Outline

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Simplified Stellar Model

Newton's Gravitation

$$\frac{d\mathcal{P}}{dr} = -G\frac{\mathcal{M}(r)\rho(r)}{r^2} \quad (1)$$
$$\frac{d\mathcal{M}}{dr} = 4\pi r^2\rho(r) \quad (2)$$

Total Energy

$$E_T = \sum_i \left(m_i + \frac{p_i^2}{2m_i}\right) + E_g + E_\gamma$$

$$E_g = -G\int_0^R \frac{\mathcal{M}(r)\rho(r)}{r} 4\pi r^2 dr$$

Integrating (1) by parts:

$$3\bar{\mathcal{P}}V = \frac{3}{5}G\frac{M^2}{R_g}$$

where have defined, R_g, gravitational radius :

$$R_g = -\frac{\frac{3}{5}GM^2}{E_g}$$

$R_g \sim 0.37R_\odot$

Star ∼ Ideal Gas

- $N\bar{T} = \bar{\mathcal{P}}V$
- $\langle K\rangle = \frac{3}{2}N\bar{T}$

$c = 1,\ \hbar = 1\ k_b = 1$

Combining all together:

$$E_T = \sum_i m_i - \frac{3}{10} G \frac{M^2}{R_g} + E_\gamma$$

$$\begin{aligned} L &= -\frac{dE_T}{dt} \\ &= -\left(\frac{d\mathcal{M}_{\text{rest}}}{dt} + \frac{3}{10} G \frac{M^2}{R_g^2} \frac{dR_g}{dt} \right) \\ &\quad -\frac{dE_\gamma}{dt} \end{aligned}$$

$L > 0$

$\frac{dE_\gamma}{dt} < 0 \ \rightarrow \ \gamma$ Diffusion

$\frac{d\mathcal{M}_{\text{rest}}}{dt} < 0 \ \rightarrow$ Exothermic Reactions

$\frac{dR_g}{dt} < 0 \ \rightarrow$ Contraction

- Nuclear energy production increases
  - $\Rightarrow T$ increases locally
  - $\Rightarrow$ Expansion(reduces$T$)
  - $\Rightarrow$reaction rate decreases.
- Nuclear production decreases
  - $\Rightarrow \mathcal{P}$ decreases
  - $\Rightarrow$Contraction.
  - $\Rightarrow T$ increases
  - $\Rightarrow$ reaction rate increases.

$c = 1$, $\hbar = 1$ $k_b = 1$

Outline

Star life-time

Gravitational time scale

$$\frac{G\mathcal{M}^2}{R} \sim L\tau_g$$

$$\tau_g \sim \frac{G\mathcal{M}^2}{RL}$$

In the case of the sun:

$$\tau_g \sim \frac{G\mathcal{M}_\odot^2}{R_\odot L_\odot} \sim 3 \times 10^9\text{y}$$

Non-Gravitational Energy

$$E_{n-g} = \mathcal{M} - \frac{G\mathcal{M}^2}{R}$$

$$E_{n-g\ \odot} \sim 1.78 \times 10^{54}\text{erg}$$

Nuclear time scale

$$4p \rightarrow^4 \text{He} + 2\nu_e + 2e^+$$

Binding Energy (^{4}He)$\sim$ **28**MeV
Fraction of mass converted into energy:

$$\epsilon = \frac{28.28\text{MeV}}{4 \times 938.27\text{MeV}} \sim 0.7\%$$

Again for the sun (having about 10% H):

$$\tau_{nuc} \simeq 0.1 \times 0.007 \frac{1.78 \times 10^{54}}{3.846 \times 10^{33}}\text{s}$$
$$\simeq 10^{10}\text{y}$$

Some Numbers

Natural Units:

$\hbar c = 197.33$ MeV $\times$ fm $= 1$

Temperature ($k_b = 1$):

1MeV $= 1.1604\times 10^{10}$K

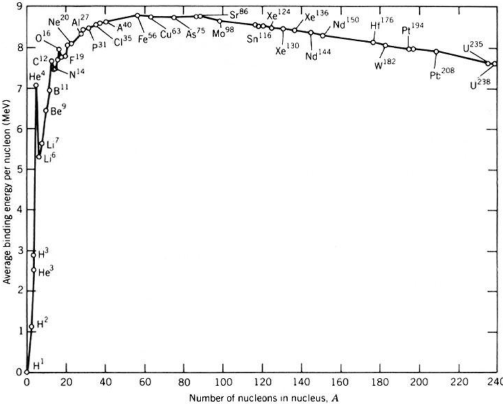
Quantity	**Value**	
G	$6.67 \times 10^{-8}\mathrm{cm}^3/\mathrm{gs}^2$	
$\mathcal{M}_\odot$	$1.99 \times 10^{33}\mathrm{g}$	$1.12 \times 10^{60}\mathrm{MeV}$
$R_\odot$	$6.96 \times 10^{10}\mathrm{cm}$	
$L_\odot$	$3.846 \times 10^{33}\mathrm{erg/s}$	
$T^c_\odot$	$1.5 \times 10^7\mathrm{K}$	$\sim 10^{-3}\mathrm{MeV}$
$\rho_\odot$	$150\ \mathrm{g/cm}^3$	
ρ_0	$3 \times 10^{14}\mathrm{g/cm}^3$	$\sim 0.16\ \mathrm{fm}^{-3}$

Stellar Evolution

Nuclear Reactions

1. $p + p \to$ He
2. ^{4}He, ^{3}He $\to$ C and O .
3. C,O and He $\to$ Ne and Mg
4. Mg+ He $\to$ ^{28}Si
5. Si $\to$ Fe (This lasts days).

Depending on Star's Mass.



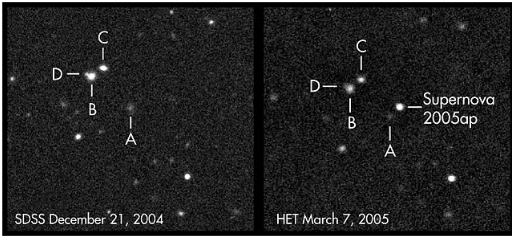
Formation of Compact Objects

Massive Stars: $\mathcal{M} \geq 8\mathcal{M}_{\odot}$

- Burns all the way to Fe (gets to stage 5).
- Fe core collapses $\leftrightarrow$ Core Collapse supernovae.
- Remnant is a Neutron Star or a Black Hole.

Low mass Stars: $\sim 1\mathcal{M}_{\odot}$

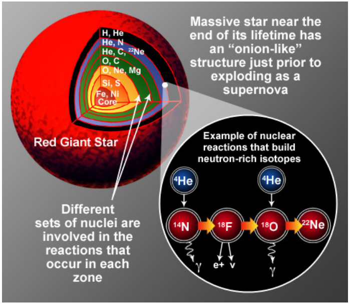
- Burns He.
- Sheds some mass.
- Never gets hot enough to burn C or O. Only until stage 2.
- Electron degeneracy provides pressure.
- No explosion.
- Remnant is a White Dwarf.



Outline

Gravitational Collapse

$\mathcal{M} > 8\mathcal{M}_{\odot}$



Strucutre of a massive star.

More or less close:

Name	Year	Distance[kpc]
SN1006	1006	2
Crab	1054	2.2
SN1181	1181	8.0
RXJ0852-4642	1300	∼0.2
Tycho	1572	7.0
Kepler	1604	10.0
CasA	1680	3.4
SN1987A	1987	50 ± 5

Before collapsing

- $\rho \sim 2 \times 10^9 \mathrm{g/cm}^3$.
- $T \sim 0.5$MeV$\sim 5 \times 10^9$K.
- Planet size: $R \sim 20$km.
- Proto-neutron Star: hot and lepton rich.

- Proto-neutron star is hot and lepton-rich.
- Iron core grows.
- Collapse to 3×10^{14}g/cm^3.
- **Density is high enough to drive electron capture**

$$e + p \to n + \nu_e$$

$$e +^A Z \to^A (Z-1) + \nu_e$$

- **In chemical equilibrium** $E_\nu \sim \mu_\nu$

$$\mu_e + \mu_n = \mu_n + E_\nu$$

- e^- captures decreasing Y_e (and $n_e = Y_e n_b$)

- **Delicate balance between pressure and gravity**
- $\mathcal{P} \sim n_e^{4/3}$
- Electron fraction:

$$Y_e = \begin{cases} 1 & \text{H} \\ {}^1/_2 & {}^4\text{He},\ {}^{12}\text{C},\ {}^{16}\text{O} \\ {}^{26}/_{56} & {}^{56}\text{Fe} \end{cases}$$

$\Rightarrow Y_e$ goes lower than ${}^1/_2$

Y_e too small to fail...

If νs continue to escape, Y_e gets too small (not enough $\mathcal{P}$) $\Rightarrow$ collapse to **Black Hole**.

Outline

νs role

νs do not leave that fast

λ_ν:= ν mean free path.
n_b:= number baryon density.
σ:=cross section

$$\lambda_\nu = \frac{1}{n_b \sigma(E)}$$

Coherent Scattering
$\nu+$ Spin zero Nucleus.

- Assume contact interaction.
- Elastic process.
- Assume Q is small.
- $Q^2 = 2E_\nu^2(1 - \cos\theta)$

q_n and q_p are weak charges of the neutron and the proton.

$\nu +^A Z$

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{G_F^2}{4\pi^2} E_\nu^2 \left(1 + \cos\theta\right) q_A^2 F(Q^2)$$

q_A:= Nucleus weak charge.
$F(Q^2)$:= Nucleus form factor.

$$F(Q^2) = \frac{1}{q_A} \int d\mathbf{r} e^{i\mathbf{Q}\cdot\mathbf{r}} \left(q_n n_n(\mathbf{r}) + q_p n_p(\mathbf{r})\right)$$

Cross section

$q_W :=$ Weak charge:=Weak Isospin$-2\sin^2\theta_W q_{em}$

$$q_u = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$$
$$q_d = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$$

$$q_p = 2q_u + q_d = \frac{1}{2} - 2\sin^2\theta_W$$
$$q_n = 2q_d + q_u = -\frac{1}{2}$$

Since $\sin^2\theta_W \sim {}^1/_2$ then $q_p \sim 0$

Nucleus total weak charge

$$q_A \sim -\frac{1}{2} \times (A - Z) = -\frac{N}{2}$$

Nucleus Form Factor

For small momentum transfer: $Q \times$ Nucleus Size $<< 1$:

$$F(Q^2) \sim 1$$

νs mean free path

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{G_F^2}{4\pi^2} E_\nu^2 \,(1+\cos\theta)\, q_A^2 F(Q^2)$$

$$\begin{aligned}\sigma_{tr} &= \int d\Omega \frac{d\sigma}{d\Omega}(1-\cos\theta)\\ &= \frac{2}{3}\frac{G_F^2}{\pi}E_\nu^2 \frac{N^2}{4}\end{aligned}$$

Consider Iron core:
^{56}**Fe,** $N = 30$
$G_F = 1.16 \times 10^{-11}\text{MeV}^{-2}$ and
$E_\nu \sim 10\text{MeV}$.

$$\begin{aligned}\sigma_{tr} &= 6.4 \times 10^{-19}\text{MeV}^{-2}\\ &= 2.49 \times 10^{-13}\text{fm}^2\end{aligned}$$

... baryon number density is:

$$\begin{aligned}n_b &= \rho_b \frac{\mathcal{N}_A}{56\text{g}}\\ &= 10^{12}\frac{\text{g}}{\text{cm}^3}\frac{6.02\times 10^{23}}{56\text{g}}\\ &\sim 1.07 \times 10^{-5}\text{fm}^{-3}\end{aligned}$$

ν trapping

$$\begin{aligned}\lambda_\nu &= \frac{1}{n_b\sigma_{tr}} \sim 3.7\times 10^{18}\text{fm}\\ &= 3.7\text{km} < R_{NS}\end{aligned}$$

Questions from last time

1. Simplest stellar model, E_γ counts only photons that do not come from nuclear reactions.
2. Star life-time Had a typo: should be 10% Hydrogen. This is the amount of H available to burn in the sun.
3. Stellar Evolution Where the ^{3}He comes from in stellar reactions? $\Rightarrow$ For all pp chains:

$$\begin{aligned} p + p &\rightarrow \mathrm{d} + e^+ + \nu_e \\ \mathrm{d} + p &\rightarrow {}^3\mathrm{He} + \gamma \end{aligned}$$

   from here different possible paths could produce ^{4}He, different pp-chains, all of them destroy ^{3}He.
4. νs role Why that form for the differential cross section?

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{G_F^2}{4\pi^2} E_\nu^2 \left(1 + \cos\theta\right) q_A^2 F(Q^2)$$

Original paper by Freedman (PRD **9** 1389 (1974))

- Since νs still in the star.
- Y_e does not fall as fast ($e-$capture is suppressed),
  $\Rightarrow$This prevents the collapse to happen faster.
- νs are radiated from the proto-neutron star ($\tau_\nu \sim$ s),
  BUT timescale of the collapse is ms $\Rightarrow$ νs are trapped.
- The supporting pressure is still decreasing.
- When $\rho_{\text{core}} > \rho_0$, $\rho_0 = 3 \times 10^{14}$g/cm^3
  the outer core free-falls onto inner core.
- The hard core repulsion makes the core to bounce
  $\Rightarrow$ a shock wave is generated.
- Shock losses its energy through scattering with ν and nuclear processes.
- νs from the core (assisted by other mechanisms) resuscitate the shock (no conclusive),
  $\Rightarrow$ expelling the massive stellar mantle.
- Proto-neutron star shrinks because of the losses of neutrinos

Role of Supernovae

Hubble image of the crab nebulae.

Image from NASA.

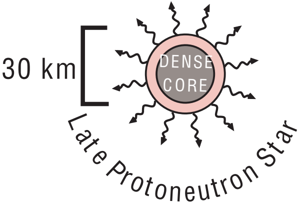
Expanding ejecta from the explosion in 1054.

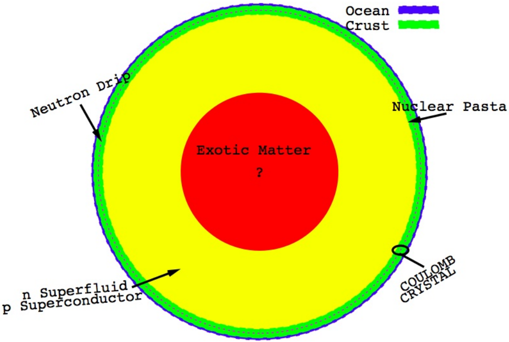
Energy released in νs:

$$\delta E = \left(-G\frac{\mathcal{M}_{GS}}{R_{GS}}\right) - \left(-G\frac{\mathcal{M}_{NS}}{R_{NS}}\right) \simeq 10^{53}\text{erg.}$$

- Stellar Nucleosynthesis. (Abundances).
- Trigger star formation.
- Accelerate cosmic rays.
- Important source of νs.
- The remnant is either a Neutron Star or a Black hole.
- Type Ia can be used as standard ruler for astronomy.

Supernovae Remnant ⇒ Neutron Star





Outline

Neutron Star cooling

Weeks after the explosion, $T \sim 10^9 - 10^{10}$ K.

$$C_V(T_i)\frac{dT_i}{dt} = -L_\nu(T_i) - L_\gamma(T_s) + \sum_k H_k$$

- In 10 to 10^2 years heat is transported by electrons into the interior,
  where it is radiated away in νs.
- $T_i \neq T_s$.

$$\begin{aligned} C_V &= \frac{4\pi}{3}R^3 c_v T_i \\ L_\nu(T_i) &= \int Q_\nu d\mathbf{r} \\ L_\gamma(T_s) &= 4\pi R^2 \sigma T_s^4 \end{aligned}$$

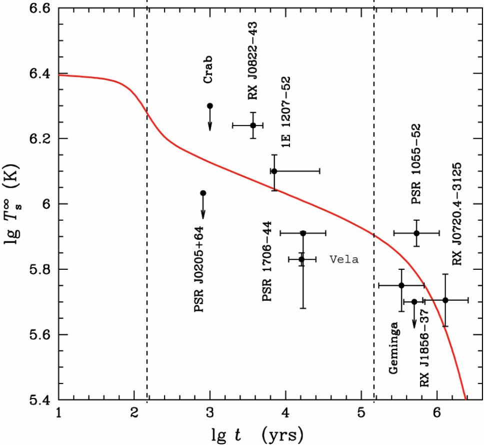
By then the star is in thermal equilibrium.

T_i:=Internal temperature.
T_s:=Surface temperature.
H_k:=Heating mechanisms (frictional heating of superfluid neutrons in the inner crust or exothermal nuclear reactions.)

Q_ν := Neutrino emissivity

Cooling Timescales

Relaxa-tion | Neutrino cooli-ing stage | Photon stage



Red line is model of Modified Urca (slow cooling) by Yakovlev & Pethick (2004)

ν cooling- emissivity

$Q_\nu := \nu$ emissivity

- Depends on specific reactions (microphysics).
- In general two forms are found:

$$Q_\nu^1 = Q_1 T_i^8$$
$$Q_\nu^2 = Q_2 T_i^6$$

- From where the ν luminosity is:

$$L_\nu^1 = \frac{4\pi R^3}{3} Q_1 T_i^8$$
$$L_\nu^2 = \frac{4\pi R^3}{3} Q_2 T_i^6$$

In the ν - cooling era: $L_\nu << L_\gamma$
Neglect other processes ($H_k \sim 0$).

$$C_V(T_i)\frac{dT_i}{dt} = -L_\nu(T_i) = \begin{cases} \frac{4\pi R^3}{3} Q_1 T_i^8 \\ \frac{4\pi R^3}{3} Q_2 T_i^6 \end{cases}$$

from where we find:

$$T_i \sim \begin{cases} t^{-1/6}, \text{ for } L_\nu^1 \\ t^{-1/4}, \text{ for } L_\nu^2 \end{cases}$$

$L_\nu^1 \Rightarrow$ slow cooling.
$L_\nu^2 \Rightarrow$ fast cooling.

But how to relate T_s and T_i?

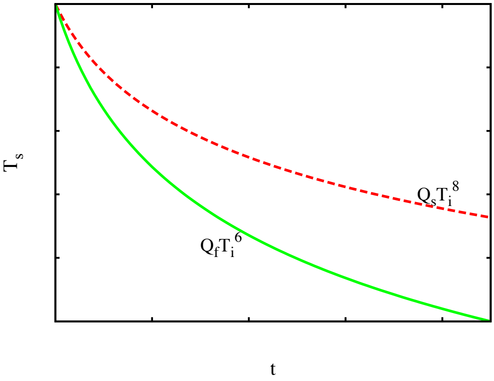
T_i and T_s

Assume a power law:

$$T_s = \kappa_{\text{env}} T_i^{\frac{1}{2}+a}$$

- Here κ_{env} and a depend on the composition of the envelope.
- It has been found $a << 1$ for most of the proposed compositions.
- Then from previous slide:

$$T_s \sim \begin{cases} t^{-1/12}, \text{ for } L_\nu^1 \\ t^{-1/8}, \text{ for } L_\nu^2 \end{cases}$$



ν emission processes

At high densities

Name	Process	Q_ν[erg/cm^{3}s]	L_ν[erg/s]
Dir. Urca	$n \to p + e + \bar{\nu}_e$	$\simeq 10^{27} T_9^6$	$10^{46} T_9^6$
	$p + e \to n + \nu_e$		
Quark Urca	$d \to u + e + \bar{\nu}_e$	$\simeq 10^{26} \alpha_c T_9^6$	$10^{41-42} T_9^6$
	$u + e \to d + \nu_e$		
Kaon Condensate	$n + K^- \to n + e + \bar{\nu}_e$	$\simeq 10^{24} T_9^6$	$10^{42} T_9^6$
	$n + e \to n + K^- + \nu_e$		
Pion condensate	$n + \pi^- \to n + e + \bar{\nu}_e$	$\simeq 10^{26} T_9^6$	$10^{44} T_9^6$
	$n + e \to n + \pi^- + \nu_e$		

At any density

Name	Process	Q_ν[erg/cm^{3}s]	L_ν[erg/s]
Mod. Urca	$n + n' \to n' + p + e + \bar{\nu}_e$	$\simeq 10^{20} T_9^8$	$10^{40} T_9^8$
	$p + e + n' \to n' + n + \nu_e$		
Bremsstrahlung	$N + N \to N + N + \nu_\ell + \bar{\nu}_\ell$	$\simeq 10^{20} T_9^8$	$10^{38} T_9^8$

$$T_n = \frac{T}{10^n \mathrm{K}}$$

Checking the Urcas

Direct Urca

$n \to p + e + \bar{\nu}_e$

$p + e \to n + \nu_e$

Effectively:

$n \to n + \nu_e + \bar{\nu}_e$

Charge Neutrality

Since gravitational attraction should win against Coulomb repulsion:

$$\frac{Ze^2}{R} \leq \frac{G(Am_b)m}{R}$$

Then net charge number:

$$Z \leq \begin{cases} 10^{-39}A, & \text{electron added} \\ 10^{-36}A, & \text{proton added} \end{cases}$$

- Composition does not change, $Y_e = \text{const}$.
- $n_p = n_e$ (charge neutrality).
- But if n_p is too small since: $p_p = (3\pi^2 n_p)^{1/3}$
  $\Rightarrow$ p_p & p_e, are too small.
- Direct Urca could only occur at high densities.

Next

- Finalize review on Cooling of Neutron Stars.
- Pulsars as Neutron Stars.
- Period of rotation, speed of sound, and causality.
- Uniform Nuclear Matter.
- Equation of State for Nuclear matter.
- Beta Equilibrium.
- Size and Mass. (Chandrasekar limit).
- Overview

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