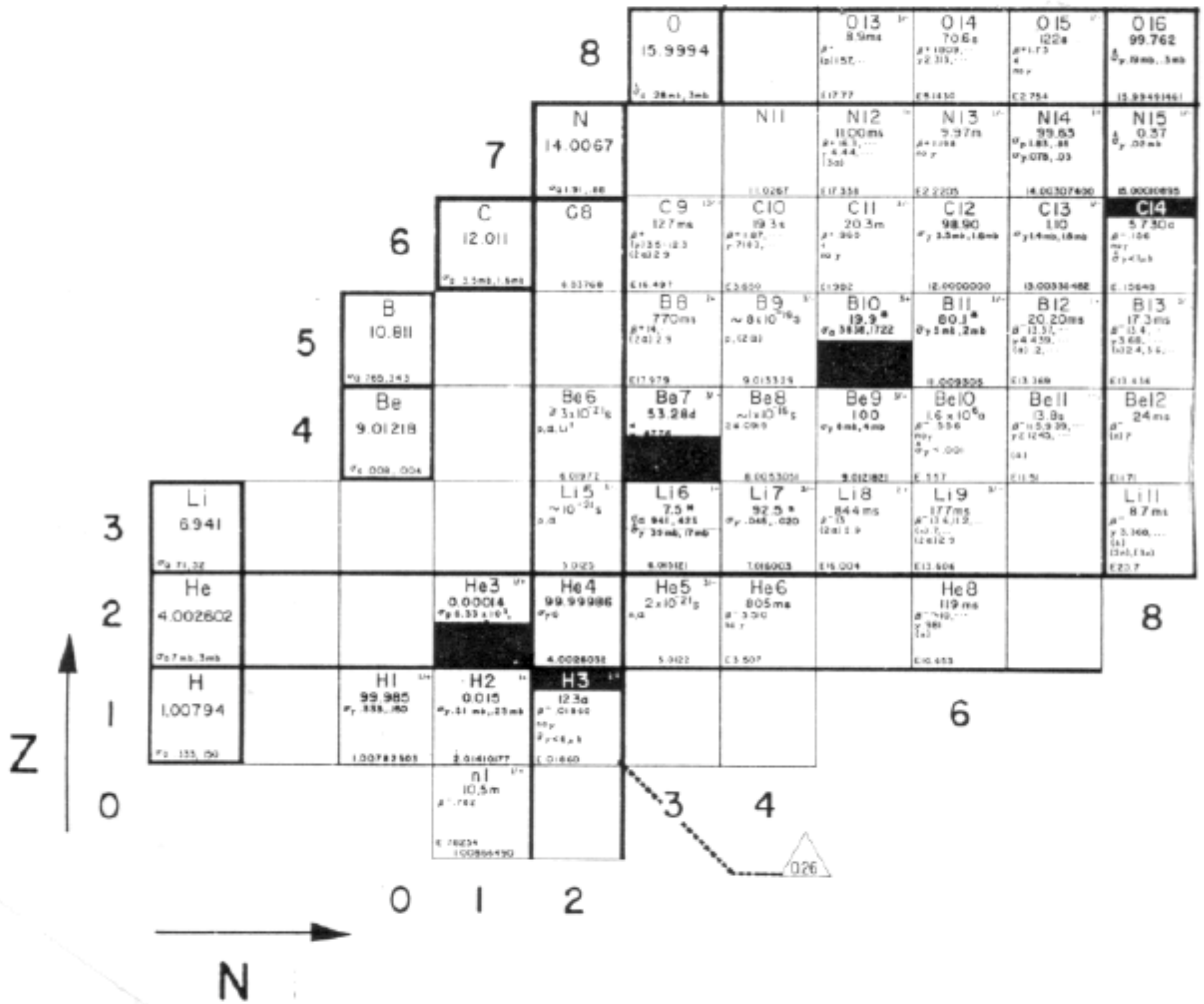
- Horizontal axis neutron number N
- Vertical axis proton number Z
- Notation for a nucleus

$$ {}^{A}_{Z}\text{Chemical symbol} \rightarrow {}^{A}\text{Chemical symbol} $$

- Since $A = Z + N$ and chemical symbol implies Z

More details

- Abundance information for stable nuclei
- Half-life for unstable nuclei
- Several possible decay modes



Towards the edges of stability

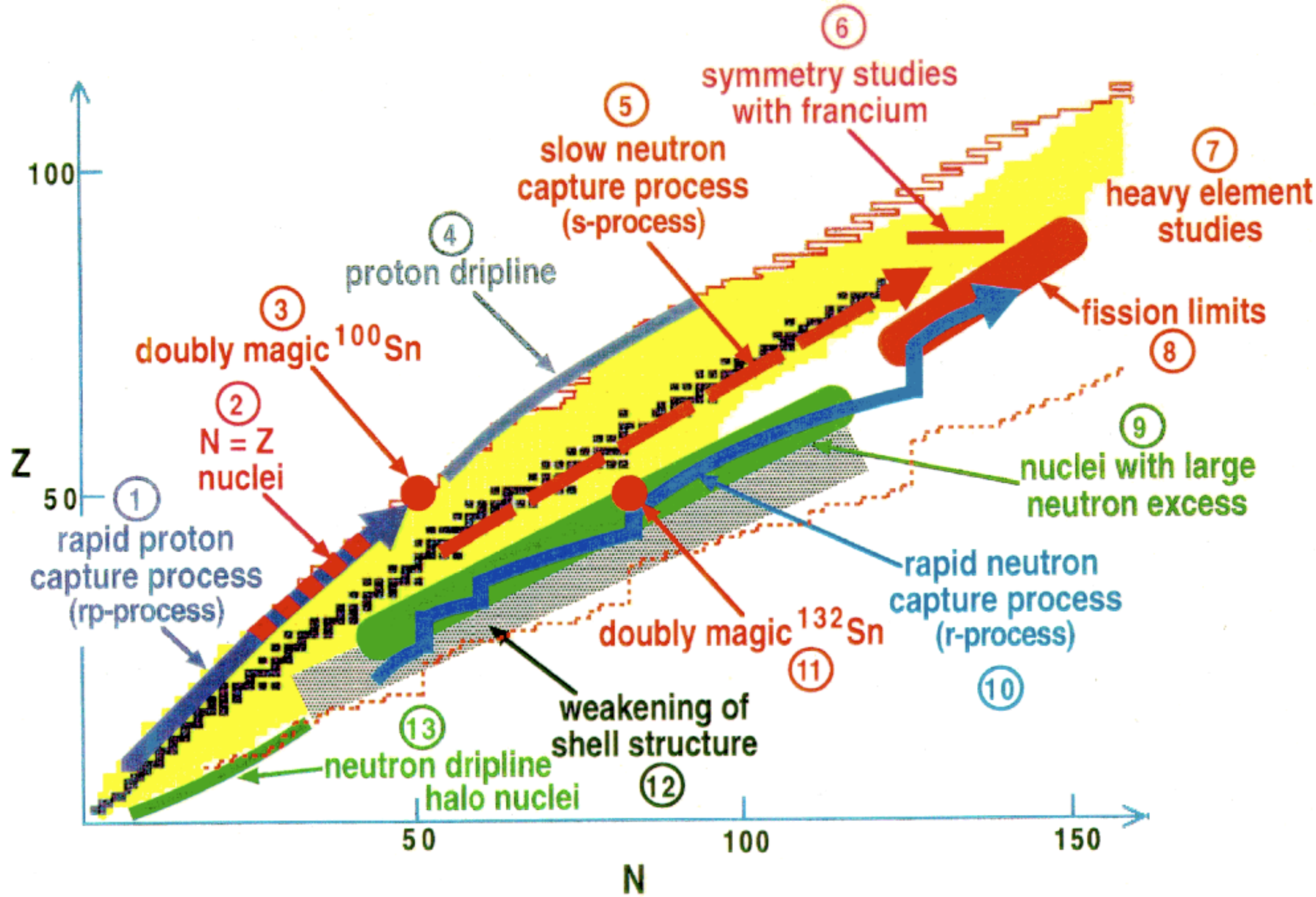


Illustration of current research associated with exotic nuclei

Binding energy

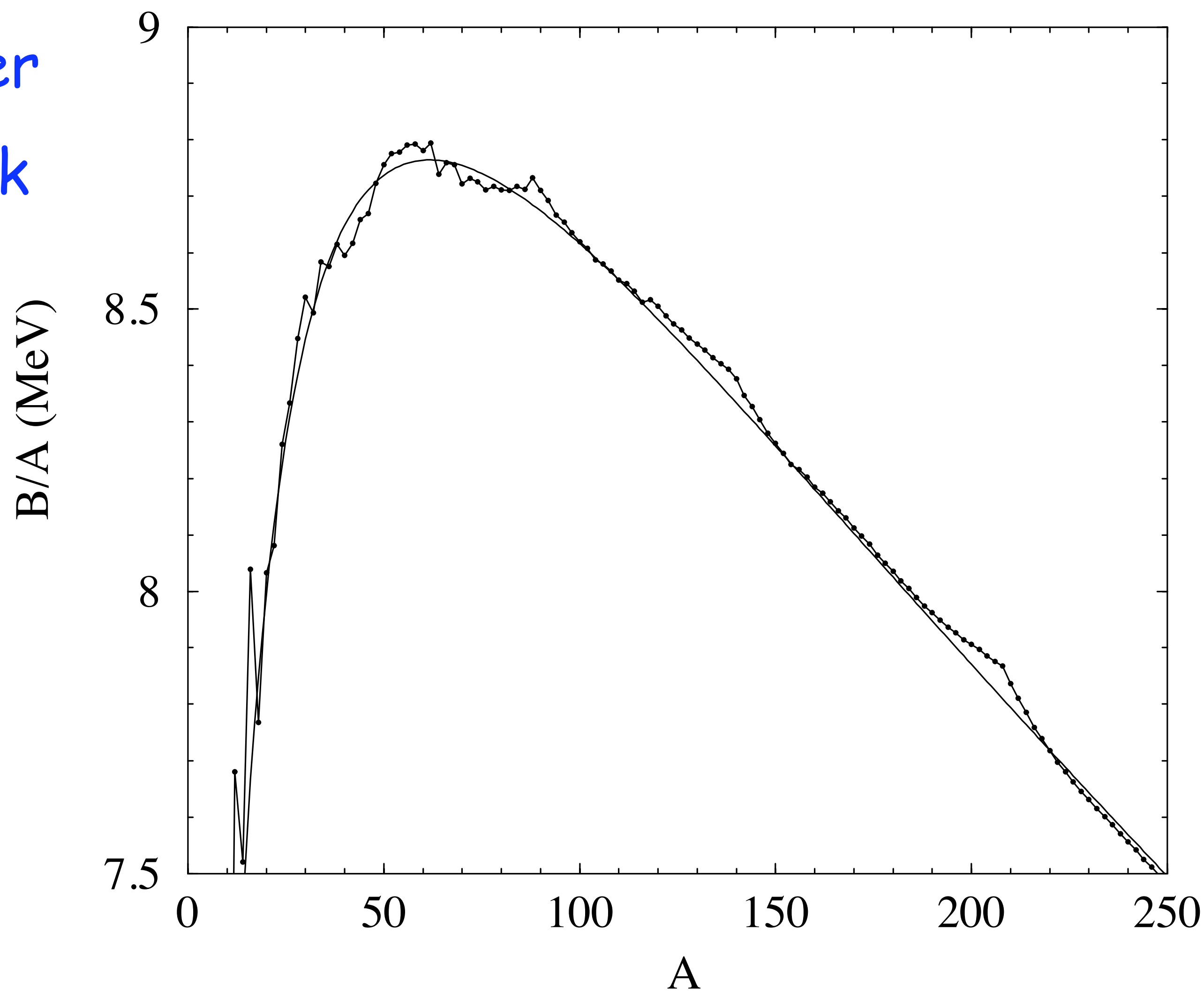
- Critical information deciding on decays/stability
- If every nucleon gets similar binding from every other nucleon --> expect total binding to be proportional to number of "bonds" = ½ A(A-1)
- So B/A ∝ c(A-1) but experiment says otherwise ---> B/A ~ constant ---> "saturation"
- Define $BE(^{A}X) = Z \cdot M_p c^2 + N \cdot M_n c^2 - M'(^{Z}X)c^2$
- actual mass
- More easily accessible: atomic binding energy

$$BE(^{A}X \text{ atom}) = Z \cdot M_{^1H} c^2 + N \cdot M_n c^2 - M(^{Z}X \text{ atom})c^2$$

- up to ~ eV the same

Binding energy per nucleon

- Often used unit: amu = atomic mass unit = 1/12 mass of ^{12}C = 1.660566×10^{-27} kg = 931.5016 MeV/c^2
- More later
- Homework



Nuclear size / densities

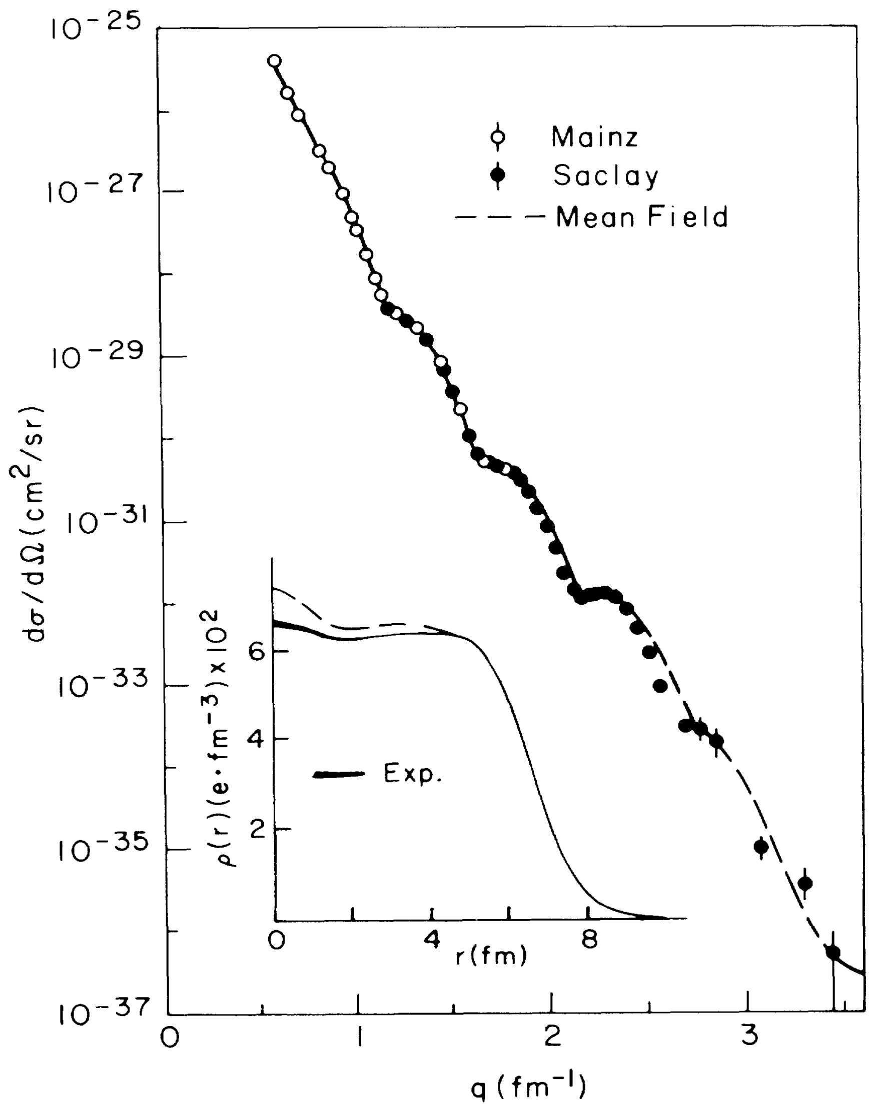
- In addition to saturation there is "charge independence" of binding --> each nucleon occupies a roughly fixed volume characterized by a certain radius r_0
- So nuclear volume

$$V = \frac{4\pi}{3} r_0^3 A = \frac{4\pi}{3} R^3$$

- leading to a nuclear radius that follows $R = r_0 A^{1/3}$
- Empirically: charge --> 1.2 fm
- matter --> 1.4 fm

Experimental determination

- Example: Phys. Rev. Lett. 58, 195 (1987)



Elastic electron scattering

- Central ideas
  - Weakly interacting probe
  - Interaction probe with target precisely known
  - Probe predominantly operates on one target particle at a time --> probe-target interaction dominated by one-body operator
  - experiment possible with sufficient accuracy
- Elastic electron scattering
  - only for stable nuclei
  - but will be available for some rare isotope beams in the future (collider set-up): Germany and Japan

Scattering from a fixed Coulomb potential

- Original experiments generated limited information about the interior charge density distribution and typical results were parametrized by

$$\rho(r) = \frac{\rho_0}{1 + \exp^{(r-R_0)/a}}$$

- introducing:
  - central density
  - radius at half density
  - diffuseness
- Analyze Schrödinger equation (nonrelativistic) even though actual experiments range from 50-1000 MeV so much larger than electron rest mass
- No need for Dirac equation to explain the physics

S. equation

- Start with $$\left\{-\frac{\hbar^2\boldsymbol{\nabla}^2}{2m}+V(\boldsymbol{r})\right\}\psi(\boldsymbol{r})=E\psi(\boldsymbol{r})$$
- Potential due to nuclear charge distribution

$$V(\boldsymbol{r})=-\hbar c\ \alpha\int d^3r'\ \frac{\rho(\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

- with fine structure constant $\alpha=\frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{\hbar c}\simeq\frac{1}{137}$
- so $\hbar c\cdot\alpha=1.44\ \mathrm{MeV\ fm}$
- and $\int d^3r\ \rho(\boldsymbol{r})=Z$
- Visualize scattering process: incoming plane wave + outgoing spherical wave modulated by function of angles

More scattering

- Solution S. equation can be written as

$$\psi(\boldsymbol{r}) = e^{i\boldsymbol{k}_i\cdot\boldsymbol{r}} + \psi_{sc}(\boldsymbol{r})$$

- Energy

$$E = \frac{\hbar^2 k^2}{2m}$$

- Elastic scattering: $k = |\boldsymbol{k}_i| = |\boldsymbol{k}_f|$

- Standard quantum analysis (read Griffiths Ch.11 or more advanced book) generates

$$\psi_{sc} \longrightarrow \frac{e^{ikr}}{r} f(\Omega) \qquad r \to \infty$$

- and cross section (probability to scatter in direction of final momentum - area unit): $\dfrac{d\sigma}{d\Omega} = |f(\Omega)|^2$

Analysis

- Insert assumed solution in S. equation