NSCL 7/20/05

Spectroscopic factors and the physics of the single-particle strength distribution in nuclei

Lecture 1: 7/18/05 Propagator description of single-particle motion and the link with experimental data

Lecture 2: 7/19/05 From diagrams to Hartree-Fock and spectroscopic factors < 1

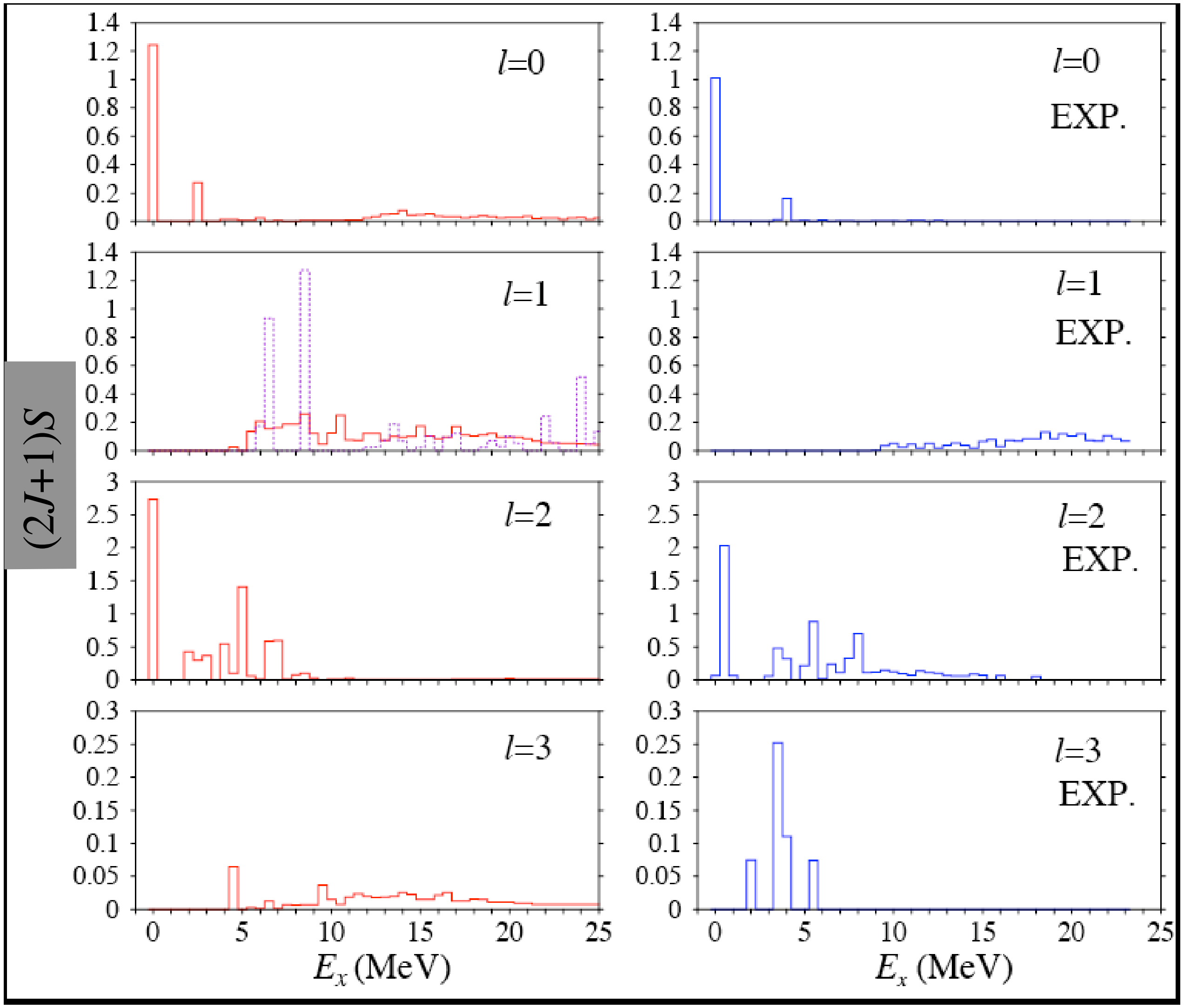
Lecture 3: 7/20/05 Influence of long-range correlations and the relation to excited states

Lecture 4: 7/21/05 Role of short-range and tensor correlations associated with realistic interactions. Prospects for nuclei with N very different from Z.

Lecture 5: 7/22/05 Saturation problem of nuclear matter

Wim Dickhoff
Washington University in St. Louis

Self-consistent 2nd order calculation with Skyrme force

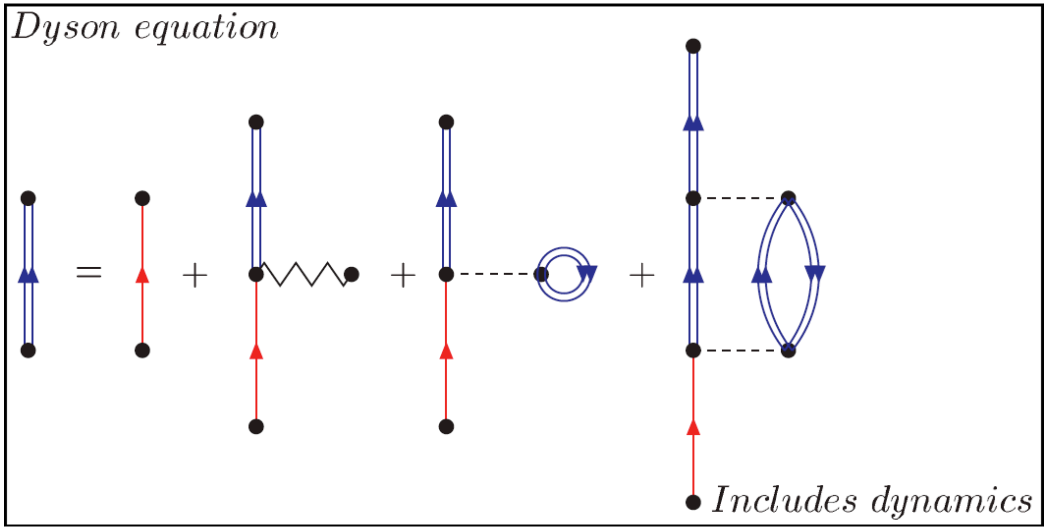


Data: ^{48}Ca(e,e´p)
Kramer NIKHEF
(1990)

Qualitatively OK
No relation with
realistic *V* yet!

Van Neck *et al*. NPA**530**,347(1991)

Self-consistent Green's functions and the energy of the ground state of atoms



Dyson(2)

Van Neck, Peirs,Waroquier
J. Chem. Phys. **115**, 15 (2001)
Dahlen & von Barth
J. Chem. Phys. **120**, 6826 (2004)

Atoms : total ground state energies (a.u.)

Method	He	Be	Ne	Mg	Ar
DFT	-2.913	-14.671	-128.951	-200.093	-527.553
HF	-2.862	-14.573	-128.549	-199.617	-526.826
CI	-2.891	-14.617	-128.733	-199.635	-526.807
Dyson(2)	-2.899	-14.647	-128.939	-200.027	-527.511
Exp.	-2.904	-14.667	-128.928	-200.043	-527.549

Outline

- Self-energy using "*G*-matrix" in second order
- Qualitative features; missing ingredients!
- Excited states and *G* ⇔ *G* and excited states
- Conserving approximations; HF ⇔ RPA *e.g.*
- E(xtended) RPA & results (Giant Resonances)
- Collective excitations in the self-energy
- Influence of "long-range"correlations
- Recent developments (Faddeev summation)
- Why does (e,e´p) yield "absolute" spectroscopic factors

How to proceed from a realistic *V*?

Must take effects of short-range and tensor correlations into account. Well known procedure: from *V* to "*G*"-matrix.

$$\langle \alpha\beta | G(E) | \gamma\delta \rangle = \langle \alpha\beta | V | \gamma\delta \rangle + \frac{1}{2} \sum_{\sigma\tau} \langle \alpha\beta | V | \sigma\tau \rangle \frac{\theta(\sigma - M)\theta(\tau - M)}{E - \varepsilon_\sigma - \varepsilon_\tau} \langle \sigma\tau | G(E) | \gamma\delta \rangle$$

Well-behaved; takes excitations outside configuration space *M* into account. Used inside *M* $\Rightarrow$ therefore this procedure doesn't **yet** completely include the effect of short-range and tensor correlations on sp motion.

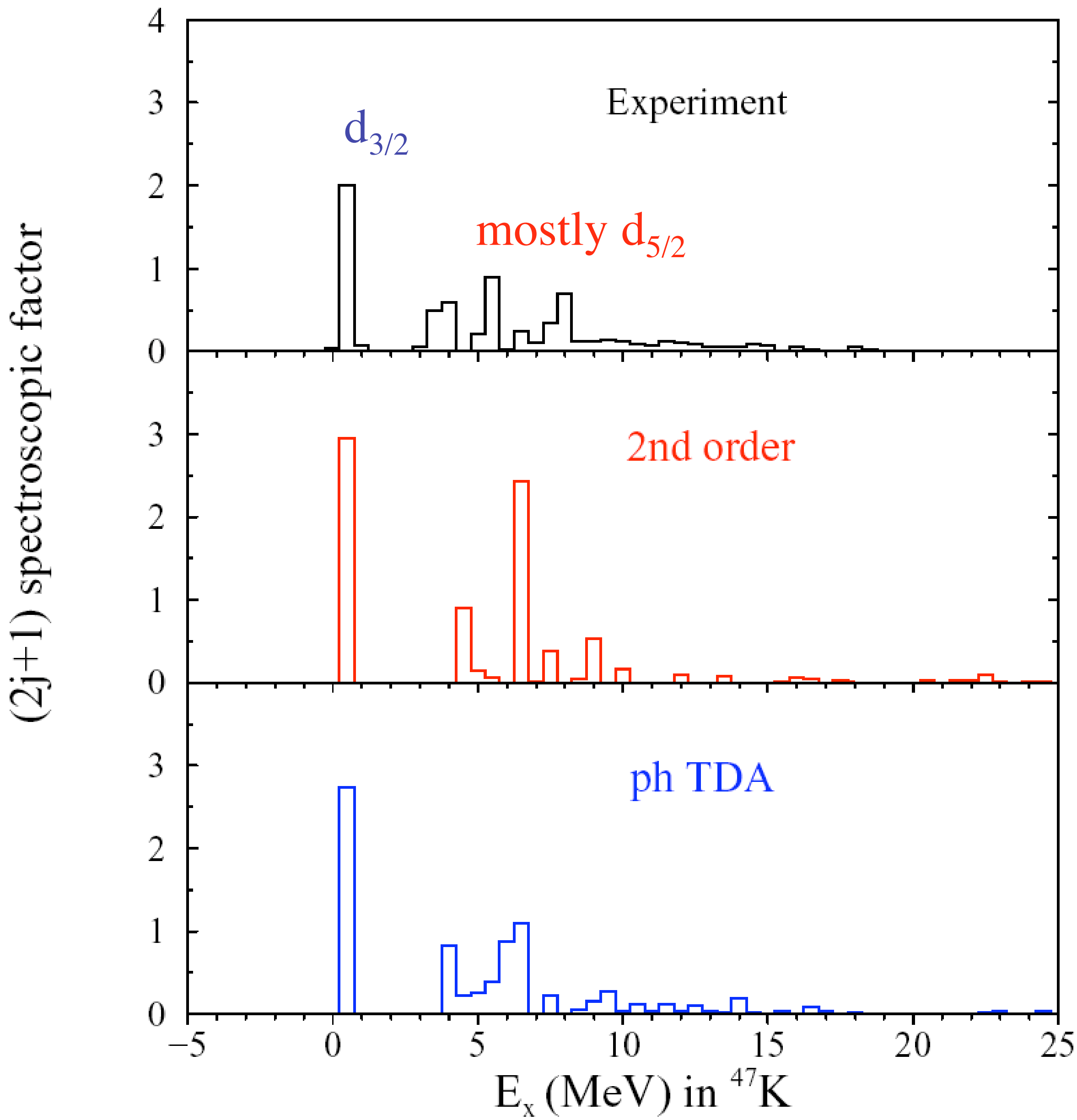
Neglect energy dependence of *G* then

$$\Sigma^{(2)}(\gamma,\delta;E) = \frac{1}{2}\left\{ \sum_{p_1 p_2 h_3} \frac{\langle \gamma h_3 | G | p_1 p_2 \rangle \langle p_1 p_2 | G | \delta h_3 \rangle}{E - (\varepsilon_{p_1} + \varepsilon_{p_2} - \varepsilon_{h_3}) + i\eta} + \sum_{h_1 h_2 p_3} \frac{\langle \gamma p_3 | G | h_1 h_2 \rangle \langle h_1 h_2 | G | \delta p_3 \rangle}{E - (\varepsilon_{h_1} + \varepsilon_{h_2} - \varepsilon_{p_3}) - i\eta} \right\}$$

Summations only inside *M*!

Spectral function ^{48}Ca (e,e´p) ^{47}K (ℓ=2)

NIKHEF data
G. Kramer, Thesis



Brand *et al*.
Nucl. Phys. **A531**, 253 (1991).
Rijsdijk *et al*.
Nucl.Phys. **A550**, 159 (1992)

Configuration space: includes three major shells above ε_F

Distribution of fragments ± 100 MeV around ε_F

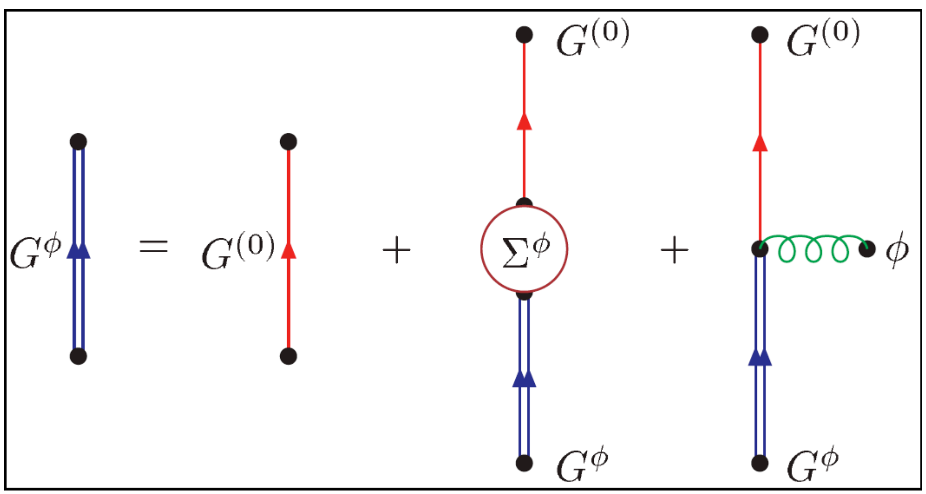
G-matrix strong enough to distribute strength in this interval

Excited states and G …
G and excited states …

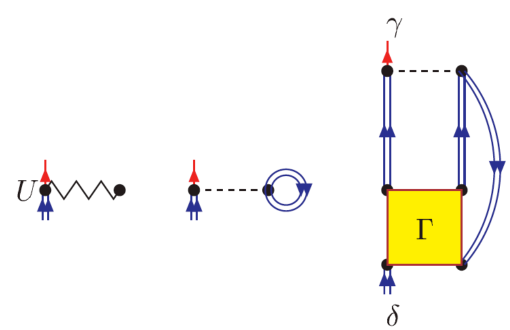
Before improving self-energy with a better description of the intermediate $2p1h$ and $2h1p$ states, it is instructive to clarify the deep relation between excited states and the sp propagator G.
$\Rightarrow$ Study time-dependent external fields that can probe excited states

$$\hat{\phi}(t) = \sum_{\gamma\delta} \langle \gamma | \phi(\vec{x},t) | \delta \rangle a_\gamma^+ a_\delta$$ So Hamiltonian reads $$\hat{H}^\phi(t) = \hat{H} + \hat{\phi}(t)$$

Equations of motion as before



Σ^ϕ as before with $G \Rightarrow G^\phi$



Conserving approximations
(Baym, Kadanoff, Pitaevskii, Ward)

Conservation laws implied by the Hamiltonian are fulfilled by imposing certain conditions on the approximate self-energy and, consequently, the vertex function Γ: in particular the issue of self-consistency is critical!

⇒ particle number, momentum, energy, …conservation

⇒ study consequences for the description of excited states

Write $G^{\phi}\left(\alpha,\overline{\beta},t-t'\right)=-\frac{i}{\hbar}\frac{\langle\Psi_0|T\left[\hat{S}a_{\alpha_F}(t)a^{+}_{\overline{\beta}_F}(t')\right]|\Psi_0\rangle}{\langle\Psi_0|T\left[\hat{S}\right]|\Psi_0\rangle}$ as an expansion in ϕ

In linear response (lowest order in ϕ):

Functional derivative of G^{ϕ} yields $\frac{\delta G^{\phi}\left(\alpha,\overline{\beta},t-t'\right)}{\delta\phi_{\gamma\overline{\delta}}\left(t''\right)}=\frac{i}{\hbar}\Pi\left(\alpha t,\beta^{-1}t';\gamma t'',\delta^{-1}t''\right)$

corresponding to the *ph* limit of the two-particle propagator.

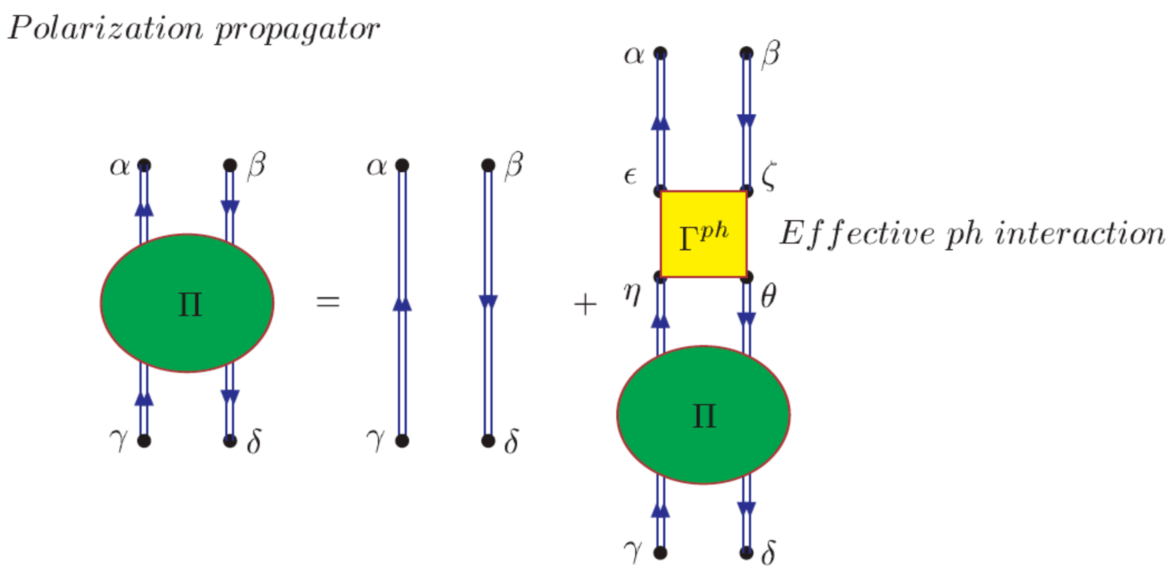
Conserving description of excited states

Fourier transform of two-time "polarization" propagator

$$\Pi\left(\alpha,\beta^{-1};\gamma,\delta^{-1};E\right)=\sum_{n\neq 0}\frac{\langle\Psi_0|a_{\bar\beta}^{+}a_{\alpha}|\Psi_n\rangle\langle\Psi_n|a_{\gamma}^{+}a_{\bar\delta}|\Psi_0\rangle}{E-\left(E_n-E_0\right)+i\eta}-\sum_{n\neq 0}\frac{\langle\Psi_0|a_{\gamma}^{+}a_{\bar\delta}|\Psi_n\rangle\langle\Psi_n|a_{\bar\beta}^{+}a_{\alpha}|\Psi_0\rangle}{E+\left(E_n-E_0\right)-i\eta}$$

contains all relevant information about excited states (location and one-body transition strength).

Integral equation for three-time polarization propagator from Dyson equation!



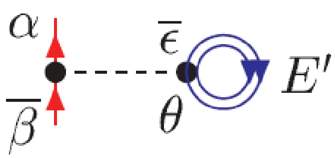
Propagators are dressed according to approximation (must be self-consistent)

Particle-hole interaction

$$\Gamma^{ph}\left(\alpha t_1, \beta^{-1} t_2, \gamma t_3, \delta t_4\right) = \frac{\delta\Sigma\left(\alpha, \overline{\beta}; t_1 - t_2\right)}{\delta G\left(\gamma, \overline{\delta}; t_3 - t_4\right)}$$

If G is conserving, so is Π
with this Γ^{ph}
Looks complicated … but …

Hartree-Fock and RPA

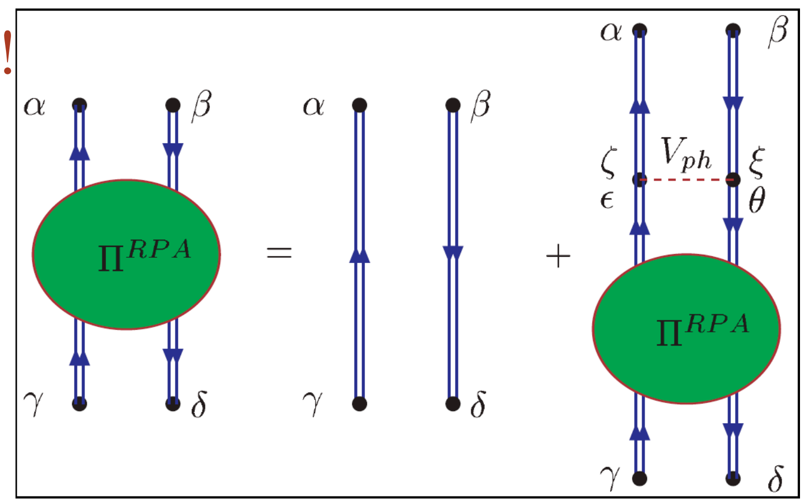


$$\Rightarrow \quad -i\hbar\delta(t-t') \sum_{\theta\overline{\epsilon}} \langle \alpha\overline{\epsilon}| V |\overline{\beta}\theta\rangle G^{HF}(\theta, \overline{\epsilon}; t - t^+)$$

Functional derivative equivalent to breaking internal propagator line so

$$\Gamma^{ph}_{HF}\left(\alpha t_1, \beta^{-1} t_2, \gamma t_3, \delta^{-1} t_4\right) = -i\hbar\delta\left(t_1 - t_2\right)\delta\left(t_1 - t_3\right)\delta\left(t_1 - t_4\right)\left\langle \alpha\overline{\delta}\middle|V\middle|\overline{\beta}\gamma\right\rangle$$

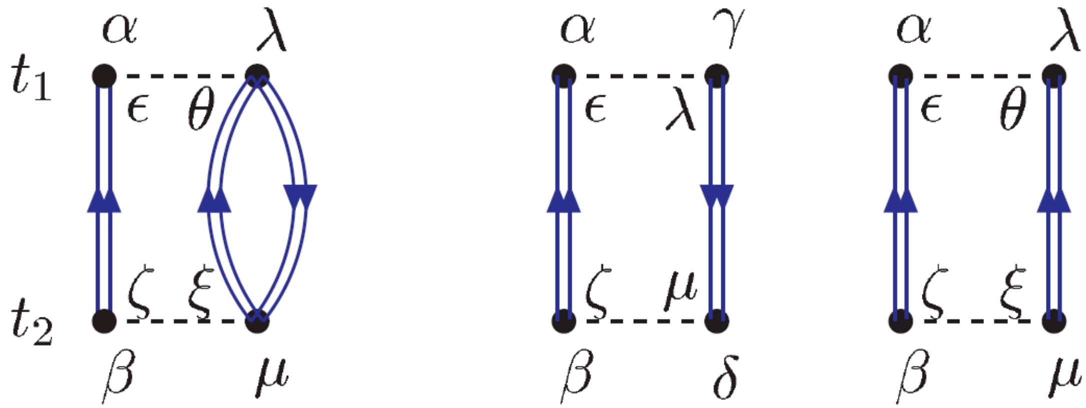
resulting in the RPA approximation to Π!!



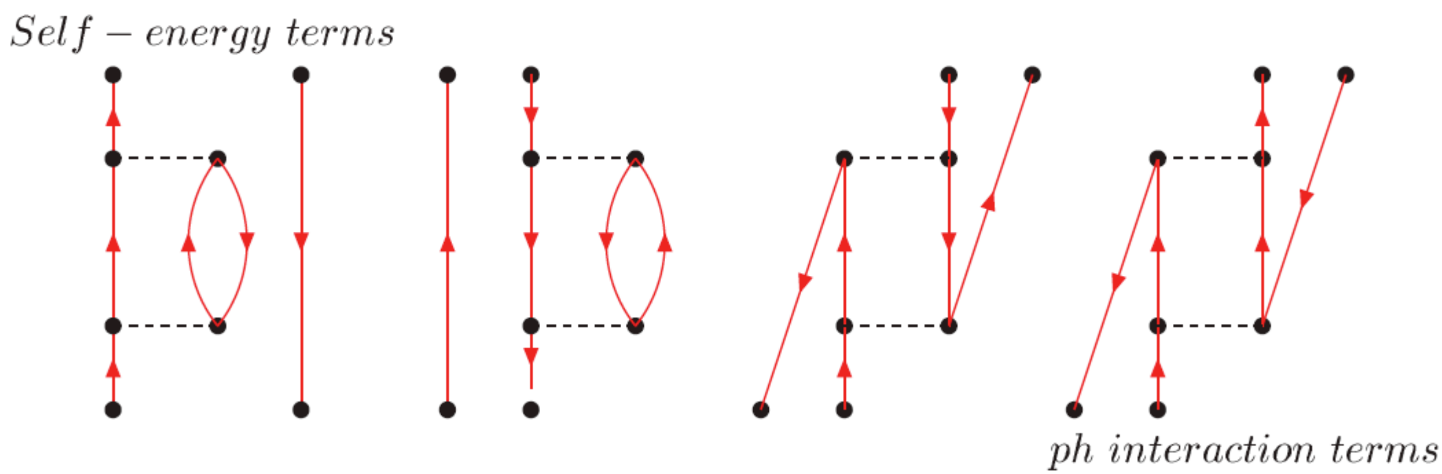
sp propagators $\Rightarrow$ HF

Beyond RPA = beyond mean-field for G

Second-order self-energy

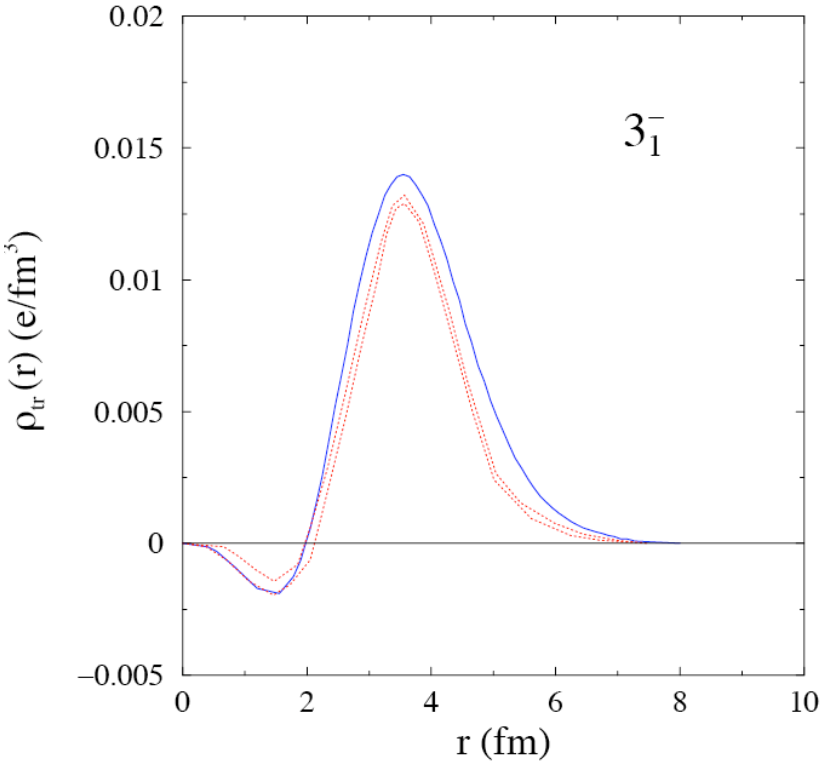


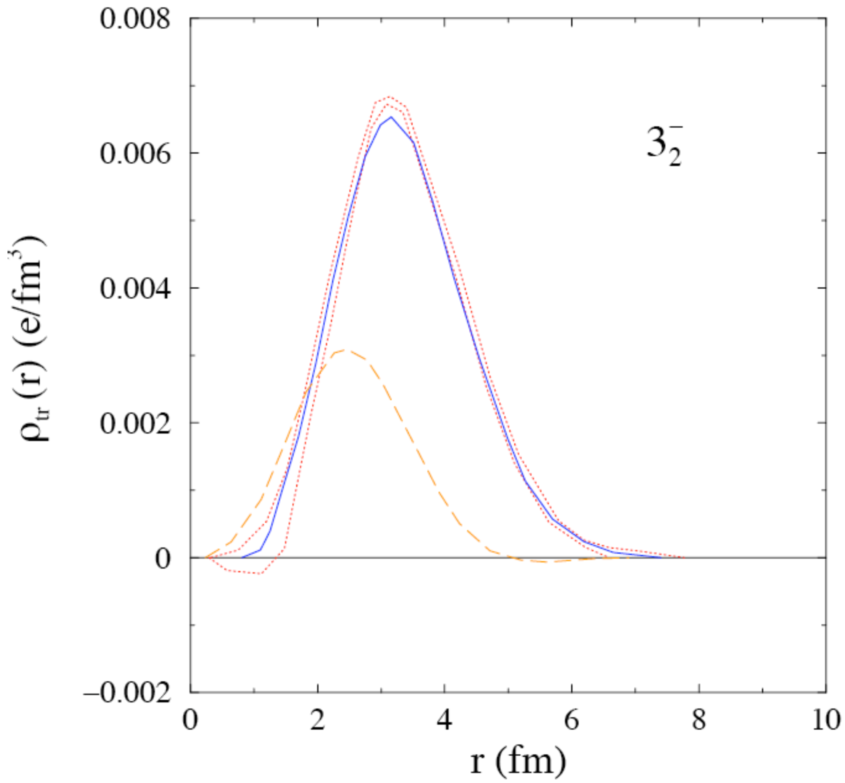
Leads to a consistent coupling of $1p1h$ to $2p2h$ configurations!



E(xtended)RPA results

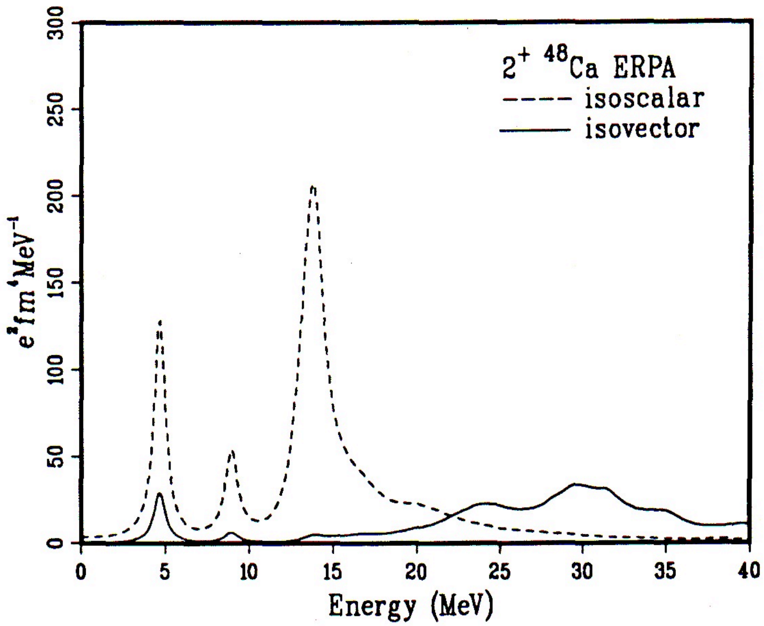
Transition densities in ^{48}Ca





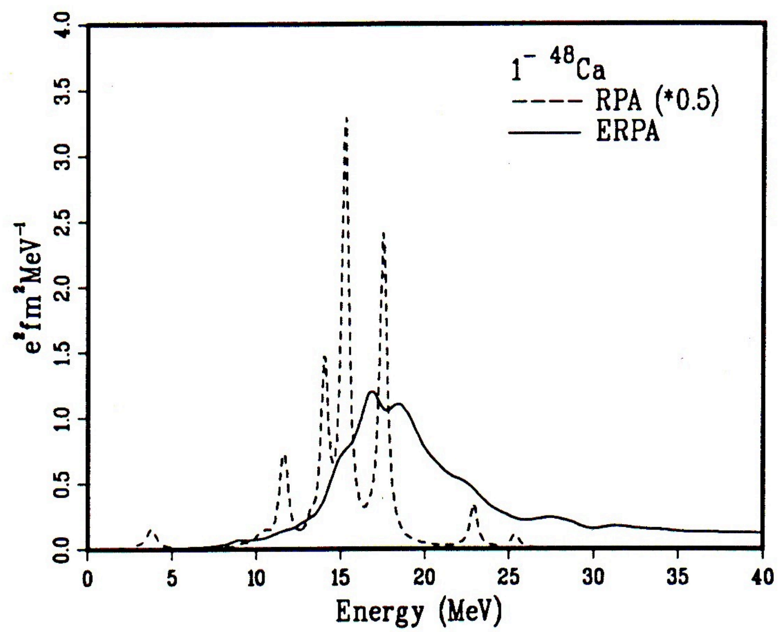
Brand *et al*. Nucl.Phys.A509, 1 (1990)

Giant Quadrupole



Giant Resonances only correct when sp fragmentation is included!

Giant Dipole



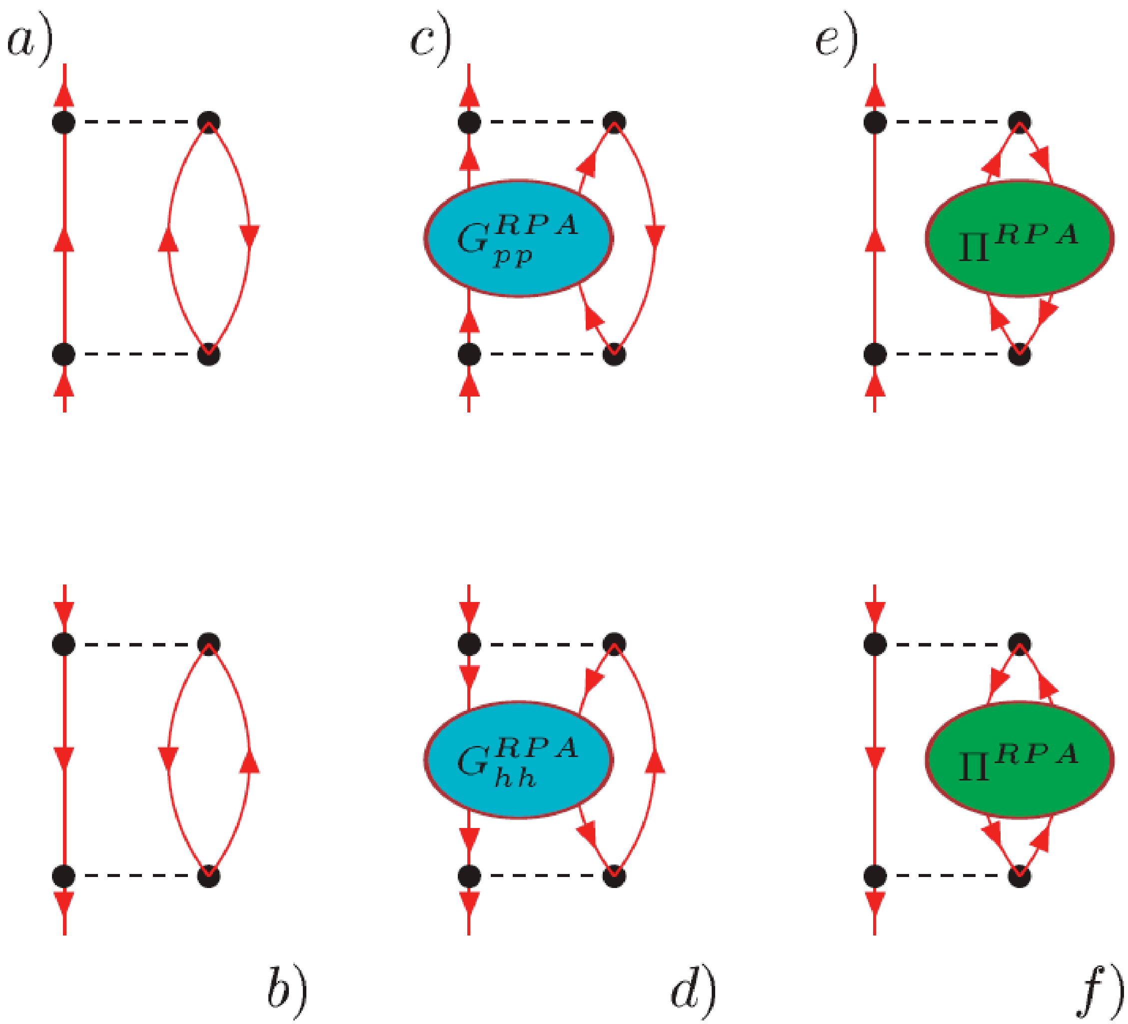
In turn:

Excited states

determine sp fragmentation

M. G. E. Brand, K. Allaart, and W. D.
Phys. Lett. **214B** , 483 (1988);
Nucl. Phys. **A509** , 1 (1990).

Long-range correlations ⇒ typical self-energy contributions



TDA or RPA

Link with excited states at low energy and their collective features

Results for TDA & RPA self-energies

Need:

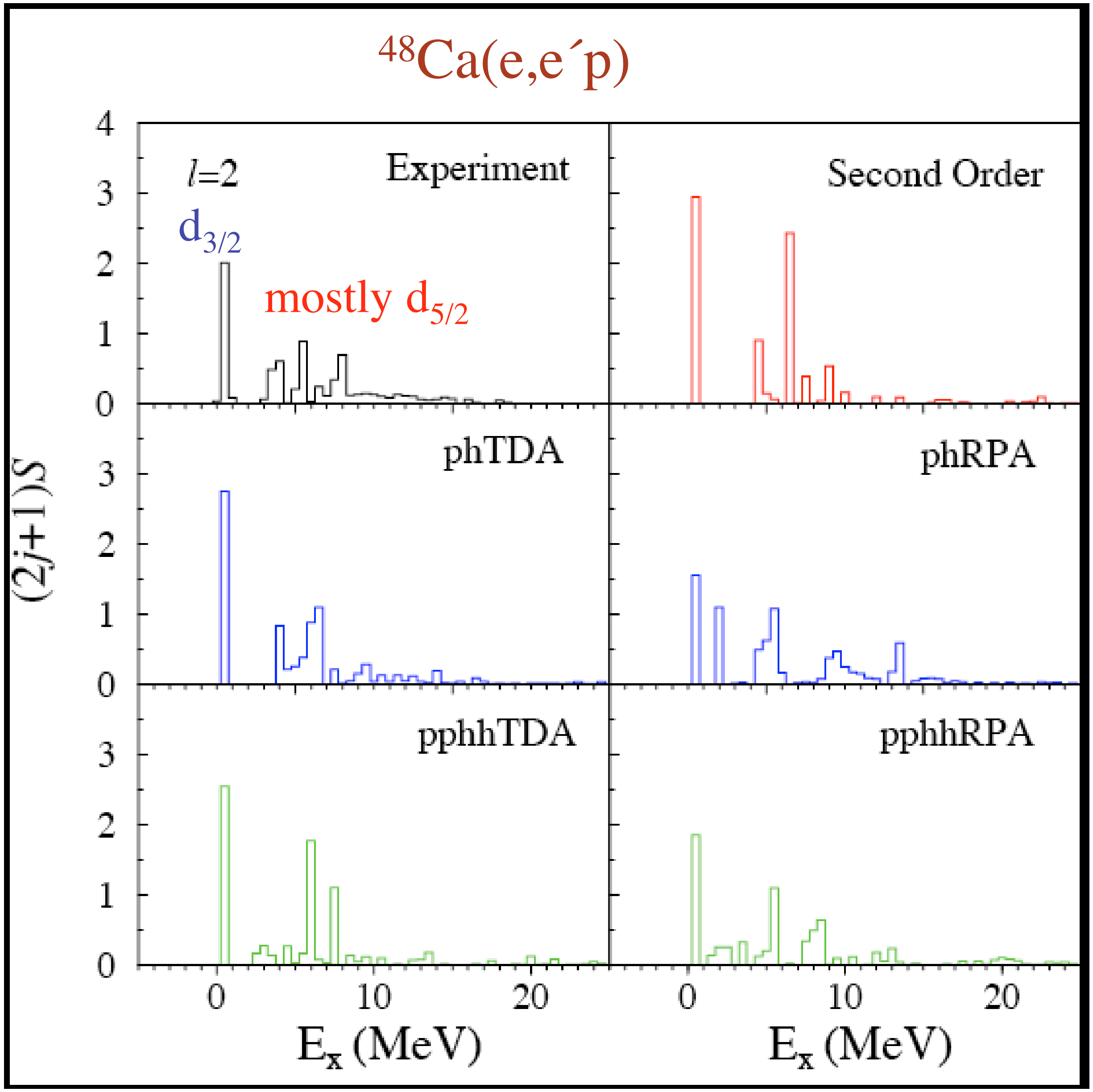
Better fragmentation at low energy
⇒ Excited states beyond RPA (unstable)

Less strength at low energy
⇒ Effect of SRC

Calculations yield:

10% strength at more bound energies!

10% depletion for states near the Fermi energy



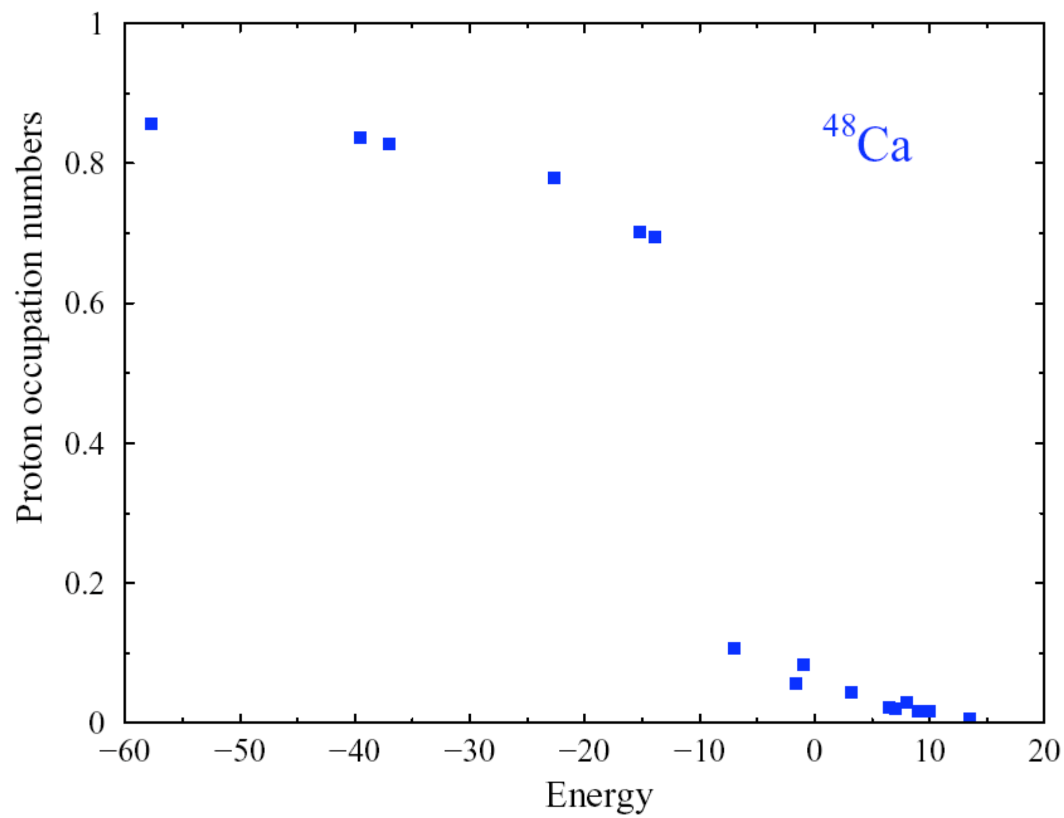
Occupation numbers ^{48}Ca

Shell	$\Sigma^{(2)}$	Σ_{ph}^{TDA}	Σ_{ph}^{RPA}	Σ_{pphh}^{TDA}	Σ_{pphh}^{RPA}
$0s\frac{1}{2}$	.967	.968	.963	.965	.952
$0p\frac{3}{2}$	.955	.956	.944	.950	.930
$0p\frac{1}{2}$	.951	.951	.939	.944	.920
$0d\frac{5}{2}$	.920	.925	.915	.898	.867
$0d\frac{3}{2}$	.877	.885	.891	.842	.780
$1s\frac{1}{2}$	.869	.860	.907	.818	.773
$0f\frac{7}{2}$	.060	.063	.048	.082	.120
$0f\frac{5}{2}$	.048	.044	.043	.064	.092
$1p\frac{3}{2}$	.033	.031	.036	.049	.063
$1p\frac{1}{2}$	.030	.028	.035	.042	.050
$0g\ 1d\ 2s$	.014	.014	.019	.018	.026
$0h\ 1f\ 2p$	.006	.006	.006	.007	.009
Total	20.053	20.093	20.125	20.165	20.370

Occupation numbers from low-energy correlations

Shell	$n(\alpha)$
$0s_{1/2}$	0.968
$0p_{3/2}$	0.956
$0p_{1/2}$	0.951
$0d_{5/2}$	0.925
$0d_{3/2}$	0.885
$1s_{1/2}$	0.860
$0f_{7/2}$	0.063
$0f_{5/2}$	0.044
$0p_{3/2}$	0.031
$0p_{1/2}$	0.028
….	

Including SRC depletion effect by treating energy dependence of *G*-matrix



Spectroscopic Strength in ^{16}O

Data: PRC49, 955 (94)

- Influence of SRC ✓✓
- Translational Invariance ✖
- Influence of LRC "✓"

  TDA for $2p1h$ and $2h1p$

  Geurts et al.

  PRC**53**, 2207 (1996)

- Influence of LRC ✓✓

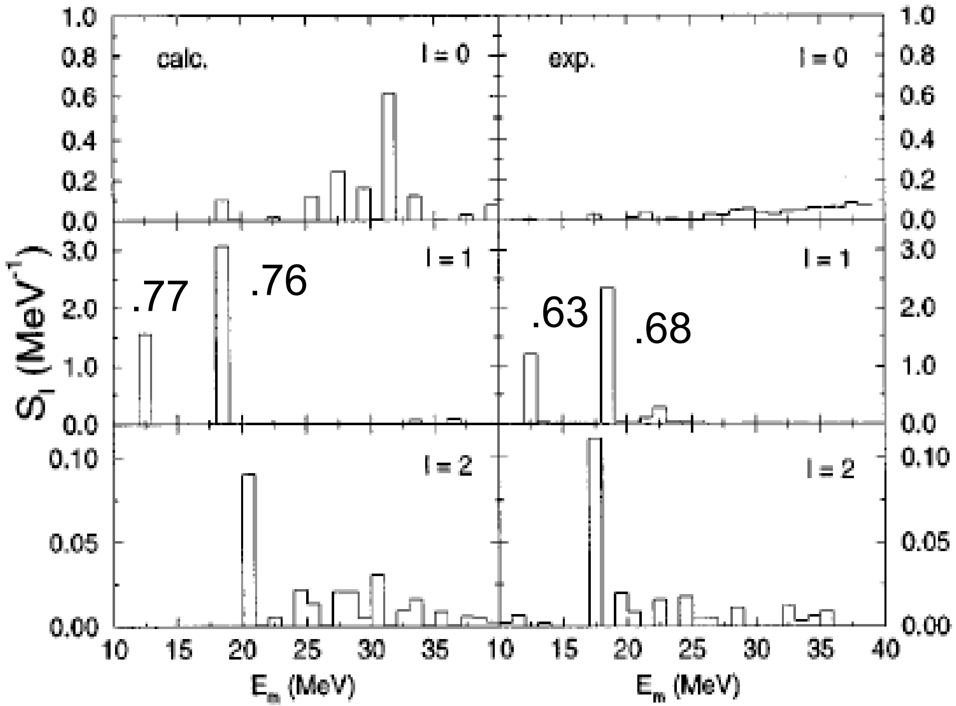
  RPA + Faddeev

  C. Barbieri and WHD,

  PRC**65**, 064313 (2002)

Still not solved because RPA is not good enough for ^{16}O!!

TDA



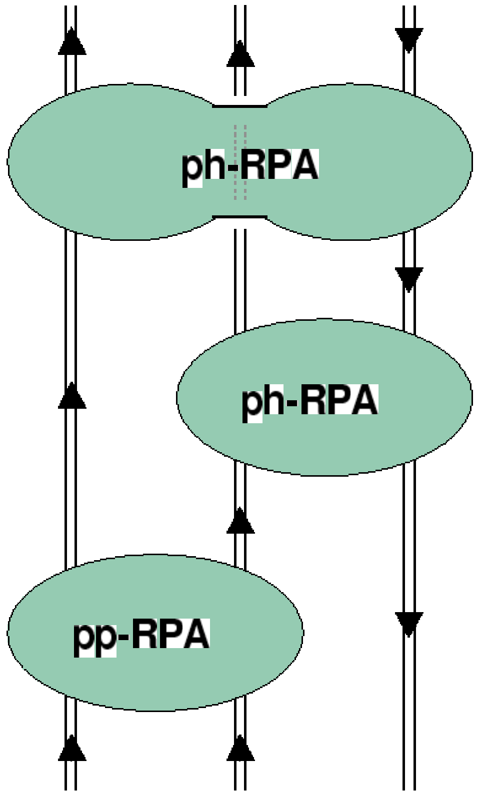
Shell	TDA	RPA
$d_{3/2}$	0.866	0.838
$s_{1/2}$	0.882	0.842
$d_{5/2}$	0.894	0.875
$p_{1/2}$	0.775	0.745
$p_{3/2}$	0.766	0.725

Faddeev technique and Long-Range Correlations

- Both pp (hh) and ph phonons are collective in nuclei using RPA
- Faddeev technique allows correct summation to all orders of these phonons
- Formalism:

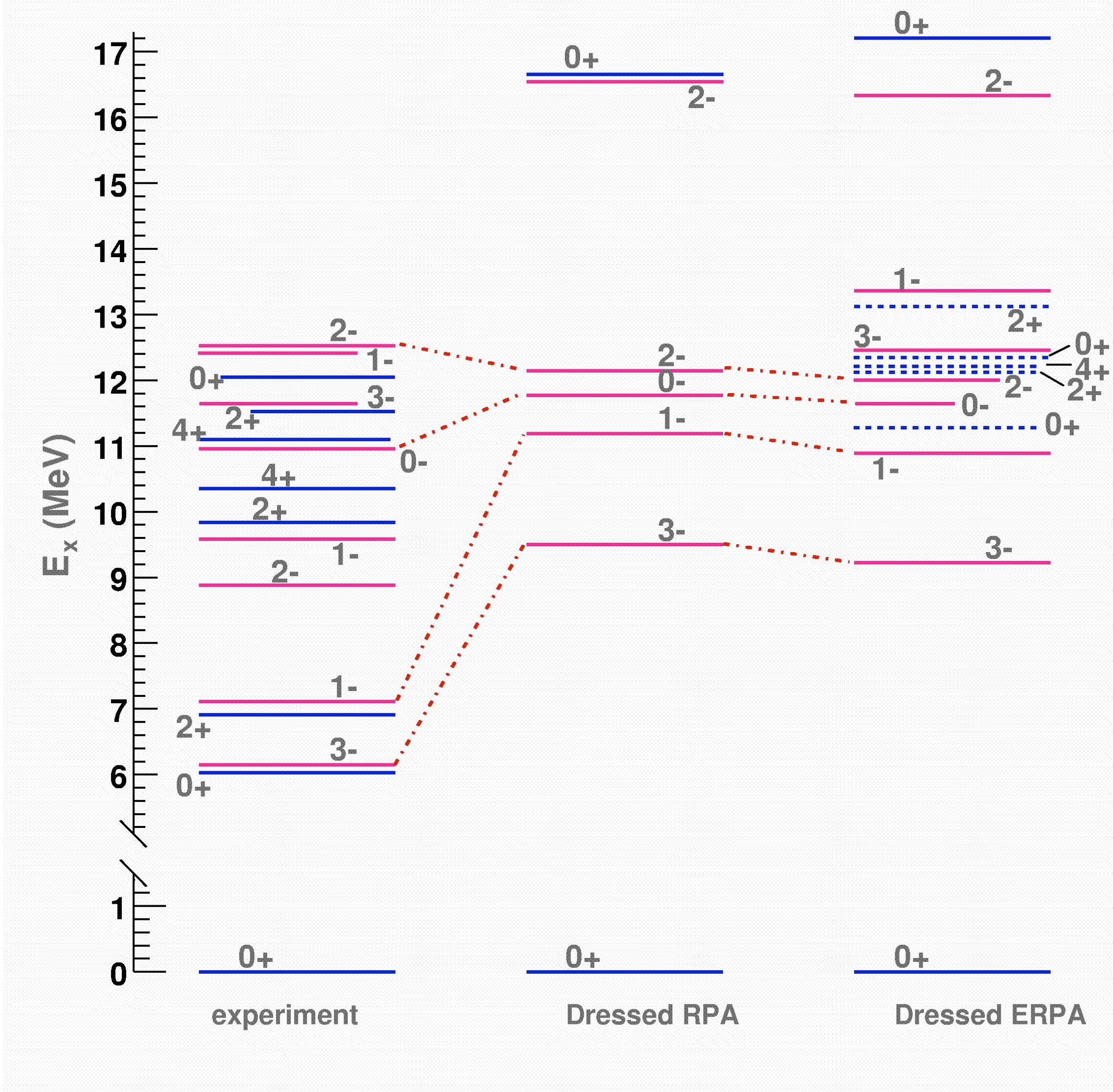
  Phys. Rev. C**63**, 034313 (2001)
- Results: for ^{16}O

  Phys. Rev. C**65,** 064313 (2002)



^{16}O spectrum

ERPA with RPA phonons
includes coupling to 2 phonon states



Long-range correlations
in nuclei are HARD
to calculate for lighter
systems!!!

C. Barbieri & WHD
Phys. Rev. C68, 014311 (2003)

Need to do better!

Relevant for (e,e´p)
and (e,e´2N)

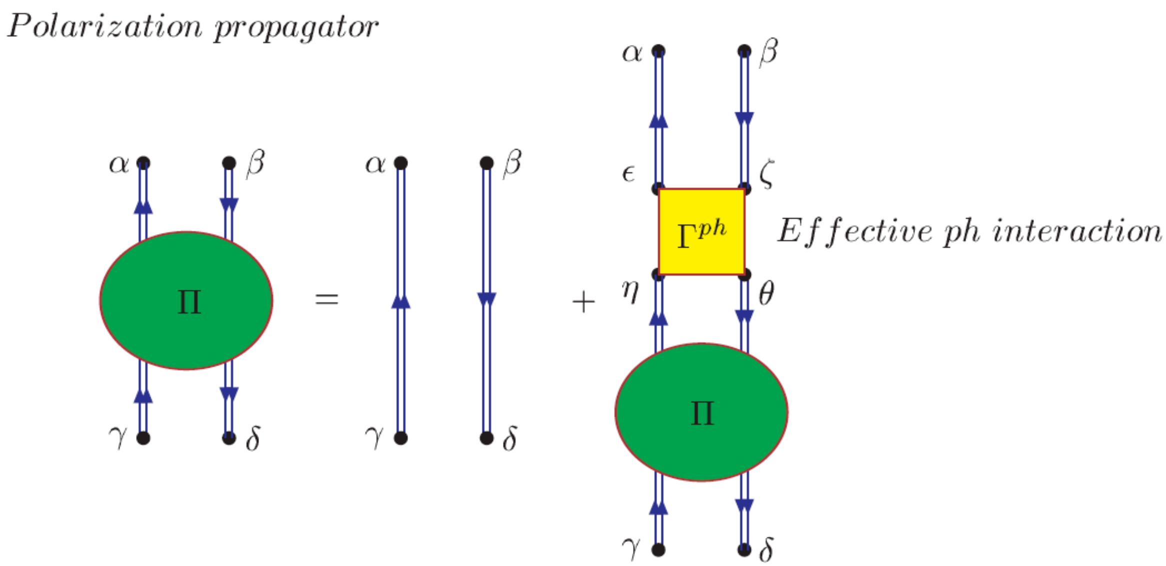
FSI and (e,e´p) ⇔ analysis

$$\hat{O} = \sum_{\alpha\beta} \langle \alpha | O | \beta \rangle a_\alpha^+ a_\beta$$

Electron Scattering ⇒ one-body operator

$$\left| \langle \Psi_n^A | \hat{O} | \Psi_0^A \rangle \right|^2 = \sum \langle \alpha | O | \beta \rangle^* \langle \gamma | O | \delta \rangle \langle \Psi_0^A | a_\beta^+ a_\alpha | \Psi_n^A \rangle \langle \Psi_n^A | a_\gamma^+ a_\delta | \Psi_0^A \rangle$$

Requires (imaginary part of) **exact** polarization propagator



Choose kinematics:

⇒ only first term

$$\langle \Psi_m^{A+1} | a_\alpha^+ | \Psi_0^A \rangle$$

⇒ Elastic scattering (phenomenology)

$$\langle \Psi_n^{A-1} | a_\beta | \Psi_0^A \rangle$$

"Absolute" spectroscopic factors √

⇒ Quasihole wave function