NSCL 7/18/05

Spectroscopic factors and the physics of the single-particle strength distribution in nuclei

Lecture 1: 7/18/05 Propagator description of single-particle motion and the link with experimental data

Lecture 2: 7/19/05 From diagrams to Hartree-Fock and spectroscopic factors < 1

Lecture 3: 7/20/05 Influence of long-range correlations and the relation to excited states

Lecture 4: 7/21/05 Role of short-range and tensor correlations associated with realistic interactions. Prospects for nuclei with N very different from Z.

Lecture 5: 7/22/05 Saturation problem of nuclear matter

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Some questions …

What does a nucleon do in the nucleus?

Is this a legitimate question?

Speculations …

How strong is the dependence on *N* and *Z*?

Energy scales: As high as a realistic V_{NN} will take you

…

Δ-isobars, pions

…

As low as the first excited state

⇒ ALL OF THEM! HOW?

⇒ Time-dependent formulation not surprising

How?

Method:	Green’s functions (Propagators) Feynman diagrams
Why:	Physical insight and useful for all many-body systems Link between experiment and theory clear Can include all energy scales Efficient with information; generates amplitudes not wave functions
(Text)Book:	Willem H Dickhoff & Dimitri Van Neck “Many-Body Theory Exposed!” Propagator description of quantum mechanics in many-body systems World Scientific (2005)
Review:	WHD & Carlo Barbieri Prog. Part. Nucl. Phys. **52**, 377-496 (2004)

Outline

- What is a propagator
- Propagator in the many-body problem
- Information contained in propagator
- Spectral functions
- Relation with experimental data
- Experimental results
- Outline of perturbation theory

What is a propagator or Green's function?

Time evolution is governed by the Hamiltonian *H*. For a single particle the state

$$|\alpha, t_0; t\rangle = e^{-\frac{i}{\hbar}H(t-t_0)}|\alpha, t_0\rangle$$

is indeed a solution of $\quad i\hbar\frac{\partial}{\partial t}|\alpha, t_0; t\rangle = H|\alpha, t_0; t\rangle$

Relation between wave function at t and t_0 can then be written as

$$\psi(\vec{r}, t) = \langle\vec{r}|\alpha, t_0; t\rangle = \langle\vec{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\alpha, t_0\rangle = \int d\vec{r}'\langle\vec{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\vec{r}'\rangle\langle\vec{r}'|\alpha, t_0\rangle$$

$$= i\hbar\int d\vec{r}'G(\vec{r}, \vec{r}'; t - t_0)\psi(\vec{r}', t_0)$$

with the propagator or Green's function defined by

$$G(\vec{r}, \vec{r}'; t - t_0) = -\frac{i}{\hbar}\langle\vec{r}|e^{-\frac{i}{\hbar}H(t-t_0)}|\vec{r}'\rangle$$

Recall Huygens' principle!

Alternative expressions

Using $\theta(t-t_0) = -\int \frac{dE'}{2\pi i} \frac{e^{-\frac{i}{\hbar}E'(t-t_0)}}{E'+i\eta}$ and $\frac{d}{dt}\theta(t-t_0) = \delta(t-t_0)$

the Fourier transform of the propagator can be written as

$$G(\vec{r},\vec{r}\,';E) = \int_{-\infty}^{\infty} d(t-t_0) e^{\frac{i}{\hbar}E(t-t_0)} G(\vec{r},\vec{r}\,';t-t_0)$$

$$= \sum_n \frac{\langle 0|a_{\vec{r}}|n\rangle\langle n|a^{+}_{\vec{r}\,'}|0\rangle}{E-\varepsilon_n+i\eta}$$

$$= \langle 0|a_{\vec{r}} \frac{1}{E-H+i\eta} a^{+}_{\vec{r}\,'}|0\rangle$$

with $H|n\rangle = \varepsilon_n|n\rangle$

Also $\langle 0|a_{\vec{r}}|n\rangle = \langle \vec{r}\,|n\rangle = u_n(\vec{r})$

So numerator yields information on wave functions and denominator on eigenvalues of *H*.

How is G calculated?

"Simple" for the case of one particle. Can proceed by splitting

$H = H_0 + V$ and using the operator identity $\dfrac{1}{A-B} = \dfrac{1}{A} + \dfrac{1}{A} B \dfrac{1}{A-B}$

for the operator $G = \dfrac{1}{E - H + i\eta}$ with $A = E - H_0 + i\eta$

and $B = V$ to obtain G in terms of $G^{(0)}$ and V:

$$
\begin{aligned}
G &= G^{(0)} + G^{(0)} V G \\
&= G^{(0)} + G^{(())} V G^{(0)} + G^{(0)} V G^{(0)} V G^{(0)} + \cdots
\end{aligned}
$$

or in a particular basis

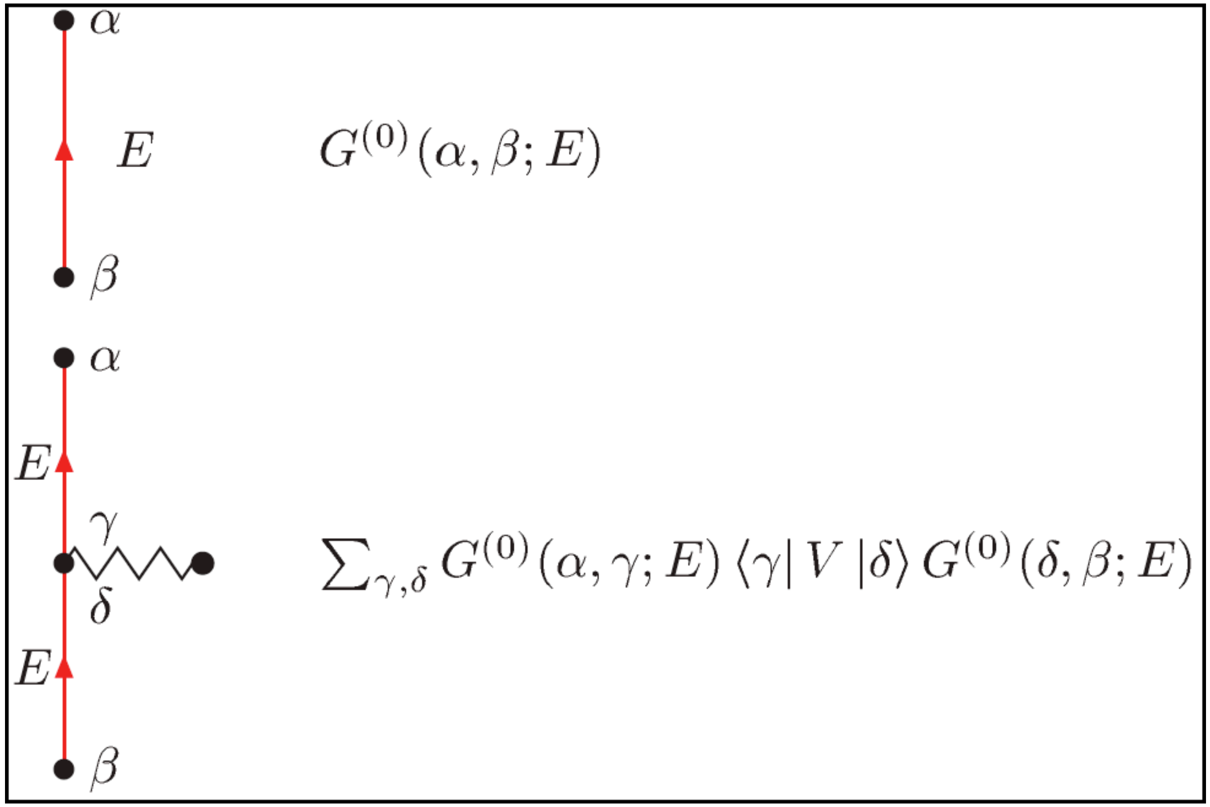
$$G(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma\delta} G^{(0)}(\alpha,\gamma;E)\langle\gamma|V|\delta\rangle G(\delta,\beta;E)$$

with $G(\alpha,\beta;E) = \langle\alpha|\dfrac{1}{E - H + i\eta}|\beta\rangle$

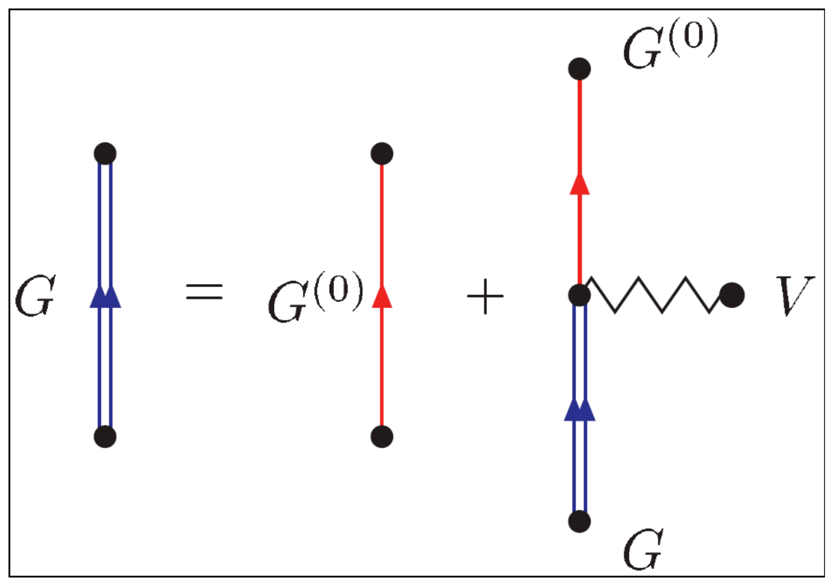
Diagrams

Lowest order

First order



All orders summed by



Single-particle propagator in the medium

Definition $$G(\alpha,\beta;t-t') = -\frac{i}{\hbar}\left\langle \Psi_0^N \right| T\left[a_{\alpha_H}(t) a_{\beta_H}^{+}(t') \right] \left| \Psi_0^N \right\rangle$$

with $$\hat{H}\left|\Psi_0^N\right\rangle = E_0^N \left|\Psi_0^N\right\rangle$$ for the exact ground state

and $$a_{\alpha_H}(t) = e^{\frac{i}{\hbar}\hat{H}t} a_\alpha e^{-\frac{i}{\hbar}\hat{H}t}$$ (Heisenberg picture)

while T orders the operators with larger time on the left including a sign change

$$G(\alpha,\beta;t-t') = -\frac{i}{\hbar}\left\{ \theta(t-t') e^{\frac{i}{\hbar}E_0^N(t-t')} \left\langle \Psi_0^N \right| a_\alpha e^{-\frac{i}{\hbar}\hat{H}(t-t')} a_\beta^{+} \left| \Psi_0^N \right\rangle \right.$$ particle

$$\left. - \theta(t-t') e^{\frac{i}{\hbar}E_0^N(t'-t)} \left\langle \Psi_0^N \right| a_\beta^{+} e^{-\frac{i}{\hbar}\hat{H}(t'-t)} a_\alpha \left| \Psi_0^N \right\rangle \right\}$$ hole

Fourier transform of G (Lehmann representation)

$$G(\alpha,\beta;E) = \sum_m \frac{\langle \Psi_0^N | a_\alpha | \Psi_m^{N+1} \rangle \langle \Psi_m^{N+1} | a_\beta^+ | \Psi_0^N \rangle}{E - \left(E_m^{N+1} - E_0^N\right) + i\eta} \quad \Leftarrow \text{Particle part}$$

$$+ \sum_n \frac{\langle \Psi_0^N | a_\beta^+ | \Psi_n^{N-1} \rangle \langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle}{E - \left(E_0^N - E_n^{N-1}\right) - i\eta} \quad \Leftarrow \text{Hole part}$$

Numerator contains information about "wave functions"

$\langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle$ and $\langle \Psi_m^{N+1} | a_\beta^+ | \Psi_0^N \rangle$

while denominator identifies eigenvalues of H for the $N\pm1$ states

Note $\hat{H} | \Psi_n^{N\pm1} \rangle = E_n^{N\pm1} | \Psi_n^{N\pm1} \rangle$

has been used for exact $N \pm 1$ states of H

Spectral functions

Probability density for the removal of a particle with quantum numbers represented by α from the ground state, while leaving the remaining system at an energy $E_n^{N-1} = E_0^N - E$

$$S_h(\alpha;E) = \sum_n \left| \langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle \right|^2 \delta\left(E - \left(E_0^N - E_n^{N-1}\right)\right)$$

for energies $E \leq \varepsilon_F^- = E_0^N - E_0^{N-1}$

Relation of "hole" spectral function to propagator

$$S_h(\alpha;E) = \frac{1}{\pi} \mathrm{Im}\, G(\alpha,\alpha;E)$$ based on $$\frac{1}{x \pm i\eta} = P\frac{1}{x} \mp i\pi\delta(x)$$

Occupation number: $$n(\alpha) = \int_{-\infty}^{\varepsilon_F^-} S_h(\alpha;E)\, dE = \langle \Psi_0^N | a_\alpha^\dagger a_\alpha | \Psi_0^N \rangle$$

Relation with experimental data

Direct knockout reaction:
Transfer a large amount of momentum and energy to a bound *N*-particle system leaving an ejected fast particle and a bound *N*-1 system. By observing the momentum of the ejected particle one can reconstruct the hole spectral function.

Initial state $\left|\Psi_i\right\rangle = \left|\Psi_0^N\right\rangle$ Final state $\left|\Psi_f\right\rangle = a_{\vec{p}}^{+}\left|\Psi_n^{N-1}\right\rangle$

External probe transfers momentum $\hat{\rho}(\vec{q}) = \sum_{\vec{p}} a_{\vec{p}}^{+} a_{\vec{p}-\vec{q}}$

Transition matrix element $\left\langle\Psi_f\left|\hat{\rho}(\vec{q})\right|\Psi_i\right\rangle \approx \left\langle\Psi_n^{N-1}\left|a_{\vec{p}-\vec{q}}\right|\Psi_0^N\right\rangle$

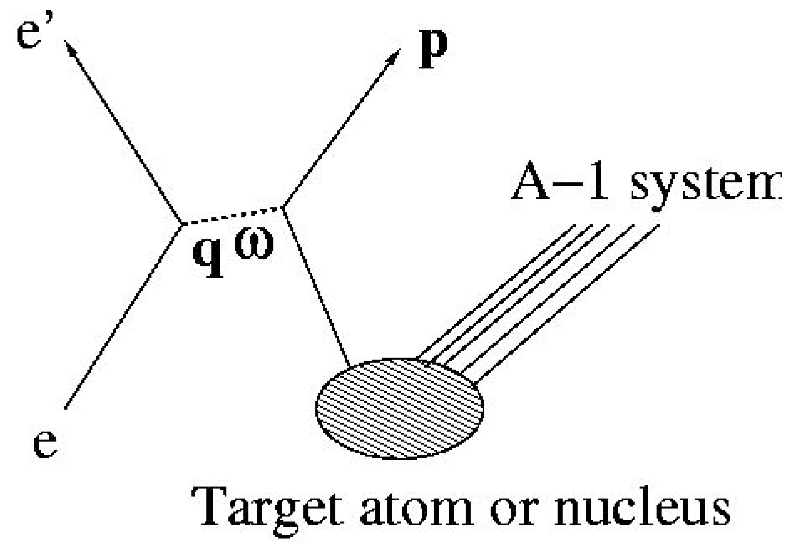
(*Plane Wave*) *Impulse Approximation* ⇒ ejected particle absorbs ***q***

Cross section from Fermi's Golden Rule

$$d\sigma \propto \sum_n \left|\left\langle\Psi_f\left|\hat{\rho}(\vec{q})\right|\Psi_i\right\rangle\right|^2 \delta\left(E + E_i - E_f\right) = S_h\left(\vec{p}_{miss}; E_{miss}\right)$$

with $\vec{p}_{miss} = \vec{p} - \vec{q}$ and $E_{miss} = \frac{\vec{p}^{\,2}}{2m} - E = E_0^N - E_n^{N-1}$

Basic idea of (e,2e) or (e,e´p)



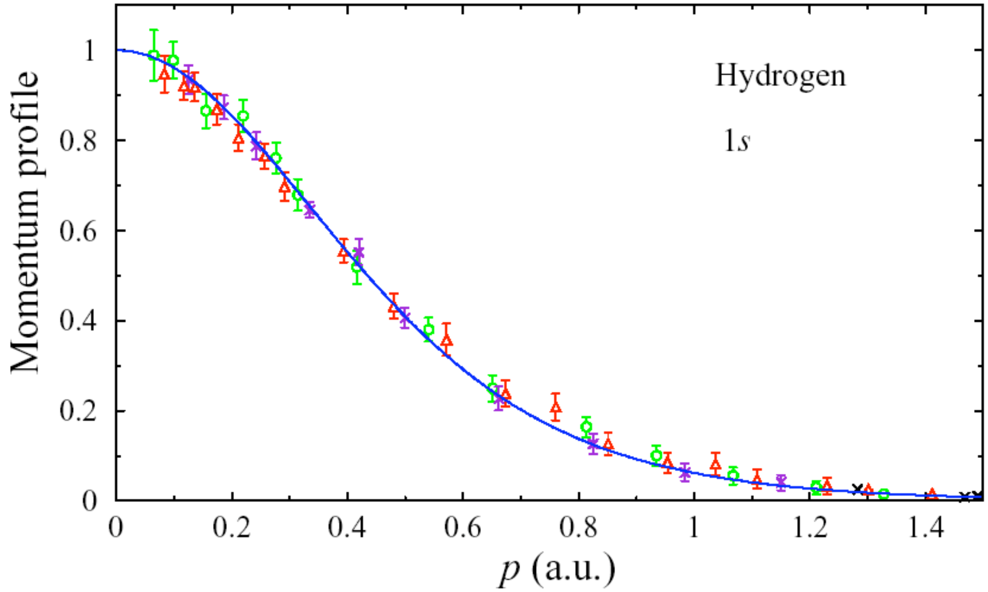
$$d\sigma_L \propto \left|\left\langle \Psi_f \left| \hat{\rho}_c(\vec{q}) \right| \Psi_i \right\rangle\right|^2 \delta(E - E_i - E_f)$$

Simplest case: $\left\langle \vec{p}, \Psi_n^{N-1} \left| \hat{\rho}_c(\vec{q}) \right| \Psi_0^N \right\rangle \Rightarrow \left\langle \Psi_n^{N-1} \left| a_{\vec{p}-\vec{q}} \right| \Psi_0^N \right\rangle$

$$\Rightarrow d\sigma_L \propto \sum_n \left\langle \Psi_0^N \left| a^+_{\vec{p}-\vec{q}} \right| \Psi_n^{N-1} \right\rangle \left\langle \Psi_n^{N-1} \left| a_{\vec{p}-\vec{q}} \right| \Psi_0^N \right\rangle \delta\left(E_{miss} - \left(E_0^N - E_n^{N-1}\right)\right)$$

Realistic case : distorted waves / more realistic description of knocked out particle

Atoms studied with the (e,2e) reaction

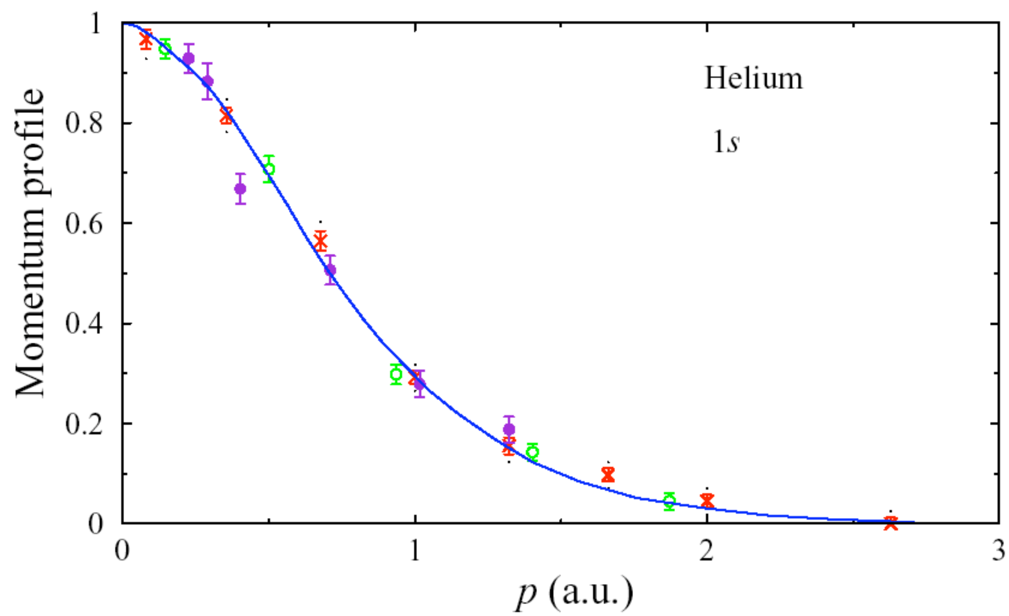


$$\varphi_{1s}(p) = 2^{3/2}\pi \frac{1}{(1+p^2)^2}$$

Hydrogen 1s wave function
“seen” experimentally
Phys. Lett. 86A, 139 (1981)

And so on for other atoms …

Helium
in Phys. Rev. A8, 2494 (1973)

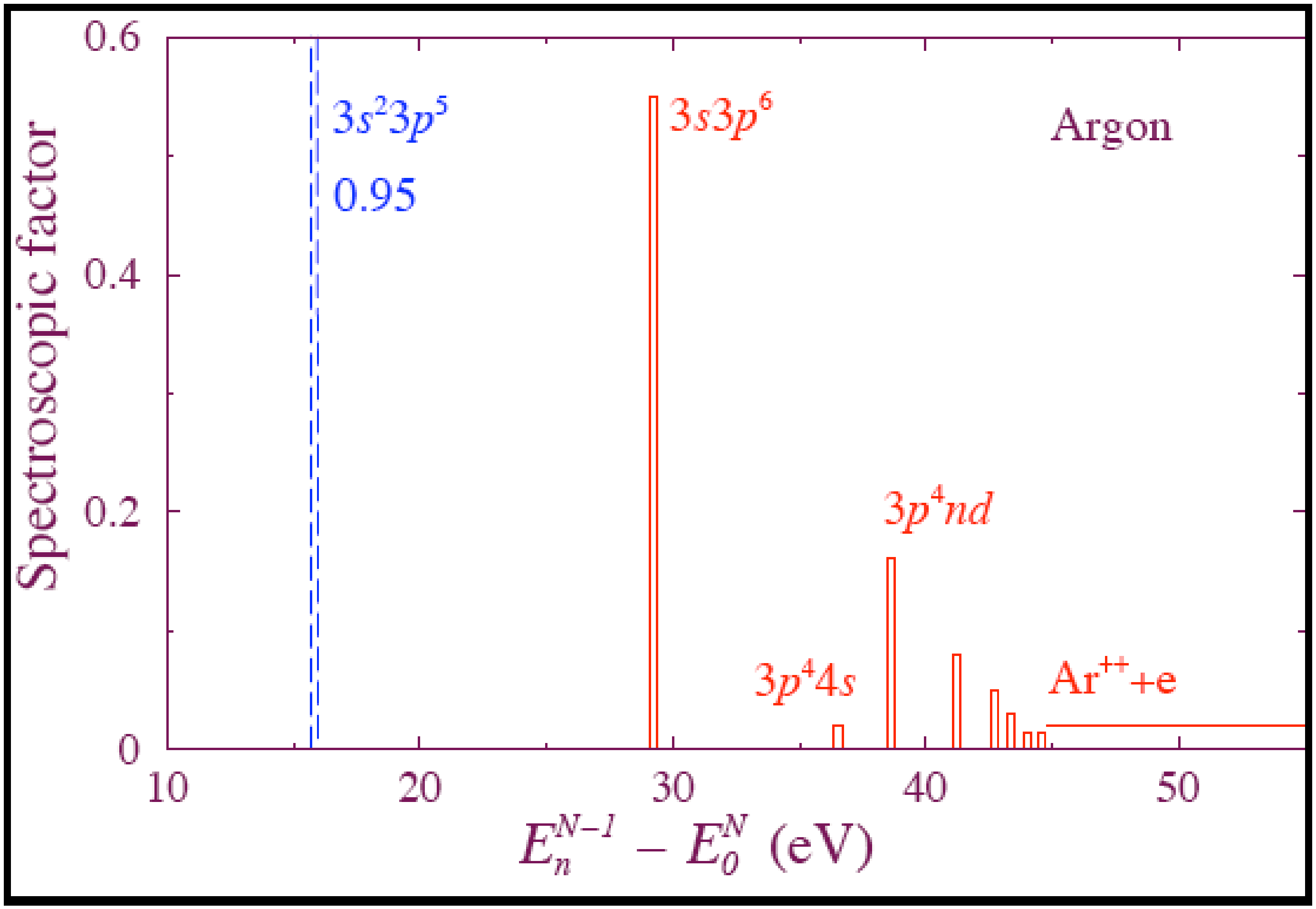


Spectroscopic factors in atoms

For a bound final N-1 state the spectroscopic factor is given by $S = \int d\vec{p} \left| \left\langle \Psi_n^{N-1} \middle| a_{\vec{p}} \middle| \Psi_0^N \right\rangle \right|^2$

For H and He the 1s electron spectroscopic factor is 1

For Ne the valence 2p electron has S=0.92 with two additional fragments, each carrying 0.04, at higher energy.

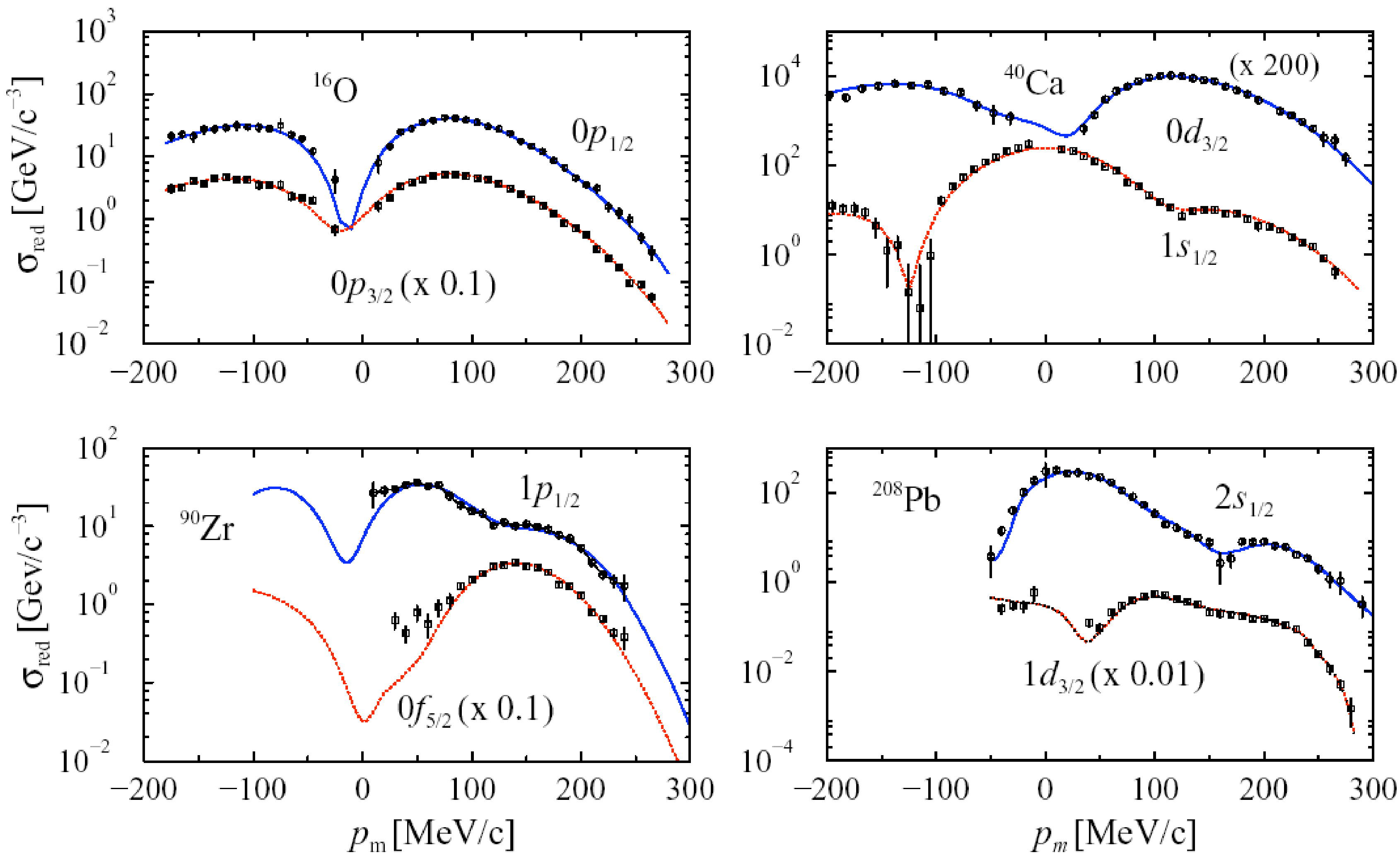


Argon
3p and 3s
strength

Closed-shell atoms
n(α) = 0 or 1

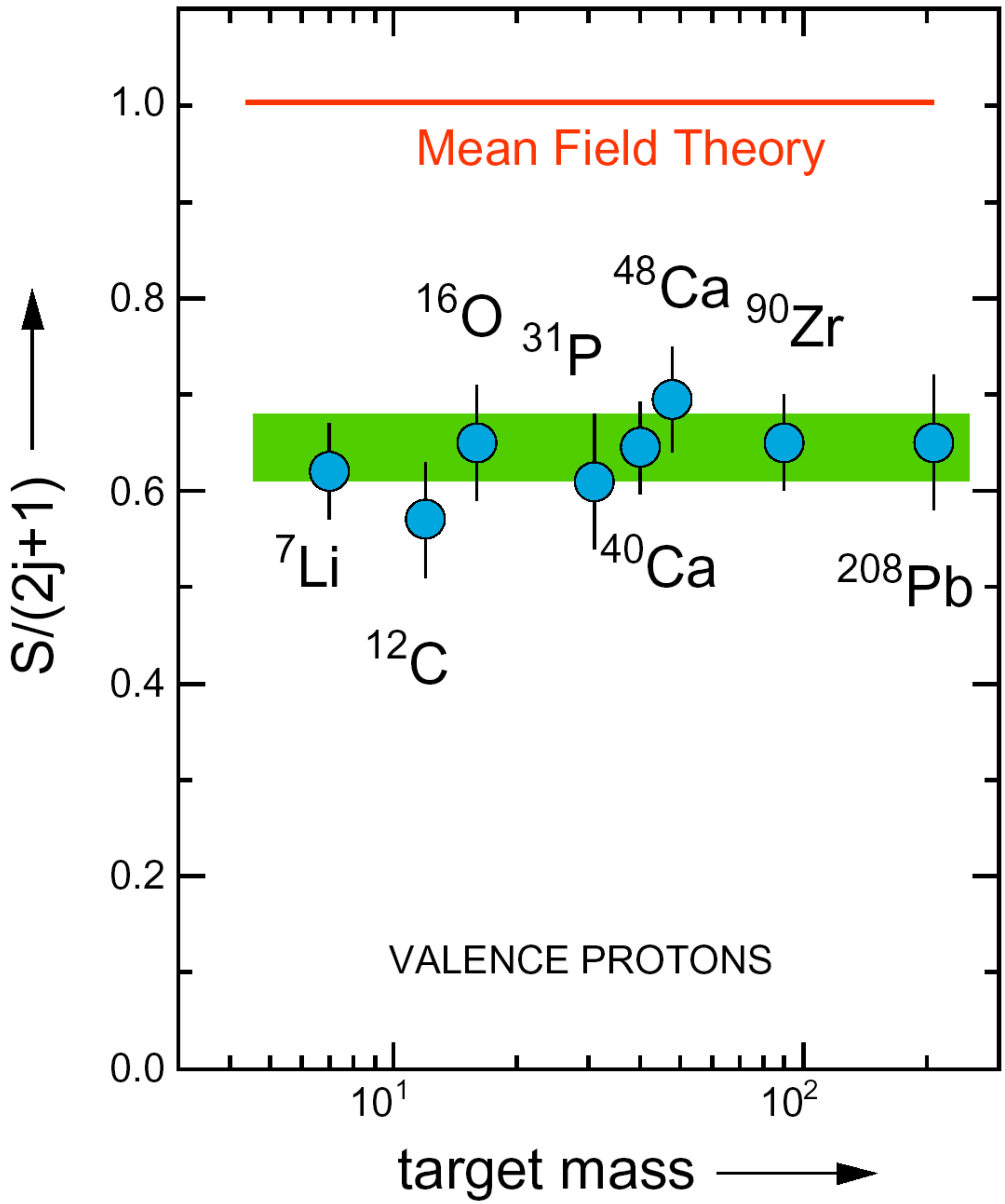
(e,e´p) cross sections for closed-shell nuclei

NIKHEF data, L. Lapikás, Nucl. Phys. A553, 297c (1993)



Except ….

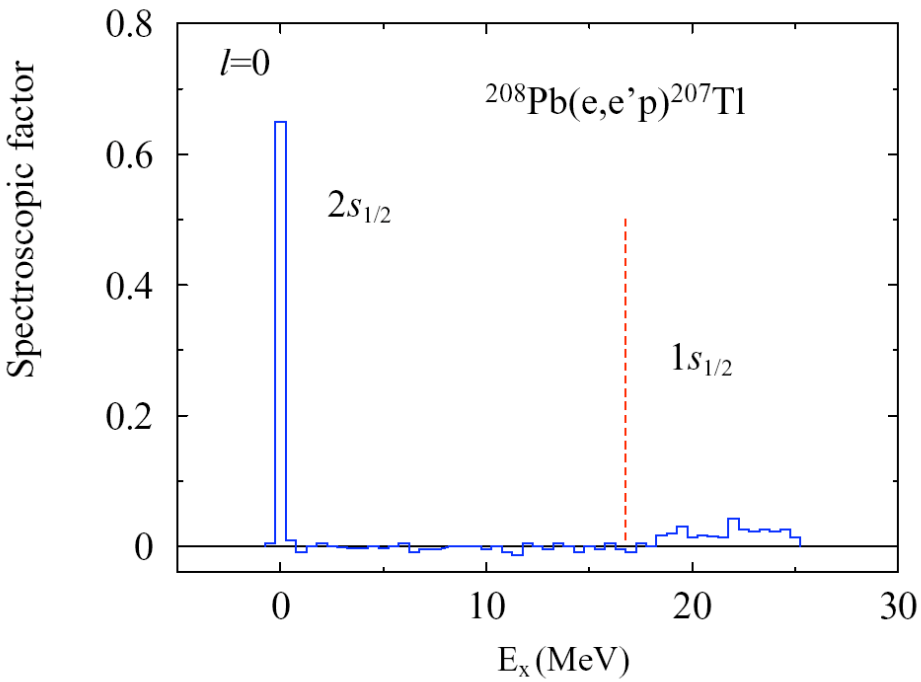
Removal probability for valence protons from NIKHEF data

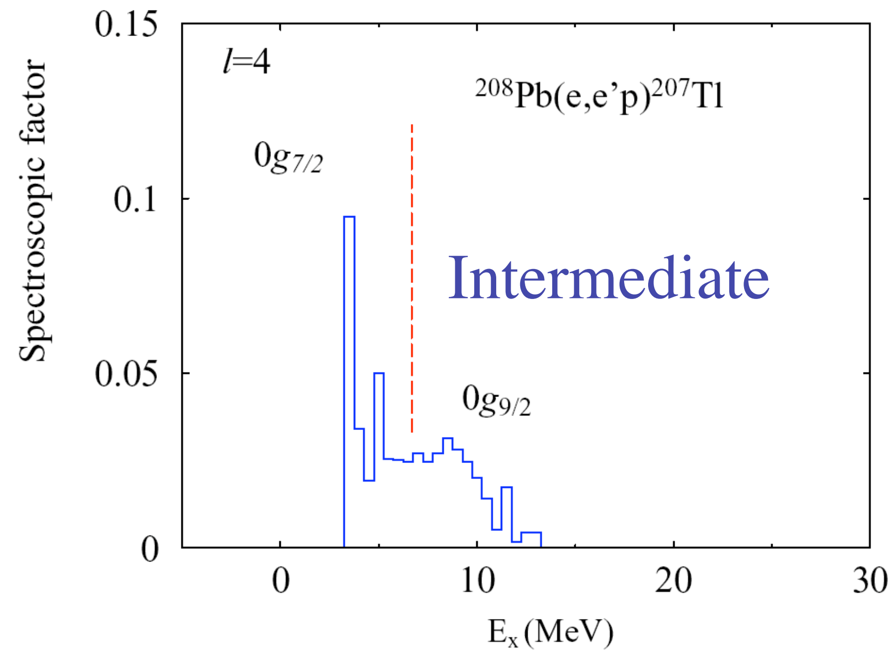


Note:

We have seen mostly data for removal of **valence protons** and …

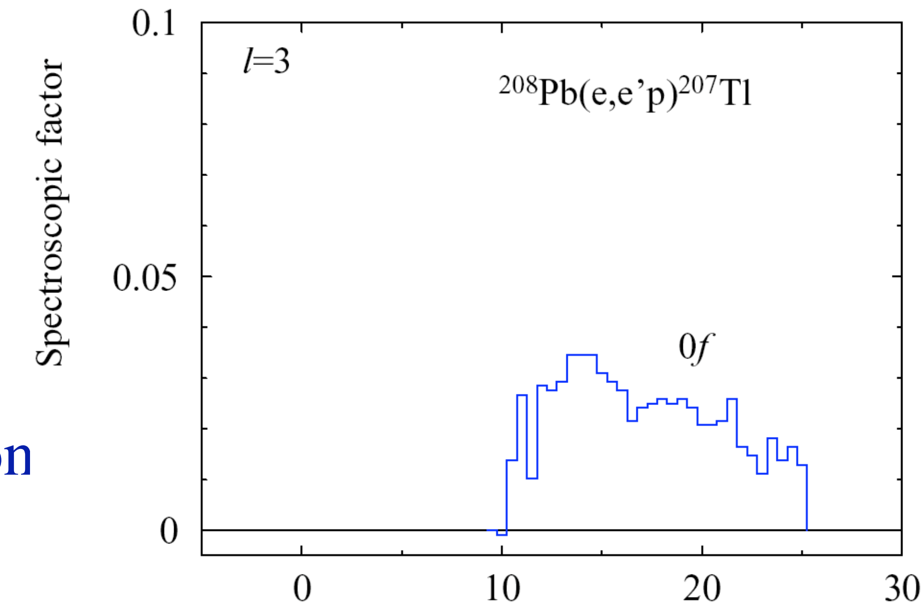
E. Quint, Ph.D.thesis NIKHEF, 1988





Quasihole strength or
spectroscopic factor $Z(2s_{1/2})$ =0.65

$n(2s_{1/2}) = 0.75$
from elastic electron scattering



Strong fragmentation
deeply-bound states

Many-body perturbation theory for *G*

- Identify solvable problem by considering $\hat{H}_0 = \hat{T} + \hat{U}$ where *U* is a suitable auxiliary potential.
- Develop expansion in $\hat{H}_1 = \hat{V} - \hat{U}$
- Employs time-evolution, Heisenberg, Schrödinger, and interaction picture of quantum mechanics.
- Once established, this expansion (expressed in Feynman diagrams) is organized in such a way that nonperturbative results can be obtained leading to the Dyson equation. The Dyson equation describes sp motion in the medium under the influence of the self-energy which is an energy-dependent complex sp potential.
- Further insight into the proper description of sp motion in the medium is obtained by studying the relation between sp and two-particle propagation. This allows the selection of appropriate choices of the relevant ingredients for the system under study.