Two particles of equal masses $m_1 = m_2 = m$ move on a frictionless surface subject to a fixed force with corresponding potential energies $U_1 = \frac{1}{2}kr_1^2$ and $U_2 = \frac{1}{2}kr_2^2$, with r_1 and r_2 the respective distance to the force center. In addition, they interact with each other via a potential energy $U_{12} = \frac{1}{2}\alpha kr^2$, where r is the distance between them and α and k are positive constants.

a) (10 points) Determine the Lagrangian in terms of the CM position $\boldsymbol{R}$ and the relative position $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$.

b) (10 points) Write down the Lagrange equations and solve them for the CM and relative coordinates X, Y and x, y.

c) (5 points) Describe the motion.

Consider problem 9.26 of the book.

a) (10 points) A baseball is thrown vertically up with initial speed v_0 from a point on the ground at colatitude θ. Use the solution of problem 34 (or 9.26 in the book) to show that the ball will return to the ground a distance

$$\frac{4\Omega v_0^3 \sin\theta}{3g^2} \tag{1}$$

to the west of its launch point.

b) (5 points) Estimate the size of this effect on the equator if $v_0 = 40$ m/s.

c) (10 points) Sketch the ball's orbit as seen from the north (by an observer fixed to the earth). Compare with the orbit of a ball dropped from a point above the equator, and explain why the Coriolis effect moves the dropped ball to the east, but the thrown ball to the west.

A rigid body consists of three equal masses (m) fastened at positions $(a, 0, 0)$, $(0, a, 2a)$, and $(0, 2a, a)$.

a) (5 points) Find the inertia tensor $\boldsymbol{I}$.

b) (15 points) Find the principal moments and a set of orthogonal principal axes.

A simple pendulum (mass M and length L) is suspended from a cart (mass m) that can oscillate on the end of a spring with force constant k, as shown in Fig. 1.

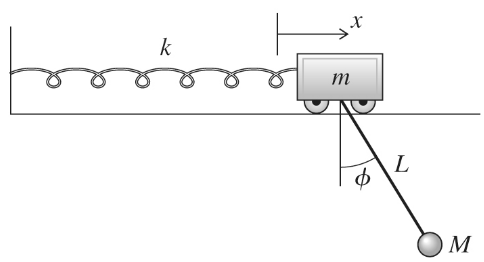

Figure 1: Picture for this problem

a) (15 points) Assuming that the angle ϕ remains small, write down the Lagrangian for this problem and determine the equations of motion for x and ϕ.

b) (15 points) Assuming that $m = M = L = g = 1$ and $k = 2$ (all in appropriate units) find the normal frequencies, and for each normal frequency find and describe the motion of the corresponding normal mode.