Physics 217 Midterm Exam

(* 1. (a) Compton scattered X-rays off of electrons;
(b) Compton observed that the X-rays scattered off of the target had wavelength peaks in two locations, one at the wavelength of the initial X-ray, and a second at some shifted wavelength λ';
(c) Compton's experiments were surprising because of the two peaks which occurred. Physicists of that time thought light behaved only wavelike and this could not explain the two peaks;
(d) The Compton effect tells us that light demonstrates particle like properties.
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(* 2. (a) Inside of the well the Hamiltonian is $\hat{H} = \frac{\hat{p}^2}{2m}$;
Outside of the well the potential is infinite,
which gives rise to the necessary boundary conditions
(there can be no wave functions outside the well): $\psi(x=a)=\psi(x=0)=0$;
(b) For the given wavefunctions, we have $\hat{H}\psi(x)=\frac{\hat{p}^2}{2m}\psi(x)=\frac{\hbar^2k^2}{2m}\psi(x)$. Hence these are the eigenstates of the Hamiltonian. The boundary conditions gives the values of k as $k=\frac{n\pi}{a}$, where n is a positive integer;
(c) From part (b) we see that the energy eigenvalues are $\frac{\hbar^2k^2}{2m}=\frac{n^2\pi^2\hbar^2}{2ma^2}$;
(d) $\hat{p}\psi(x)=\frac{-i\hbar n\pi}{a}\sqrt{\frac{2}{a}}\cos(kx)\neq p\psi(x)$. This works because $\psi(x)=\sin(kx)$ is a linear combination of momentum eigenfunctions (e^{ikx}) meaning that $\psi(x)$ itself is not a momentum eigenfunction;
(e) Adding the normal time dependence we get $\Psi(x,t)=\frac{1}{2}\psi_1(x)e^{-iE_1t/\hbar}-\frac{\sqrt{3}}{2}\psi_2(x)e^{-iE_2t/\hbar}$. The time-dependent Schrodinger equation states $i\hbar\frac{\partial}{\partial t}\Psi=-\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2}\Psi$. Putting our wavefunction into this equation,
the left hand side looks like $i\hbar\frac{\partial}{\partial t}\Psi=\frac{E_1}{2}\psi_1(x)e^{-iE_1t/\hbar}-\frac{\sqrt{3}\,E_2}{2}\psi_2(x)e^{-iE_2t/\hbar}$. The right hand side looks like $-\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2}\Psi=\frac{1}{2}\frac{\hbar^2k_1^2}{2m}\psi_1(x)e^{-iE_1t/\hbar}-\frac{\sqrt{3}}{2}\frac{\hbar^2k_2^2}{2m}\psi_2(x)e^{-iE_2t/\hbar}=\frac{E_1}{2}\psi_1(x)e^{-iE_1t/\hbar}-\frac{\sqrt{3}\,E_2}{2}\psi_2(x)e^{-iE_2t/\hbar}$. We see that the left and right sides of the equation are equivalent thus this wavefunction satisfies the time-dependent Schrodinger equation.
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(* 3. (a) $\int_{-\infty}^{\infty} |\Psi|^2 dx = \int_{-\infty}^{0} 2qe^{2qx} dx + \int_0^{\infty} 2qe^{-2qx} dx = 2$. Thus this wavefunction is not normalized. In order to normalize the wavefunction, we need to multiply it by $\frac{1}{\sqrt{2}}$, obtaining the wavefunction $\Psi(x, t=0) = \sqrt{q}\, e^{-q|x|}$ which is normalized. The units of q are inverse length. (the argument of the exponential function should be unitless, and x has the unit of length, hence q has the unit inverse length);

(b) $\tilde{\psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{q}\, e^{-q|x|} e^{ikx} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} \sqrt{q}\, e^{(q+ik)x} dx + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \sqrt{q}\, e^{(-q+ik)x} dx =$

$\sqrt{\frac{q}{2\pi}} \left(\frac{1}{q+ik} - \frac{1}{-q+ik}\right) = \sqrt{\frac{q}{2\pi}} \frac{2q}{q^2+k^2} = q^{\frac{3}{2}} \sqrt{\frac{2}{\pi}} \frac{1}{q^2+k^2}$.

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