There are a total of 100 points on this exam. Attempt all problems. Partial credit will be given, wherever possible, for attempted problems provided that all the work is shown clearly. Just explaining first in words how you plan to do the problem could get you some credit but to get full credit you need to justify your answers.

You may use your book, your first 6 graded problem sets, and the notes you made in class but no worked-out problems. You can use a calculator if necessary, but not read stored text from it. Write your name and ID on the exam, since it will be collected when the exam is over together with your answers.

Name:

University ID:

1) A metal ball (mass m) with a hole through it is threaded on a frictionless vertical rod. A massless string (length ℓ) attached to the ball runs over a massless, frictionless pulley and supports a block of mass M, as shown in Fig. 1.

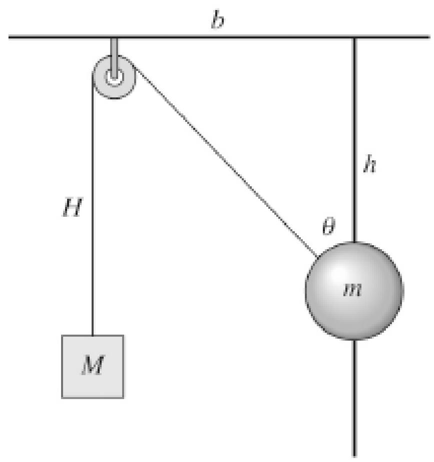

Figure 1: Picture for problem 1

a) (10 points) Determine the potential energy $U(\theta)$ due to gravity by adding the contributions due to m and M. The potential energy can be expressed in terms of h and H. Eliminate these two variables in favor of θ and the constants b and ℓ. Assume that the ball and the pulley have negligible size.

b) (10 points) By differentiating $U(\theta)$ find whether the system has an equilibrium position, and for what values of m and M equilibrium can occur. Discuss the stability of any equilibrium position.

2) Consider the usual angle ϕ as generalized coordinate for a simple pendulum of length ℓ that is suspended from the ceiling of an elevator that is accelerating upwards with constant acceleration a.

a) (5 points) Determine the velocity of the point of support in the frame fixed to earth so both x and y components.

b) (5 points) Determine the corresponding velocity components of the bob with respect to the elevator.

c) (5 points) Determine the velocity of the bob with respect to the frame fixed to earth using the velocities of parts a) and b).

d (10 points) Determine the potential energy and construct the Lagrangian using the angle ϕ of the bob.

e) (5 points) Determine the equation of motion.

f) (5 points) Determine the angular frequency of small oscillations.

3) Consider a simple pendulum shown in Fig. 2 with mass m and length ℓ. Its point of support P is attached to the edge of a wheel with center

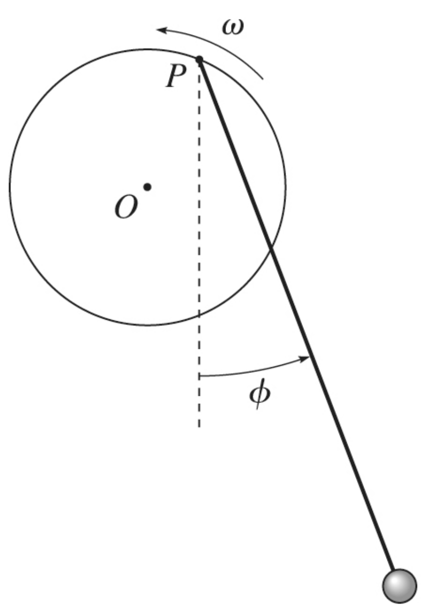

Figure 2: Picture for problem 3

O and radius R that is forced to rotate at a fixed angular velocity ω. At $t = 0$ the point P is level with O on the right. Several steps are identified below to ultimately generate the differential equation for the angle $\phi(t)$ which is an appropriate choice as a generalized coordinate.

a) (5 points) Determine the position, *i.e.* the x and y component, of the point P as a function of time.

b) (10 points) Determine the position of the bob at time t (also x and y components explicitly including the angle ϕ).

c) (10 points) Determine the velocity of the bob by differentiating the result of part b) and construct the corresponding kinetic energy.

d) (10 points) Determine U as a function of ϕ and write down the Lagrangian.

e) (5 points) Derive the equation of motion for the angle ϕ.

f) (5 points) Check whether your answer makes sense in the special case $\omega = 0$.