NSCL 8/02/05

Spectroscopic factors and the physics of the single-particle strength distribution in nuclei

Lecture 1: 7/18/05	Propagator description of single-particle motion and the link with experimental data
Lecture 2: 7/19/05	From diagrams to Hartree-Fock and spectroscopic factors < 1
Lecture 3: 7/20/05	Influence of long-range correlations and the relation to excited states
Lecture 4: 7/27/05	Role of short-range and tensor correlations associated with realistic interactions.
Lecture 5: 7/28/05	Summary of results. Prospects for nuclei with N very different from Z.
Lecture 6: 8/02/05	Saturation problem of nuclear matter

Wim Dickhoff
Washington University in St. Louis

The two “most elusive” numbers in nuclear physics

- What are these numbers?
- In what sense are they elusive?
- What is the history?
- Three-body forces? Relativity? Give up?
- What has been learned from (e,e´p)?
- What really decides the saturation density?
- Nuclear Matter with SRC? No LRC?
- Conclusions

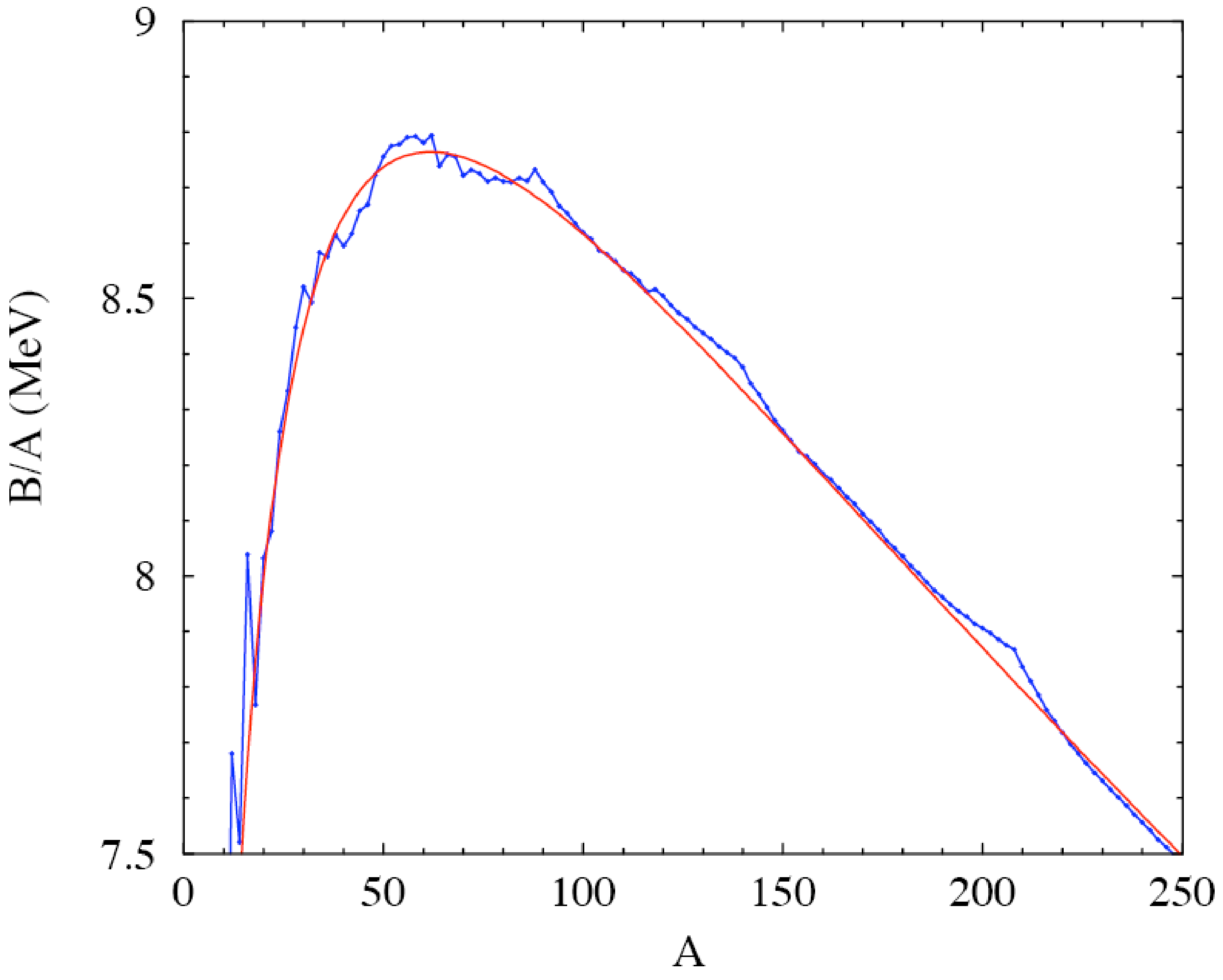
Empirical Mass Formula

Global representation of nuclear masses (Bohr & Mottelson)

$$B = b_{vol}A - b_{surf}A^{2/3} - \tfrac{1}{2}b_{sym}\frac{(N-Z)^2}{A} - \tfrac{3}{5}\frac{Z^2e^2}{R_c}$$

- Volume term $b_{vol} = 15.56$ MeV
- Surface term $b_{surf} = 17.23$ MeV
- Symmetry energy $b_{sym} = 46.57$ MeV
- Coulomb energy $R_c = 1.24\,A^{1/3}$ fm
- Pairing term must also be considered

Empirical Mass Formula



Plotted: most stable nucleus for a given A

Central density of nuclei

Multiply charge density at the origin by A/Z

$\Rightarrow$ Empirical density = 0.16 nucleons / fm^3

$\Rightarrow$ Equivalent to $k_F = 1.33\ \mathrm{fm}^{-1}$

Nuclear Matter

N = Z

No Coulomb

$A \Rightarrow \infty$, $V \Rightarrow \infty$ but $A/V = \rho$ fixed

“Two most important numbers”

$b_{vol} = 15.56$ MeV and $k_F = 1.33\ \mathrm{fm}^{-1}$

Historical Perspective

• First attempt using scattering in the medium	*Brueckner* 1954
• Formal development (linked cluster expansion)	*Goldstone* 1956
• Low-density expansion	*Galitskii* 1958
• Reorganized perturbation expansion (60s)	*Bethe & students*
involving ordering in the number of hole lines	*BBG*-expansion
• Variational Theory vs. Lowest Order *BBG* (70s)	*Clark, Pandharipande*
• Variational results & next hole-line terms (80s)	*Day, Wiringa*
• Three-body forces? Relativity? (80s)	*Urbana, CUNY*
• Confirmation of three hole-line results (90s)	*Baldo et al.*
• New insights from experiment	***NIKHEF* Amsterdam**
about what nucleons are up to in the nucleus (90s & 00s)	***JLab***

Old pain and suffering!

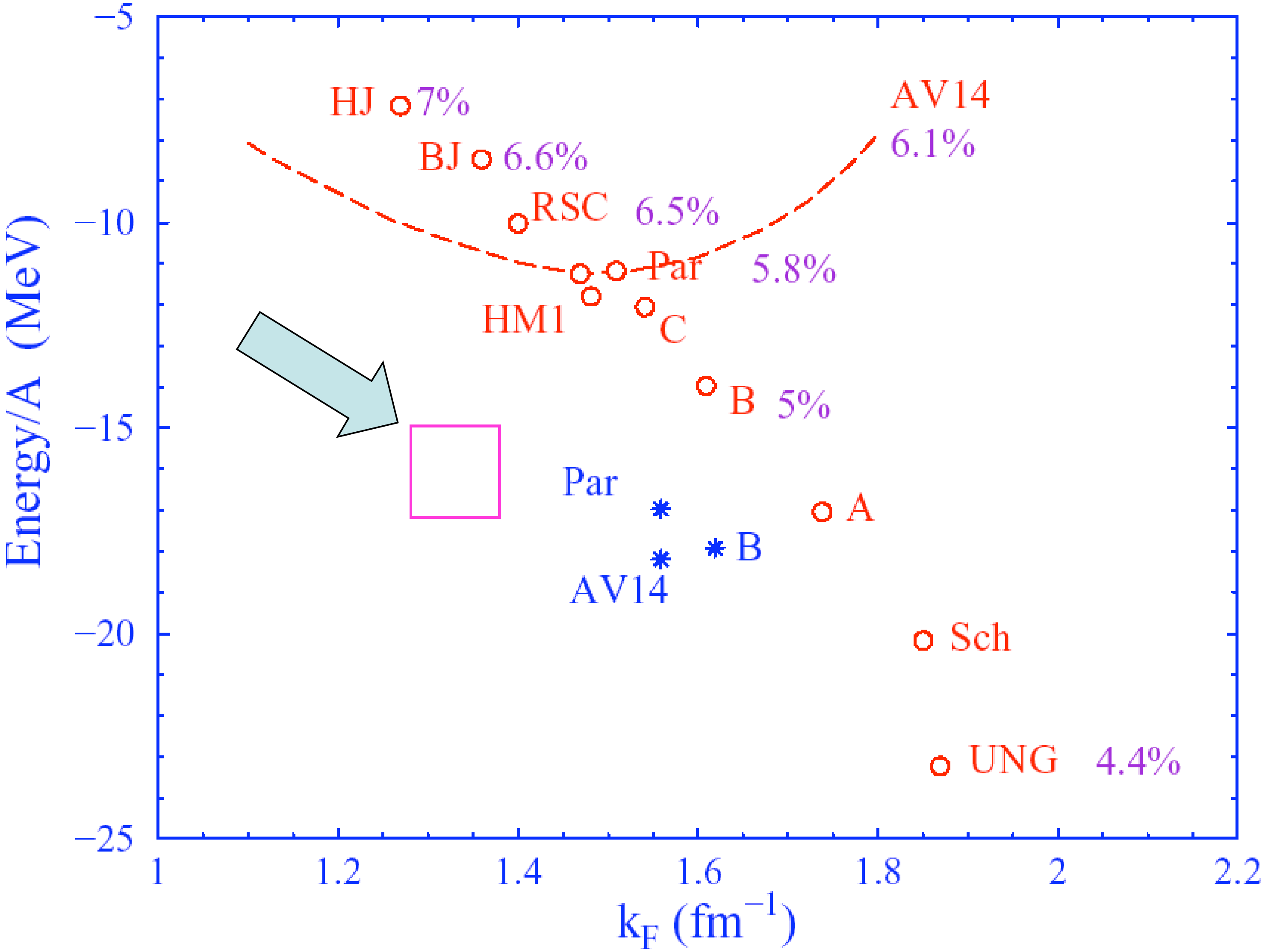
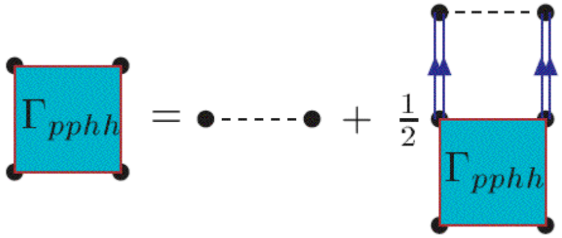


Figure adapted from Marcello Baldo (Catania)

Lowest-order Brueckner theory (two hole lines)



G^f_{BG} angle-average of

$$G^f_{BG}(k_1,k_2;E) = \frac{\theta(k_1-k_F)\theta(k_2-k_F)}{E-\varepsilon(k_1)-\varepsilon(k_2)+i\eta}$$

$$\langle k\ell|G^{JST}(K,E)|k'\ell'\rangle = \langle k\ell|V^{JST}|k'\ell'\rangle + \frac{1}{2}\sum_{\ell''}\int_0^\infty \frac{dq}{(2\pi)^3}q^2\langle k\ell|V^{JST}|q\ell''\rangle G^f_{BG}(q;K,E)\langle q\ell''|G^{JST}(K,E)|k'\ell'\rangle$$

Spectrum $$\varepsilon_{BHF}(k) = \frac{\hbar^2k^2}{2m} + \Sigma_{BHF}(k;\varepsilon_{BHF}(k))$$

$k < k_{\mathrm{F}} \Rightarrow$ standard choice
all $k \Rightarrow$ continuous choice

Self-energy $$\Sigma_{BHF}(k;E) = \frac{1}{\nu}\sum_{m,m'}\int\frac{d^3k'}{(2\pi)^3}\theta(k_F-k')\left\langle \vec{k}\vec{k}'mm'\middle|G\left(\vec{k}+\vec{k}';E+\varepsilon_{BHF}(k')\right)\middle|\vec{k}\vec{k}'mm'\right\rangle$$

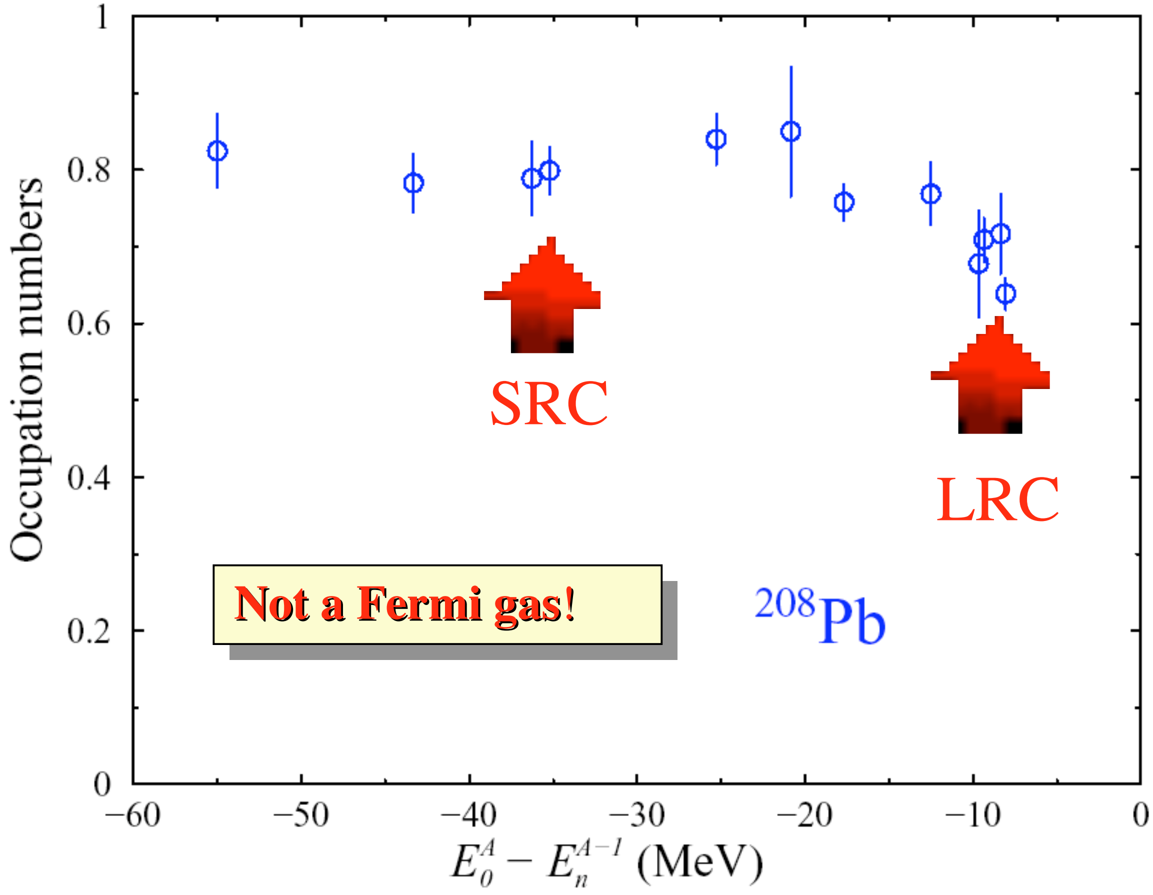
Energy

$$\frac{E}{A} = \frac{4}{\rho}\int\frac{d^3k}{(2\pi)^3}\theta(k_F-k)\frac{\hbar^2k^2}{2m}$$

$$+\frac{1}{2\rho}\sum_{m,m'}\int\frac{d^3k}{(2\pi)^3}\theta(k_F-k)\int\frac{d^3k'}{(2\pi)^3}\theta(k_F-k')\left\langle \vec{k}\vec{k}'mm'\middle|G\left(\vec{k}+\vec{k}';\varepsilon_{BHF}(k)+\varepsilon_{BHF}(k')\right)\middle|\vec{k}\vec{k}'mm'\right\rangle$$

M. van Batenburg (thesis, 2001) & L. Lapikás from ^{208}Pb (e,e´p) ^{207}Tl

Occupation of deeply-bound proton levels from **EXPERIMENT**



Not a Fermi gas!

Up to 100 MeV
missing energy
and
270 MeV/c
missing momentum

Covers the whole
mean-field domain
for the FIRST time!!

Confirmation of theory

Where are the last protons? Answer is coming!

Jlab data

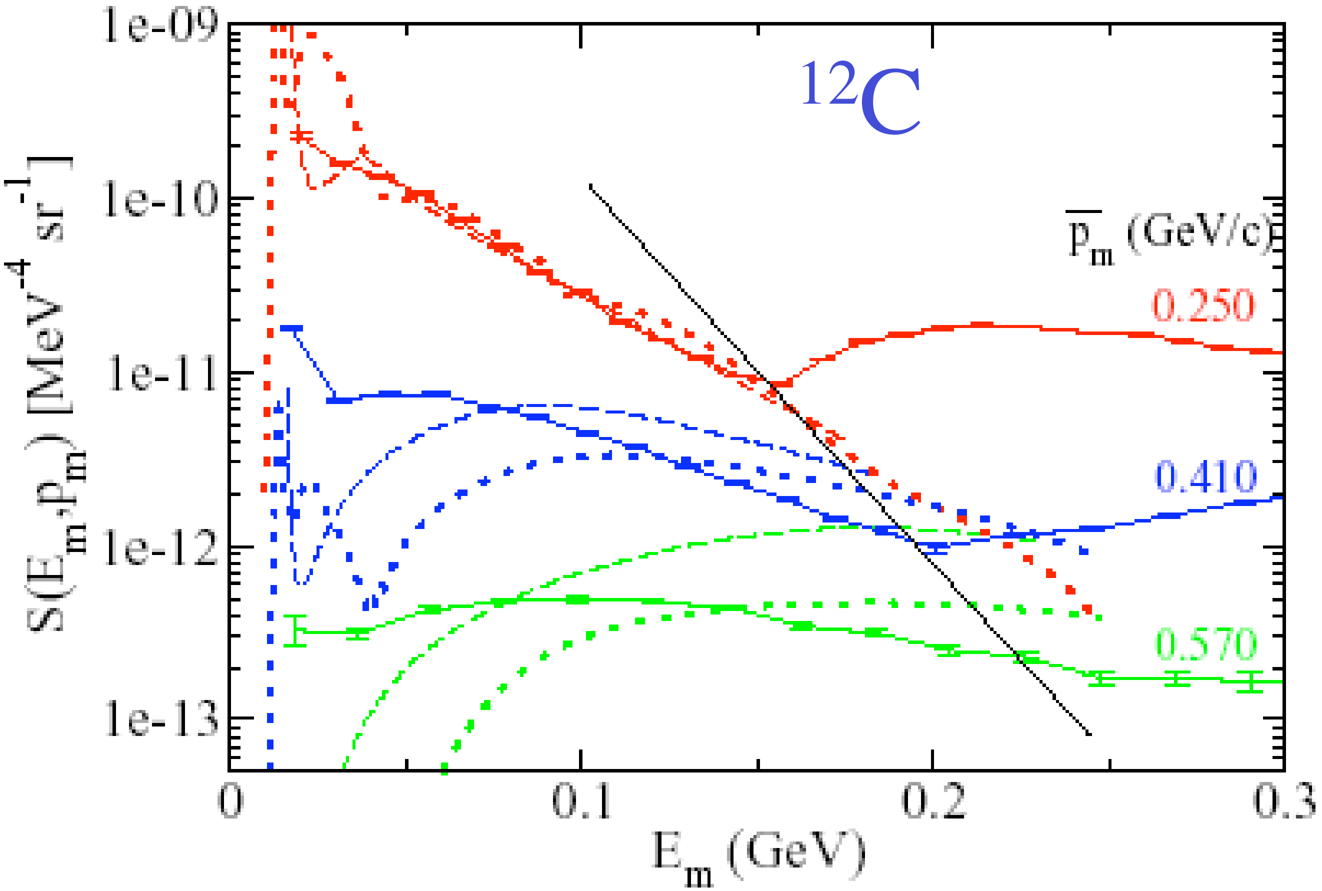
PRL**93**,182501 (2004)

Rohe et al.

Location of high-momentum componen

integrated

strength OK!



There are high-momentum components in the nuclear ground state!

Energy Sum Rule (Migdal, Galitskii, Koltun ...)

Finite nuclei

$$E_0^A = \langle \Psi_0^A | \hat{H} | \Psi_0^A \rangle = \frac{1}{2}\sum_{\alpha\beta} \langle \alpha | T | \beta \rangle n_{\alpha\beta} + \frac{1}{2}\sum_{\alpha} \int_{-\infty}^{\varepsilon_F^-} dE \; E \, S_h(\alpha; E)$$

$$n_{\alpha\beta} = \langle \Psi_0^A | a_\alpha^+ a_\beta | \Psi_0^A \rangle = \frac{1}{\pi} \int_{-\infty}^{\varepsilon_F^-} dE \, \mathrm{Im}\, G(\beta, \alpha; E)$$

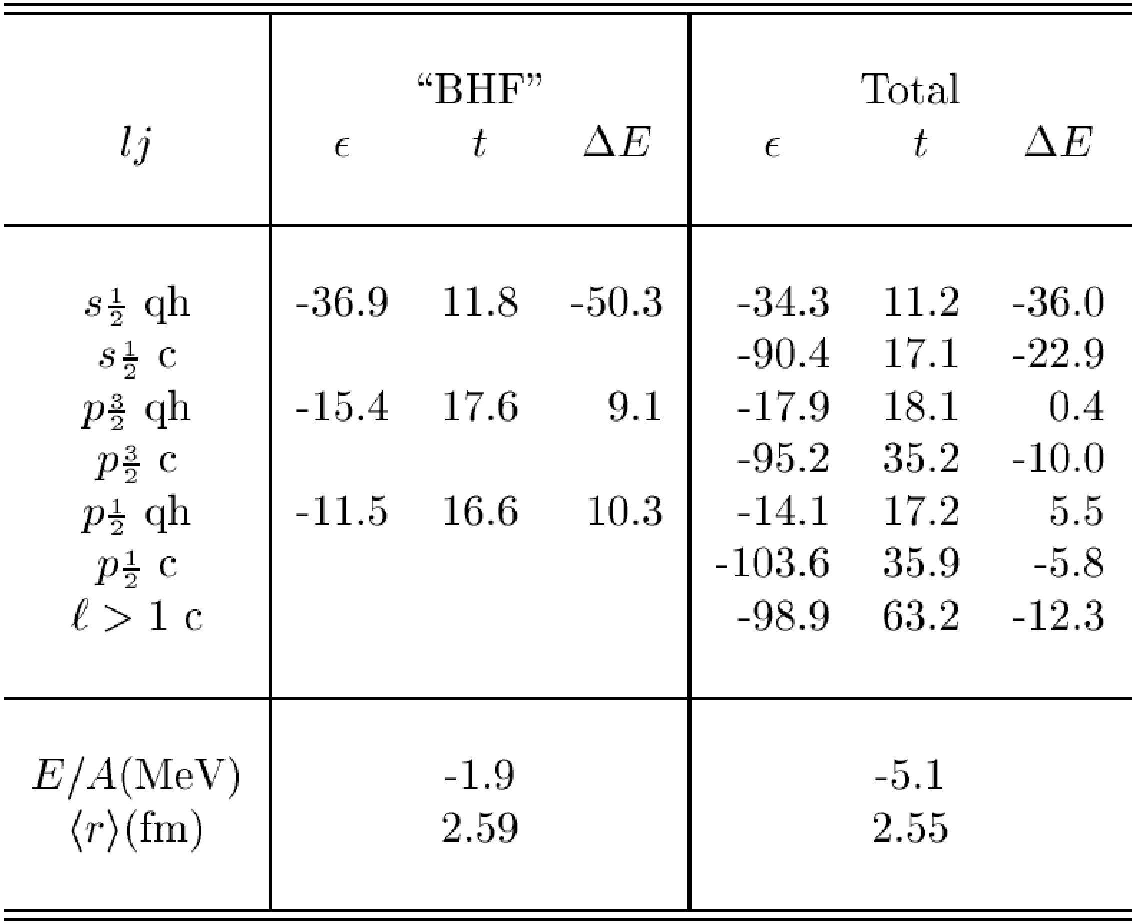
$$S_h(\alpha; E) = \sum_n \left| \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \right|^2 \delta\left(E - \left(E_0^A - E_n^{A-1} \right) \right) = \frac{1}{\pi} \mathrm{Im}\, G(\alpha, \alpha; E)$$

Nuclear matter

$$\frac{E}{A} = \frac{1}{2} \left\{ \frac{4}{\rho} \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\varepsilon_F} dE \left(\frac{\hbar^2 k^2}{2m} + E \right) S_h(k; E) \right\}$$

- Presumes only two-body interactions!
- Correct description of experimental spectral function should yield good *E/A*!!

Where does binding come from (really)?



lj	"BHF" ϵ	t	ΔE	Total ϵ	t	ΔE
$s\frac{1}{2}$ qh	-36.9	11.8	-50.3	-34.3	11.2	-36.0
$s\frac{1}{2}$ c				-90.4	17.1	-22.9
$p\frac{3}{2}$ qh	-15.4	17.6	9.1	-17.9	18.1	0.4
$p\frac{3}{2}$ c				-95.2	35.2	-10.0
$p\frac{1}{2}$ qh	-11.5	16.6	10.3	-14.1	17.2	5.5
$p\frac{1}{2}$ c				-103.6	35.9	-5.8
$\ell > 1$ c				-98.9	63.2	-12.3
E/A(MeV)		-1.9			-5.1	
$\langle r \rangle$(fm)		2.59			2.55	

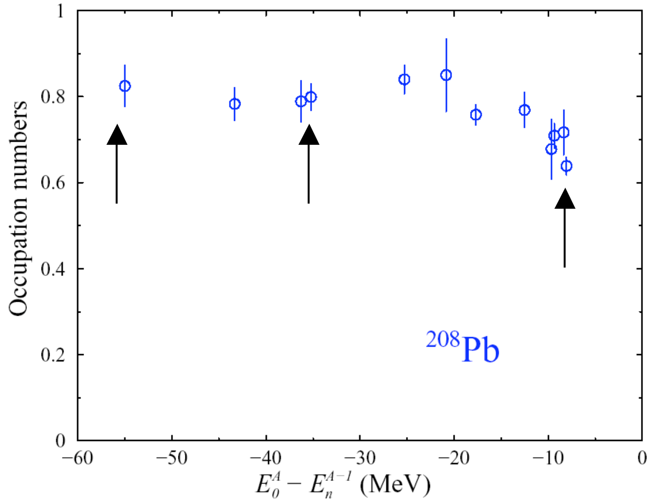
^{16}O PRC51,3040(1995)

Quasiholes contribute 37% to the total energy
High-momentum nucleons (continuum) contribute 63%
but represent **only** about 10% of the particles!!

Saturation density and SRC

- Saturation density related to nuclear charge density at the origin. Data for ^{208}Pb ⇒ $A/Z * \rho_{ch}(0) = 0.16\ fm^{-3}$
- Charge at the origin determined by protons in *s* states
- Occupation of 0*s* and 1*s* totally dominated by SRC as can be concluded from recent analysis of ^{208}Pb(e,e´p) data and theoretical calculations of occupation numbers in nuclei and nuclear matter.
- Depletion of 2*s* proton also dominated by SRC:

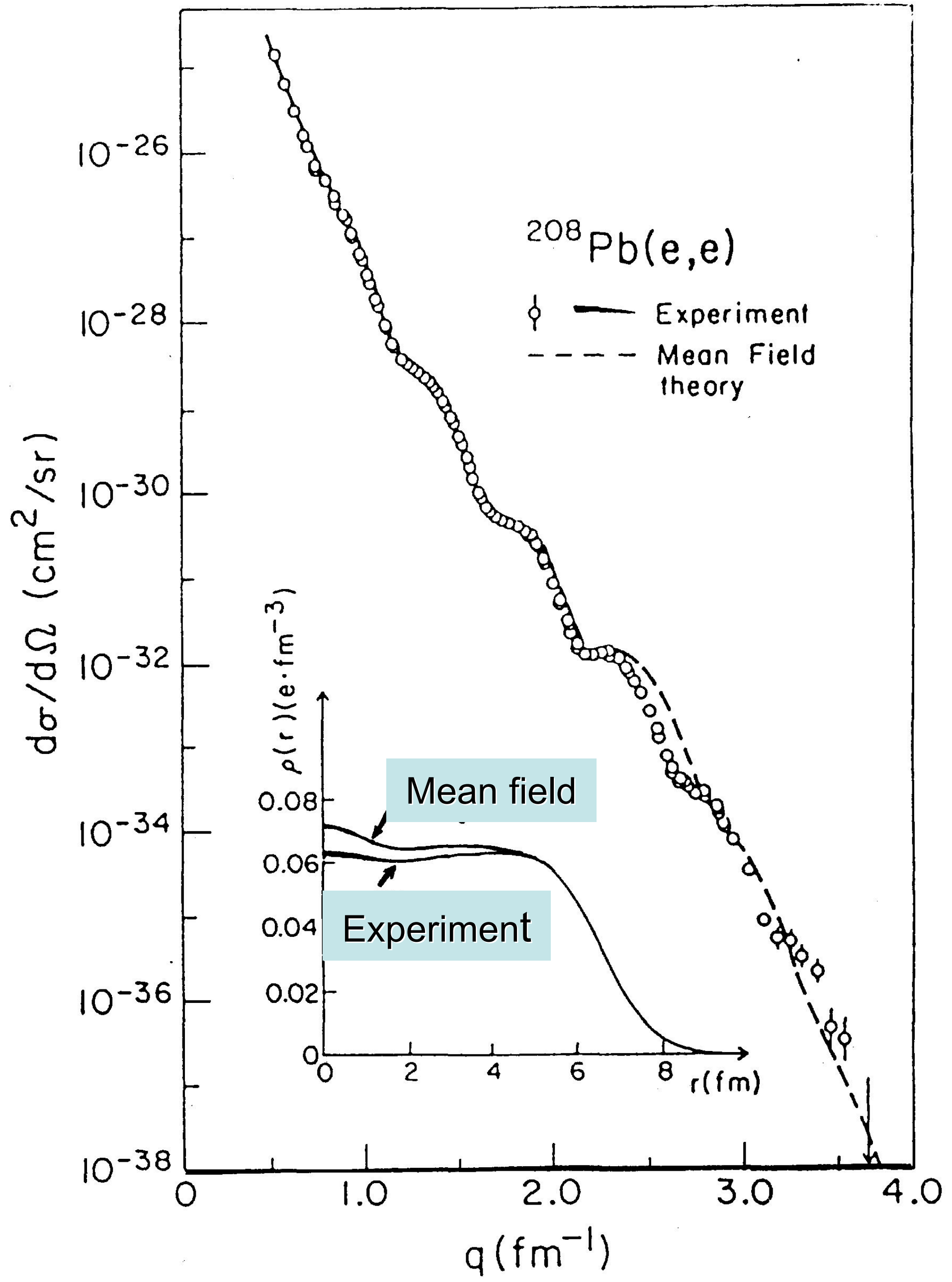
  15% of the total depletion of 25% ($n_{2s} = 0.75$)



- Conclusion: **Nuclear saturation dominated by SRC**

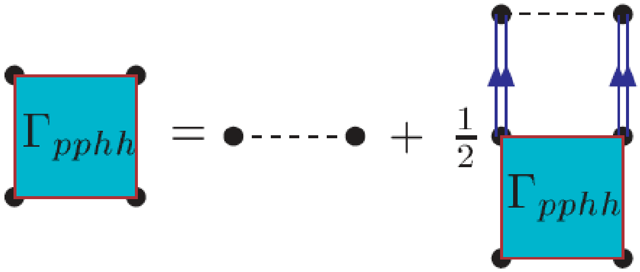
  and therefore high-momentum components

Elastic electron scattering from ^{208}Pb

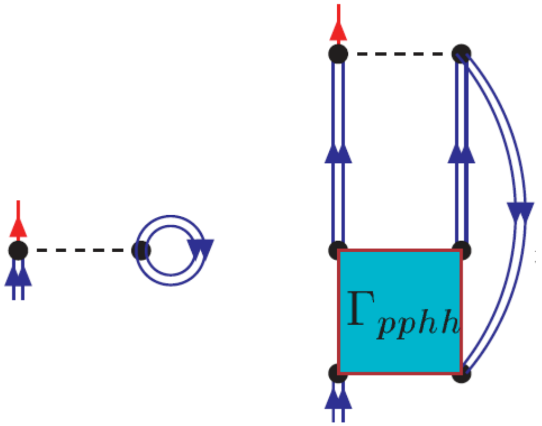


B. Frois *et al.*
Phys. Rev. Lett. **38**, 152 (1977)

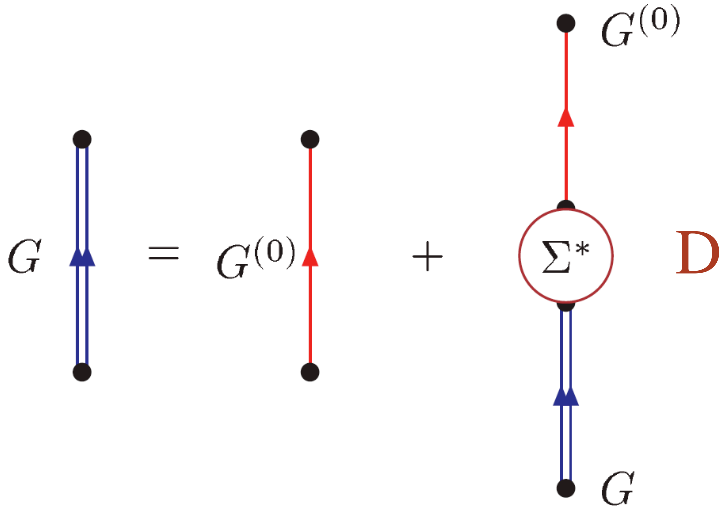
Self-consistent treatment of SRC in nuclear matter



Interaction



Self-energy

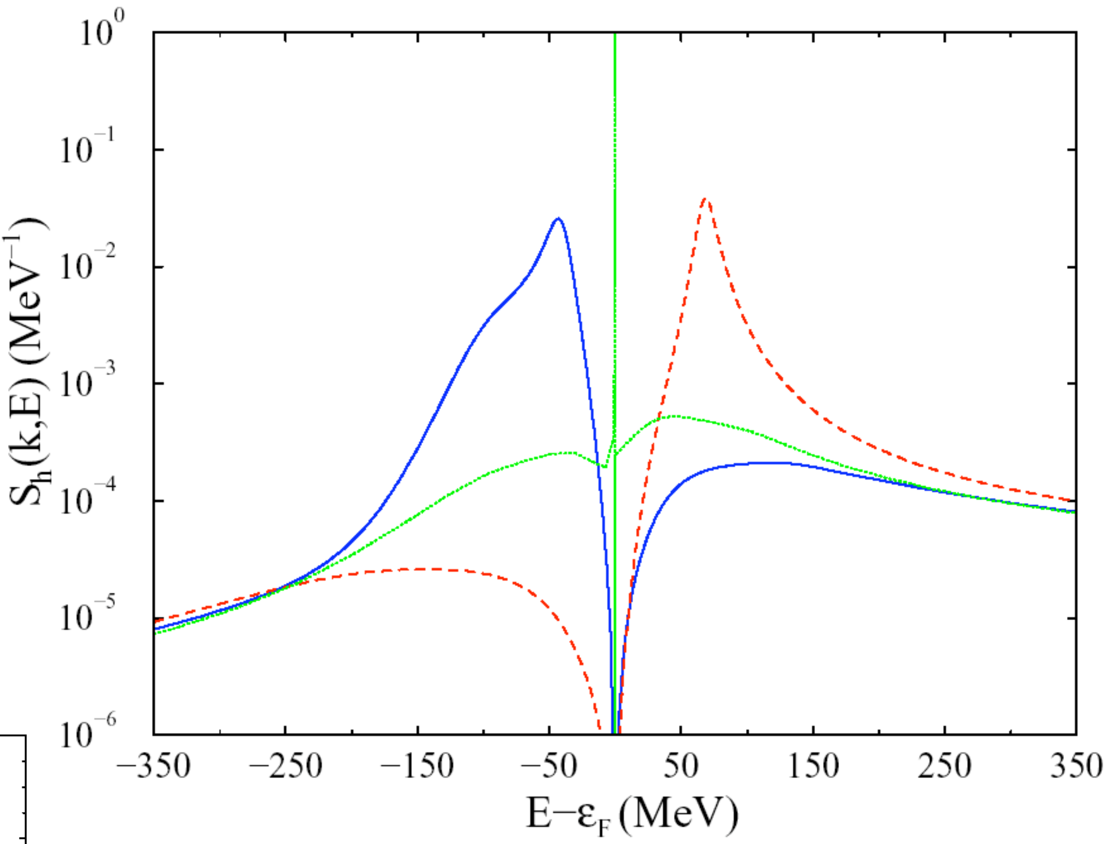


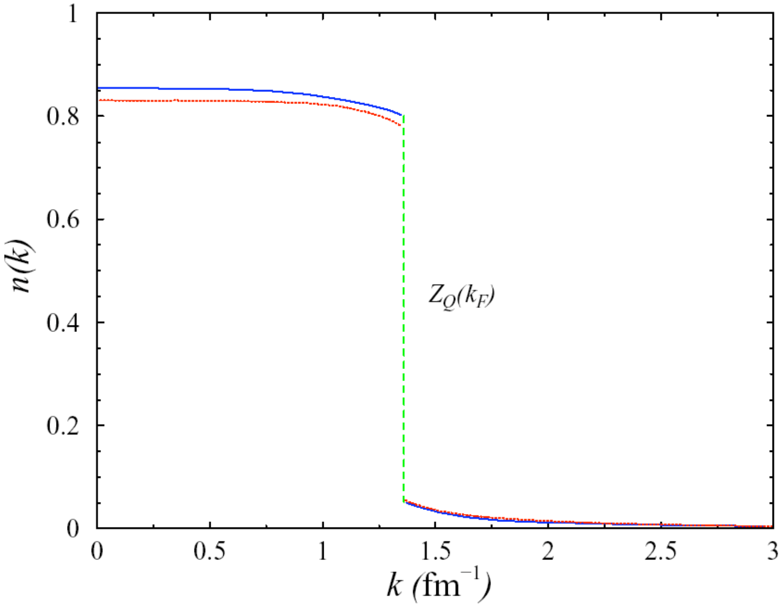
Dyson equation

Results from Nuclear Matter

2nd generation (2000)

- Spectral functions for $k = 0, 1.36, \& \ 2.1\ \mathrm{fm}^{-1}$
- Common tails on both sides of ε_F





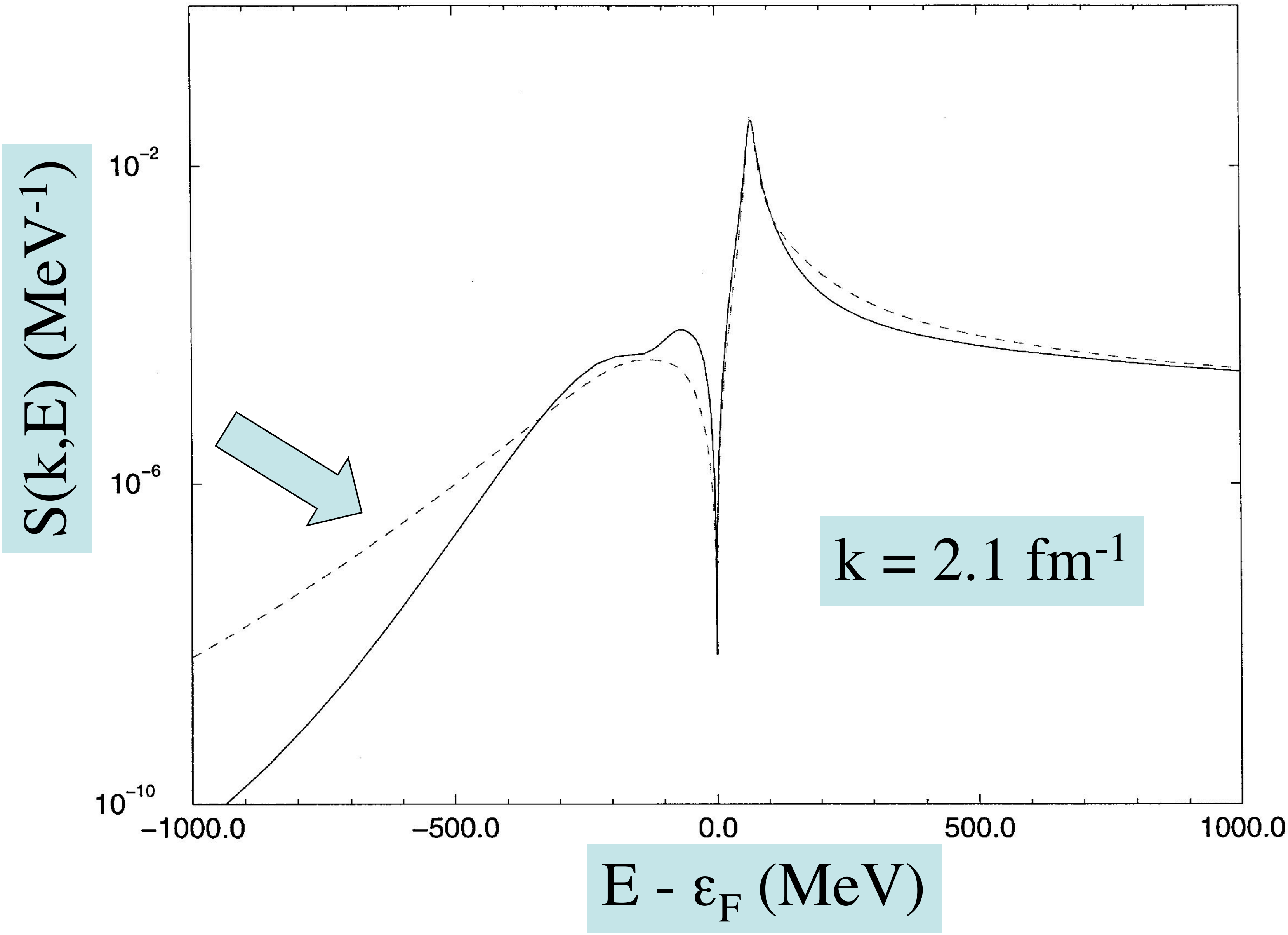
Momentum distribution :

only minor changes

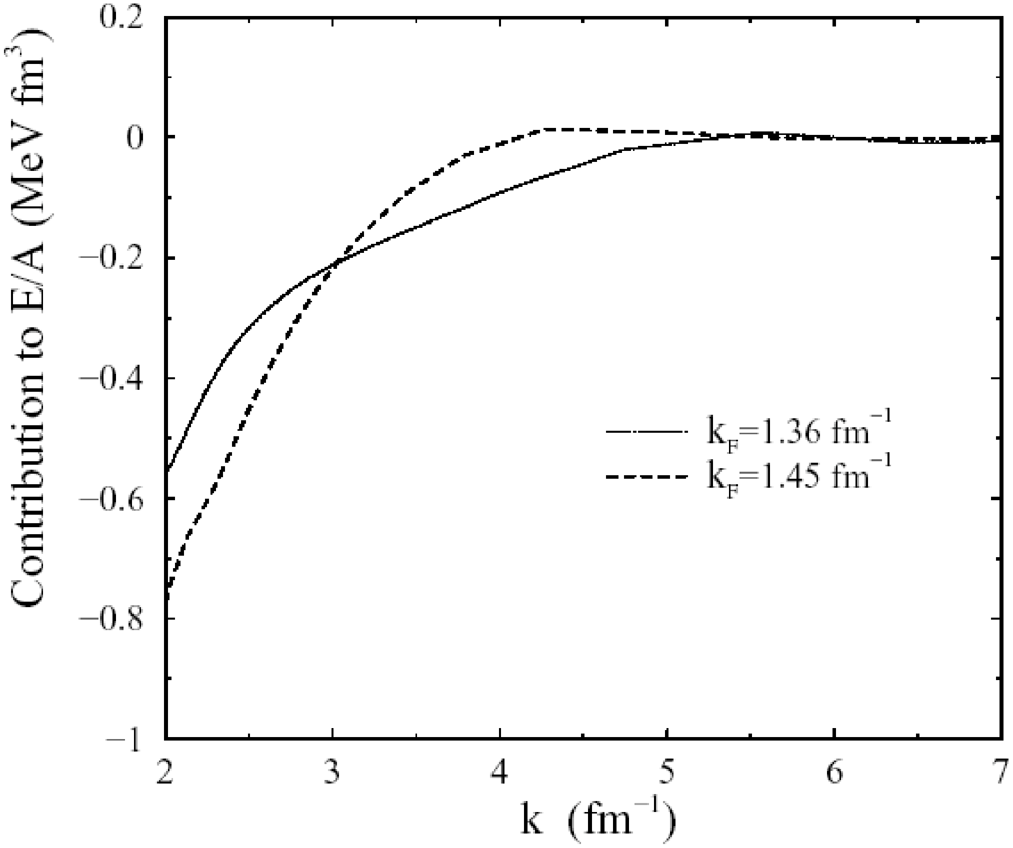
occupation in nuclei

depleted similarly!?!

Self-consistent spectral functions



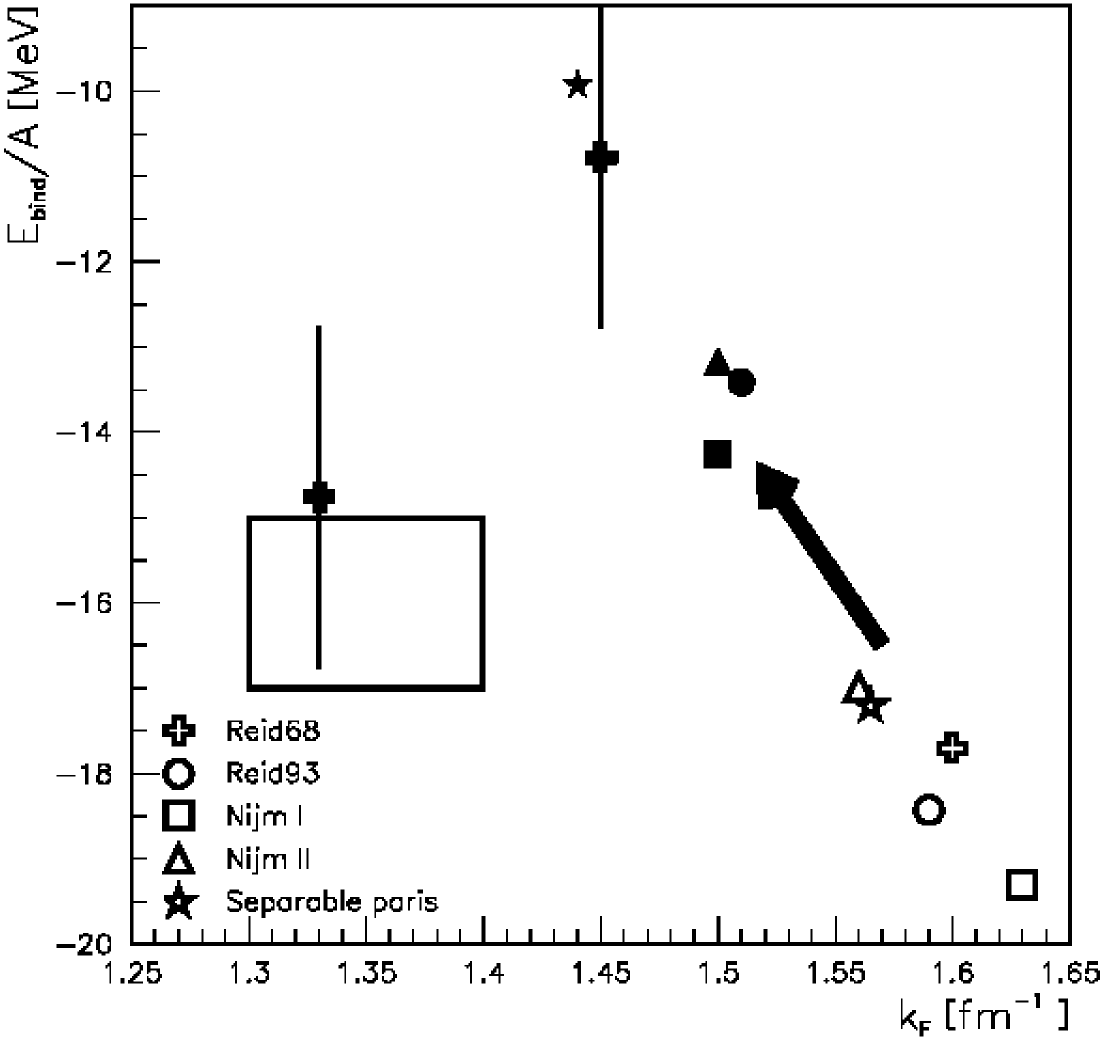
Saturation with self-consistent spectral functions in nuclear matter ⇒ reasonable saturation properties



Contribution to the energy per particle before integration over the single-particle momentum at high momentum for two densities

Saturation of Nuclear Matter
Ladders and self-consistency for Nuclear Matter

- Ghent group
  Dewulf, Van Neck &
  Waroquier
- St. Louis
  Stoddard, WD

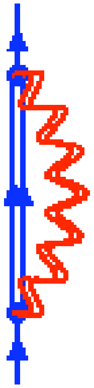


Phys. Rev. Lett. **90**, 152501 (2003)

Self-consistent spectral functions

- Distribution below ε_F broadens for high momenta and develops a common tail at high missing energy
- Slight increase in occupation $k < k_F$ to 85% at $k_F = 1.36$ fm^{-1} compared to Phys. Rev. C**44**, R1265 (1991) & Nucl. Phys. **A555**, 1 (1993)
- Self-consistent treatment of Pauli principle
- Interaction between dressed particles weaker (reduced cross sections for both pn and nn)
- Pairing instabilities disappear in all channels
- Saturation with lower density than before and reasonable binding
- **Contribution of long-range correlations excluded**

Self-consistent Green's functions and the energy of the ground state of the electron gas



GW approximation

G	self-consistent sp propagator
W	screened Coulomb interaction
	⇒ RPA with dressed sp propagators

Electron gas : -XC energies (Hartrees)

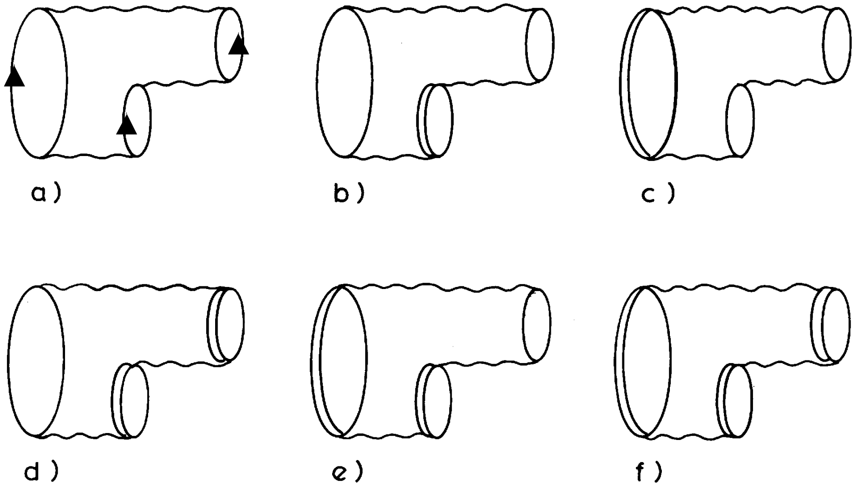
Method	$r_s = 1$	$r_s=2$	$r_s=4$	$r_s=5$	$r_s=10$	$r_s=20$	Reference
QMC	0.5180	0.2742	0.1464	0.1197	0.0644	0.0344	CA80
	0.5144	0.2729	0.1474	0.1199	0.0641	0.0344	OB94;OHB99
GW	0.5160	0.2727	0.1450	0.1185	0.0620	0.032	GG01
		0.2741	0.1465				HB98
RPA	0.5370	0.2909	0.1613	0.1340	0.0764	0.0543	

What about long-range correlations in nuclear matter?

- Collective excitations in nuclei very different from those in nuclear matter
- Long-range correlations normally associated with small q
- Contribution to the energy like $dq\, q^2 \Rightarrow$ very small (except for e-gas)
- Contributions of collective excitations to the binding energy of nuclear matter dominated by pion-exchange induced excitations?!?

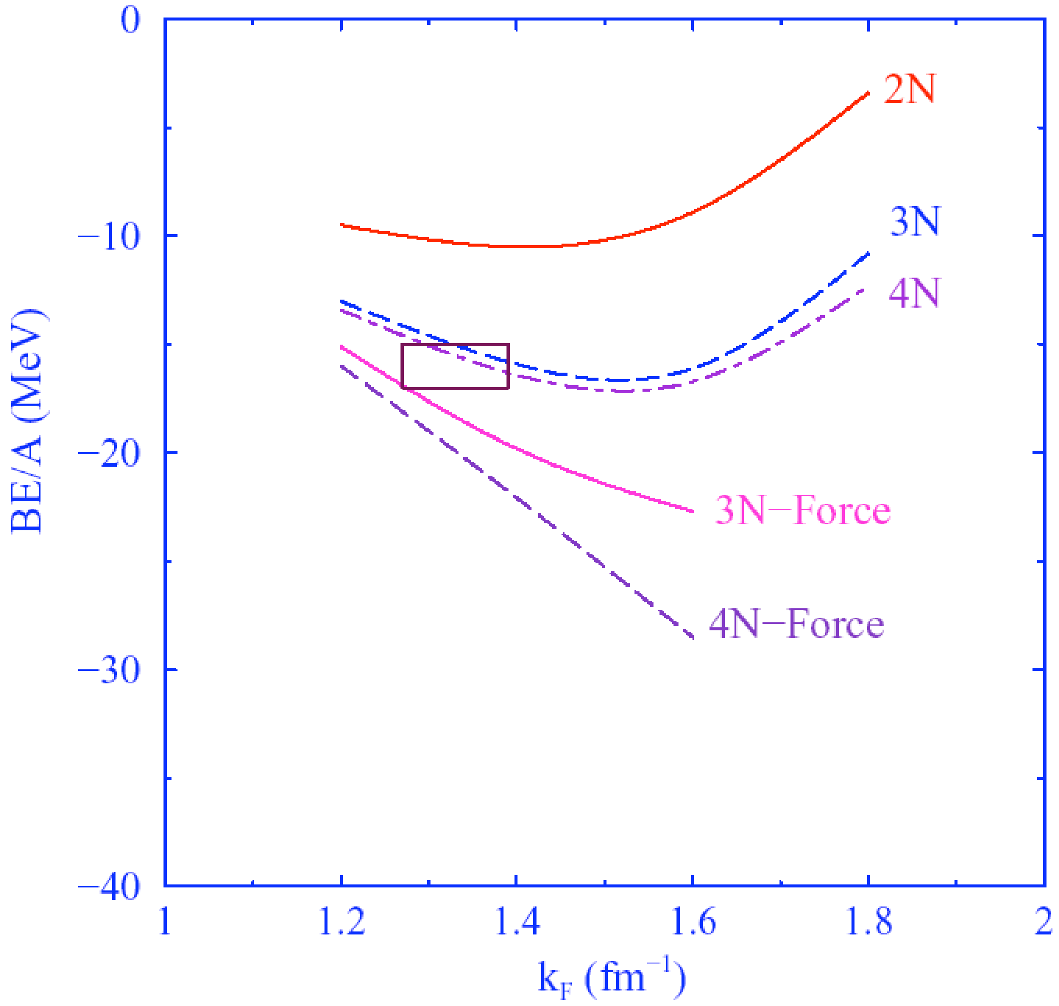
Inclusion of Δ-isobars as “3N-” and “4N-force”

Nucl. Phys. **A389**, 492 (1982)



k_F [fm^{-1}]	1.0	1.2	1.4	1.6
third order				
a)	-0.303	-1.269	-3.019	-5.384
b)	-0.654	-1.506	-2.932	-5.038
c)	-0.047	-0.164	-0.484	-1.175
d)	0.033	0.095	0.220	0.447
e)	-0.104	-0.264	-0.589	-1.187
f)	0.041	0.137	0.385	0.962
Sum	-1.034	-2.971	-6.419	-11.375

Inclusion of Δ-isobars as 3N- and 4N-force



2N,3N, and 4N from
B.D.Day, PRC24,1203(81)

Rings with Δ-isobars :

Nucl. Phys. **A389**, 492 (1982)

PPNPhys **12**, 529 (1983)

⇒ No sensible convergence with Δ-isobars

Nuclear Saturation without π-collectivity

- Variational calculations treat LRC (on average) and SRC simultaneously (Parquet equivalence) so **difficult** to separate LRC and SRC
- Remove 3-body ring diagram from Catania hole-line expansion calculation ⇒ about the correct saturation density
- Hole-line expansion doesn’t treat Pauli principle very well
- Present results improve treatment of Pauli principle by self-consistency of spectral functions => more reasonable saturation density and binding energy acceptable

- Neutron matter: pionic contributions must be included (Δ)

Pion collectivity: nuclei vs. nuclear matter

- Pion collectivity leads to pion condensation at higher density in nuclear matter (including Δ-isobars) => Migdal ...
- No such collectivity observed in nuclei ⇒ LAMPF / Osaka data

- Momentum conservation in nuclear matter dramatically favors amplification of π-exchange interaction at fixed *q*
- In nuclei the same interaction is sampled over all momenta Phys. Lett. **B146**, 1(1984)

$$V_\pi(q) = -\frac{f_\pi^2}{m_\pi^2}\,\frac{q^2}{m_\pi^2 + q^2}$$

Needs further study

⇒ **Exclude collective pionic contributions to nuclear matter binding energy**

Two Nuclear Matter Problems

The usual one

- With π-collectivity and only nucleons
- Variational + CBF and

  three hole-line results presumed OK (for *E*/*A*) but not directly relevant for comparison with nuclei!
- **NOT OK** if Δ-isobars are included
- Relevant for neutron matter

The relevant one?!

- Without π-collectivity
- Treat only SRC
- But at a sophisticated level by using self-consistency
- Confirmation from Ghent results ⇒ Phys. Rev. Lett. **90**, 152501 (2003)
- 3N-forces difficult ⇒ π ...
- Relativity?

Comments

Relativity

- Saturation depends on NNσ-coupling in medium but underlying correlated two-pion exchange behaves differently in medium
- $m^* \rightarrow 0$ with increasing ρ opposite in liquid ^{3}He

  appears unphysical
- Dirac sea under control?
- sp strength overestimated too many nucleons for $k<k_F$

Three-body forces

- Microscopic models yield only attraction in matter and more so with increasing ρ
- Microscopic background of phenomenological repulsion in 3N-force (if it exists)?
- 4N-, etc. forces yield increasing attraction with ρ
- Needed in light nuclei and attractive!
- Mediated by π-exchange
- Argonne group can't get nuclear matter right with new 3N-force

Conclusions

- Good understanding of role of short-range correlations
  - Depletion of Fermi sea: nuclear matter OK for nuclei
    - Confirmed by experiment
  - High-momentum components
    - # of protons experimentally confirmed
- Long-range correlations crucial for distribution of sp strength

- Energy per particle from self-consistent Green's functions

- Better understanding of nuclear matter saturation
  ⇒ SRC dominate (don't treat LRC from pions)

- We know what protons are up to in nuclei!!