One-electron atom → ground state

$$|n=1\ l=0\ m=0\rangle \equiv |100\rangle \rightarrow E_{10} \text{ energy}$$

Electric field →

$$V = -e\,|\vec{E}|\,z \quad \rightarrow \text{uniform electric field}$$

$$|100\rangle_{\text{perturbed}} = |100\rangle + \sum_{nlm} |nlm\rangle \frac{\langle nlm|-e|\vec{E}|z|100\rangle}{E_{10}-E_{nl}}$$

Note $l=1\ m=0$ because $z = r\cos\theta \propto r Y_{10}$ → no sum over l & $m=0$

$\{$ energy $|nlm\rangle$

Expectation value

$$\underset{\text{pert.}}{\langle 100|}\, ez\, |100\rangle_{\text{pert}}$$

$$= \left\{\langle 100| + \sum_{n'} C^{*}_{n'l=1} \langle n'l=1 m=0|\right\} ez \left\{|100\rangle + \sum_{n} |nl=1 m=0\rangle C_{nl=1}\right\}$$

with $C_{nl=1} = \dfrac{\langle nl=1\ m=0|\,(-e|\vec{E}|z)\,|100\rangle}{E_{10}-E_{nl=1}}$

$\langle 100|z|100\rangle = 0$ as discussed before e.g. parity.

as is true for $\langle n'l=1\ m=0|z|nl=1\ m\rangle = 0$

$$\rightarrow \underset{\text{pert}}{\langle 100|}ez\,|100\rangle_{\text{pert}} = \underbrace{-2}_{\text{cross terms}} e^2|\vec{E}| \sum_{n} \frac{|\langle 100|z|nl=1\ m=0\rangle|^2}{E_{10}-E_{nl=1}}$$

$$\Delta = e^2|\vec{E}|^2 \sum_{nl=1} \frac{|\langle 100|z\,|nl=1\ m=0\rangle|^2}{E_{10}-E_{nl=1}}$$