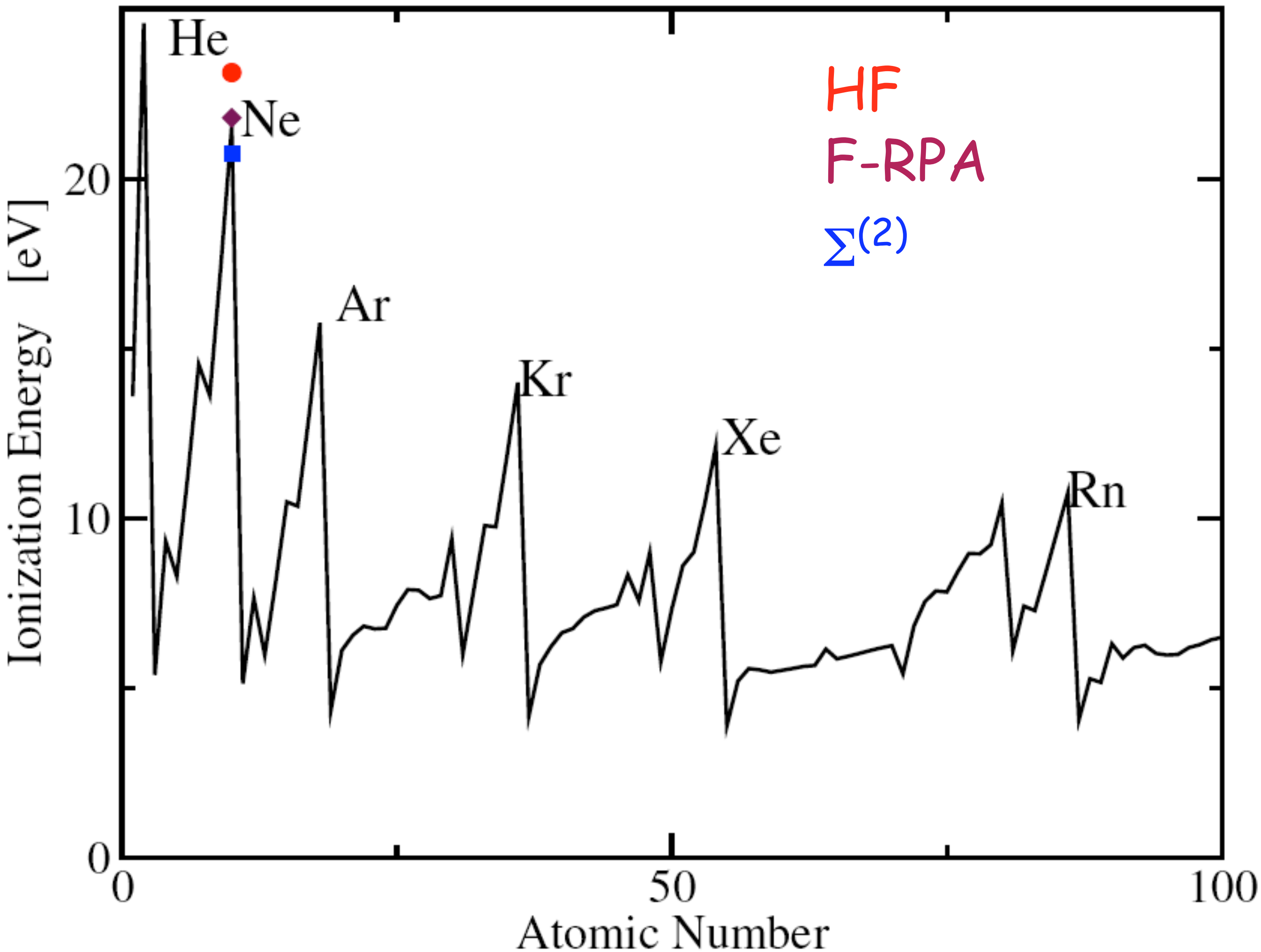
Periodic table



F-RPA, DOM and QP-DFT

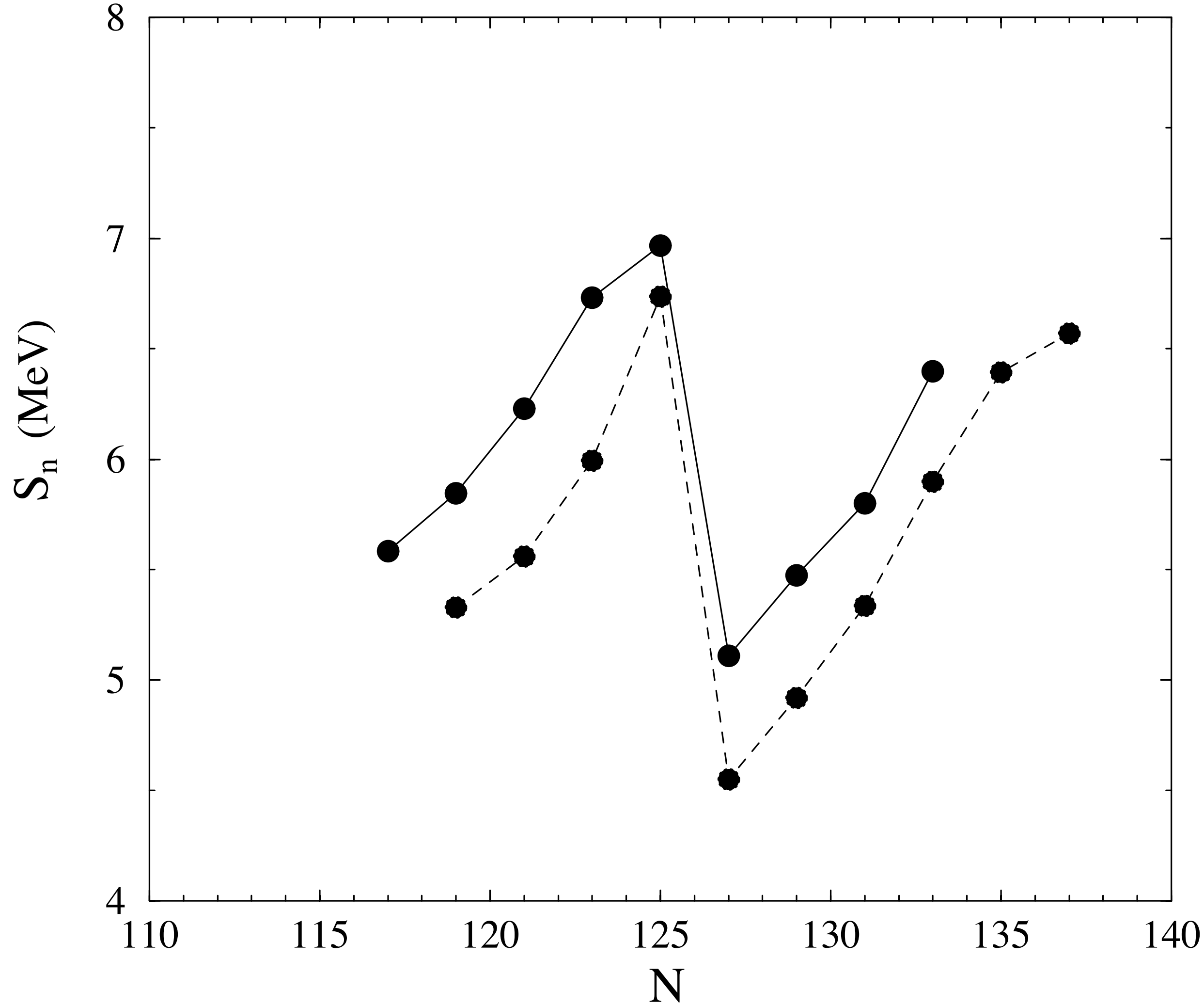
Nucleons in nuclei

- Atoms: shell closures at 2,10,18,36,54,86
- Similar features observed in nuclei
- Notation:
  - # of neutrons N
  - # of protons Z
  - # of nucleons $A = N + Z$
- Equivalent of ionization energy: separation energy
  - for protons $S_p(N,Z) = B(N,Z) - B(N,Z-1)$
  - for neutrons $S_n(N,Z) = B(N,Z) - B(N-1,Z)$
  - binding energy

$$M(N,Z) = \frac{E(N,Z)}{c^2} = N\ m_n\ +\ Z\ m_p\ -\ \frac{B(N,Z)}{c^2}$$

Shell closure at N=126

- Odd-even effect: plot only even Z

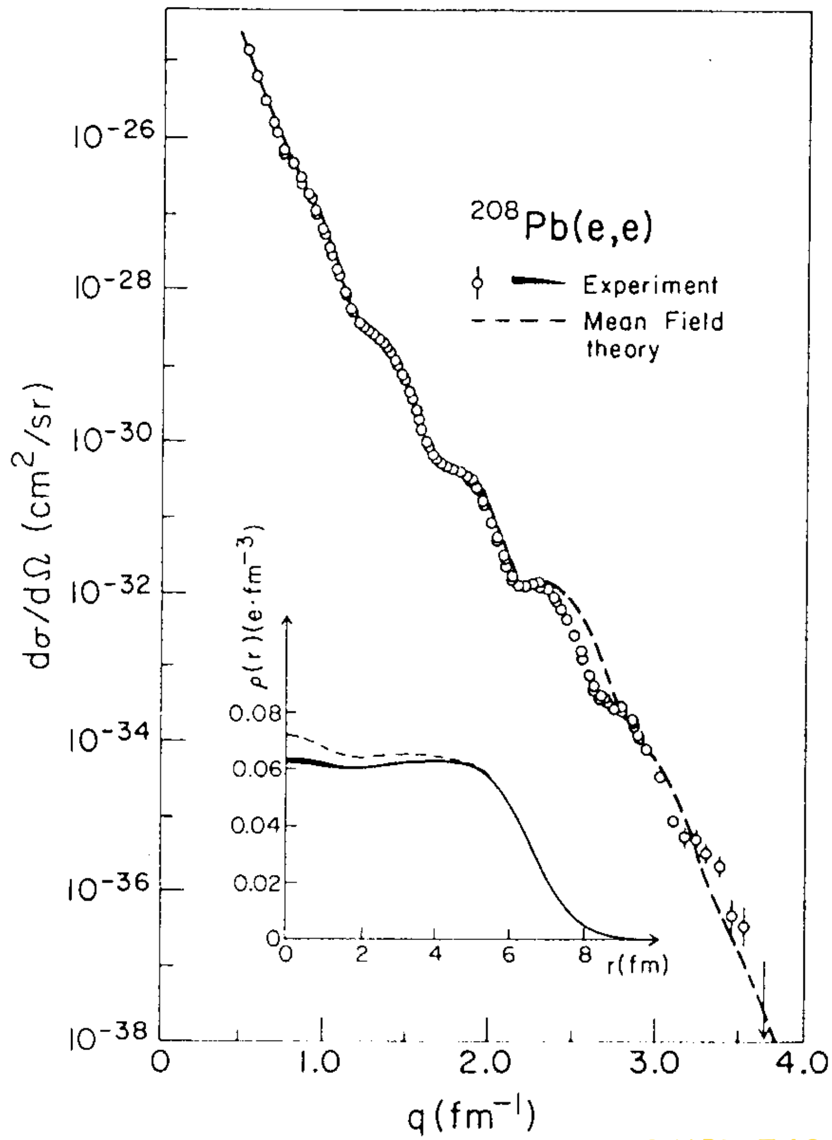


Solid: N-Z=41
Dashed: N-Z=43

- Also at other values N and Z: 2, 8, 20, 28, 50, and 82
- "Magic numbers"

Empirical potential

- Analogy to atoms suggests finding a sp potential ⇒ shells + IPM
- Difference(s) with atoms?
- Properties of empirical potential
  - overall?
  - size?
  - shape?
- Consider nuclear charge density



Frois & Papanicolas, Ann. Rev. Nucl. Part. Sci. **37**, 133 (1987)

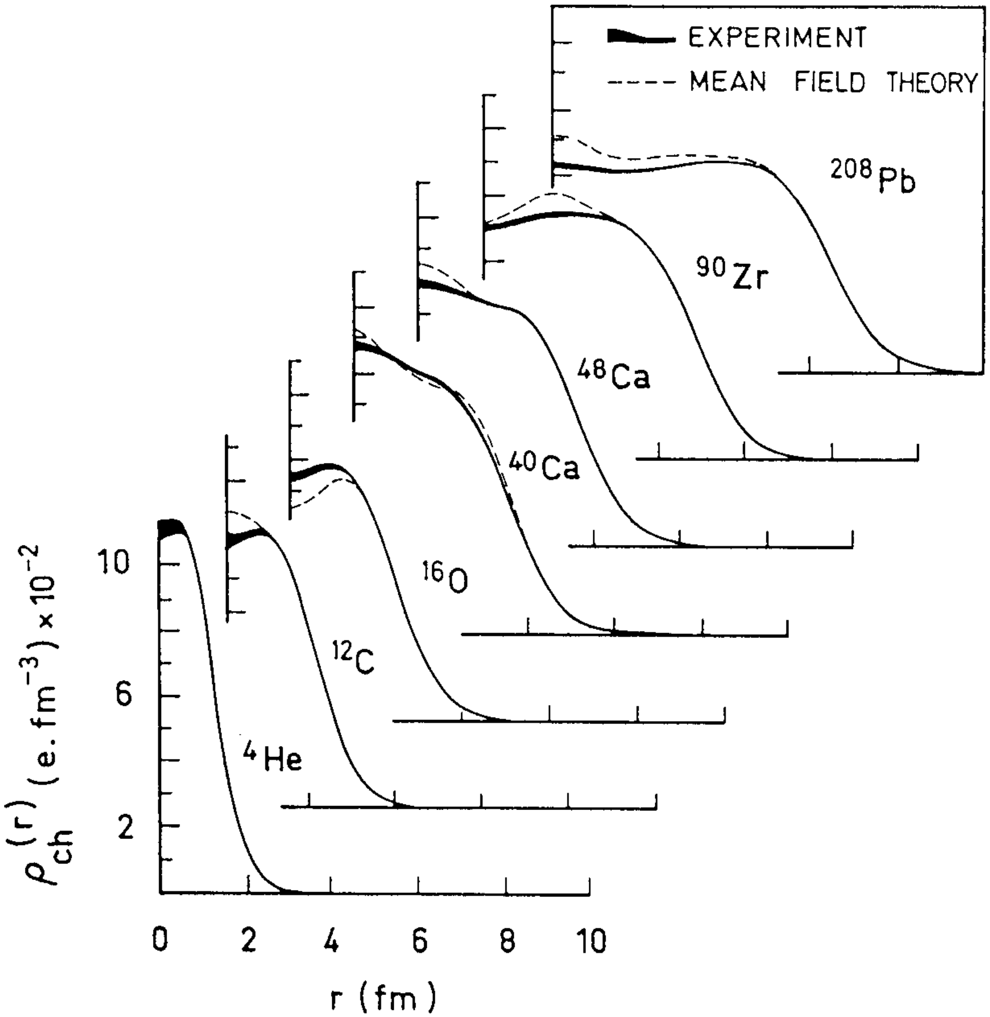
Nuclear density distribution

- Central density (A/Z* charge density) about the same for nuclei heavier than ^{16}O, corresponding to 0.16 nucleons/fm^3
- Important quantity
- Shape roughly represented by
- $$\rho_{ch}(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-c}{z}\right)}$$

$$c \approx 1.07 A^{\frac{1}{3}}\,\mathrm{fm}$$

$$z \approx 0.55\mathrm{fm}$$

- Potential similar shape



Empirical potential

- Bohr & Mottelson Vol.1

$$U = V f(r) + V_{\ell s} \left(\frac{\boldsymbol{\ell} \cdot \boldsymbol{s}}{\hbar^2} \right) r_0^2 \frac{1}{r} \frac{d}{dr} f(r)$$

- Central part roughly follows shape of density

$$f(r) = \left[1 + \exp\left(\frac{r - R}{a} \right) \right]^{-1}$$

- Woods-Saxon form
- Depth $V = \left[-51 \pm 33 \left(\frac{N - Z}{A} \right) \right] \text{ MeV}$

  + neutrons

  - protons

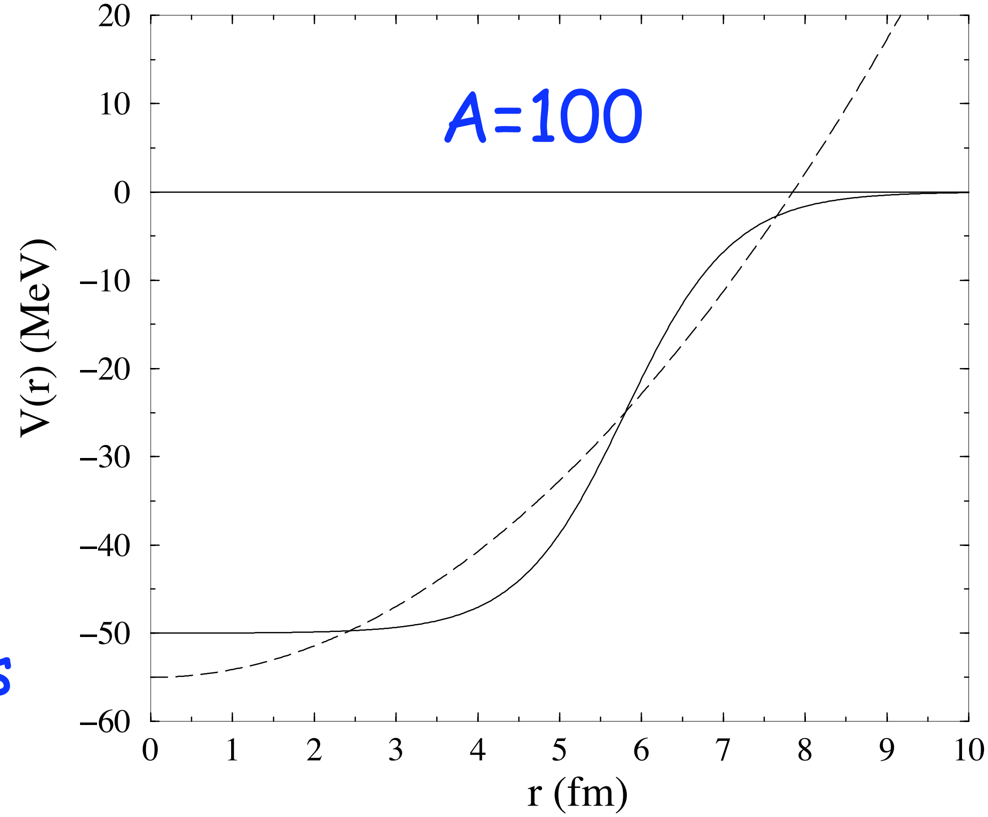
- radius $R = r_0 \; A^{1/3}$ with $r_0 = 1.27 \text{ fm}$
- diffuseness $a = 0.67 \text{ fm}$

Analytically solvable alternative

- Woods-Saxon (WS) generates finite number of bound states
- IPM: fill lowest levels ⇒ nuclear shells ⇒ magic numbers
- reasonably approximated by 3D harmonic oscillator

$$U_{HO}(r) = \frac{1}{2}m\omega^2 r^2 - V_0$$

$$H_0 = \frac{\boldsymbol{p}^2}{2m} + U_{HO}(r)$$



- Eigenstates in spherical basis

$$H_{HO}\left|n\ell m_\ell m_s\right\rangle = \left[\hbar\omega(2n+\ell+\tfrac{3}{2}) - V_0\right]\left|n\ell m_\ell m_s\right\rangle$$

Harmonic oscillator

- Filling of oscillator shells
- # of quanta $N = 2n + \ell$

N	n	ℓ	# of particles	"magic #"	parity
0	0	0	2	2	+
1	0	1	6	8	-
2	1	0	2		+
2	0	2	10	20	+
3	1	1	6		-
3	0	3	14	40	-
4	2	0	2		+
4	1	2	10		+
4	0	4	18	70	+

Need for another type of sp potential

- 1949 Mayer and Jensen suggest the need of a spin-orbit term
- Requires a coupled basis

$$|n(\ell s)jm_j\rangle = \sum_{m_\ell m_s} |n\ell m_\ell m_s\rangle \left(\ell\ m_\ell\ s\ m_s|\ j\ m_j\right)$$

- Use $\boldsymbol{\ell}\cdot\boldsymbol{s} = \frac{1}{2}(\boldsymbol{j}^2 - \boldsymbol{\ell}^2 - \boldsymbol{s}^2)$ to show that these are eigenstates

$$\frac{\boldsymbol{\ell}\cdot\boldsymbol{s}}{\hbar^2}|n(\ell s)jm_j\rangle = \tfrac{1}{2}\left(j(j+1) - \ell(\ell+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right)|n(\ell s)jm_j\rangle$$

- For $j = \ell + \frac{1}{2}$ eigenvalue $\frac{1}{2}\ell$
- while for $j = \ell - \frac{1}{2}$ $-\frac{1}{2}(\ell + 1)$
- so SO splits these levels! and more so with larger ℓ

Inclusion of SO potential and magic numbers

- Sign of SO?

$$V_{\ell s}\left(\frac{\boldsymbol{\ell}\cdot\boldsymbol{s}}{\hbar^2}\right) r_0^2 \frac{1}{r}\frac{d}{dr} f(r)$$

$$V_{\ell s} = -0.44V$$

- Consequence for

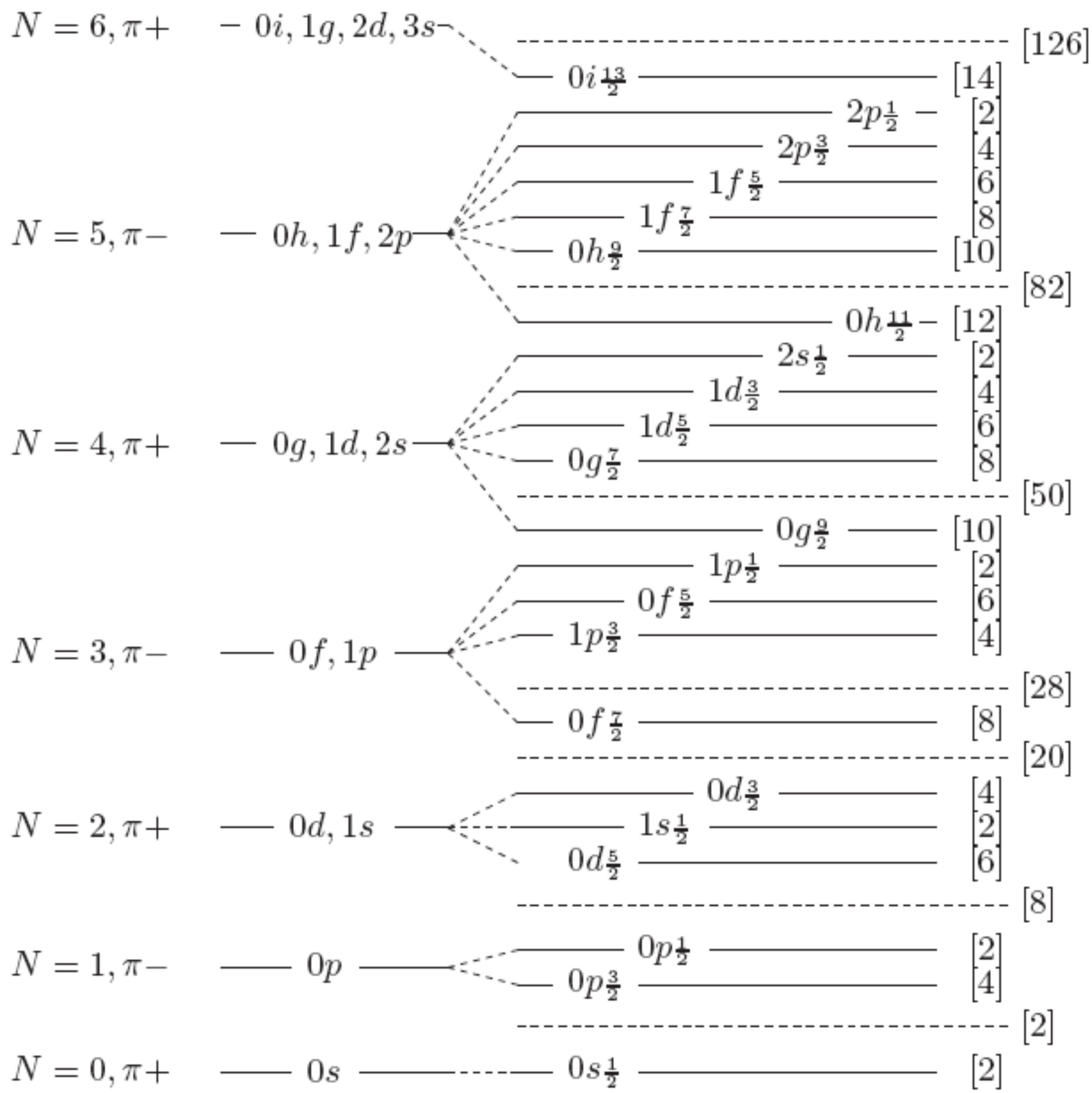
$$0f\tfrac{7}{2}$$

$$0g\tfrac{9}{2}$$

$$0h\tfrac{11}{2}$$

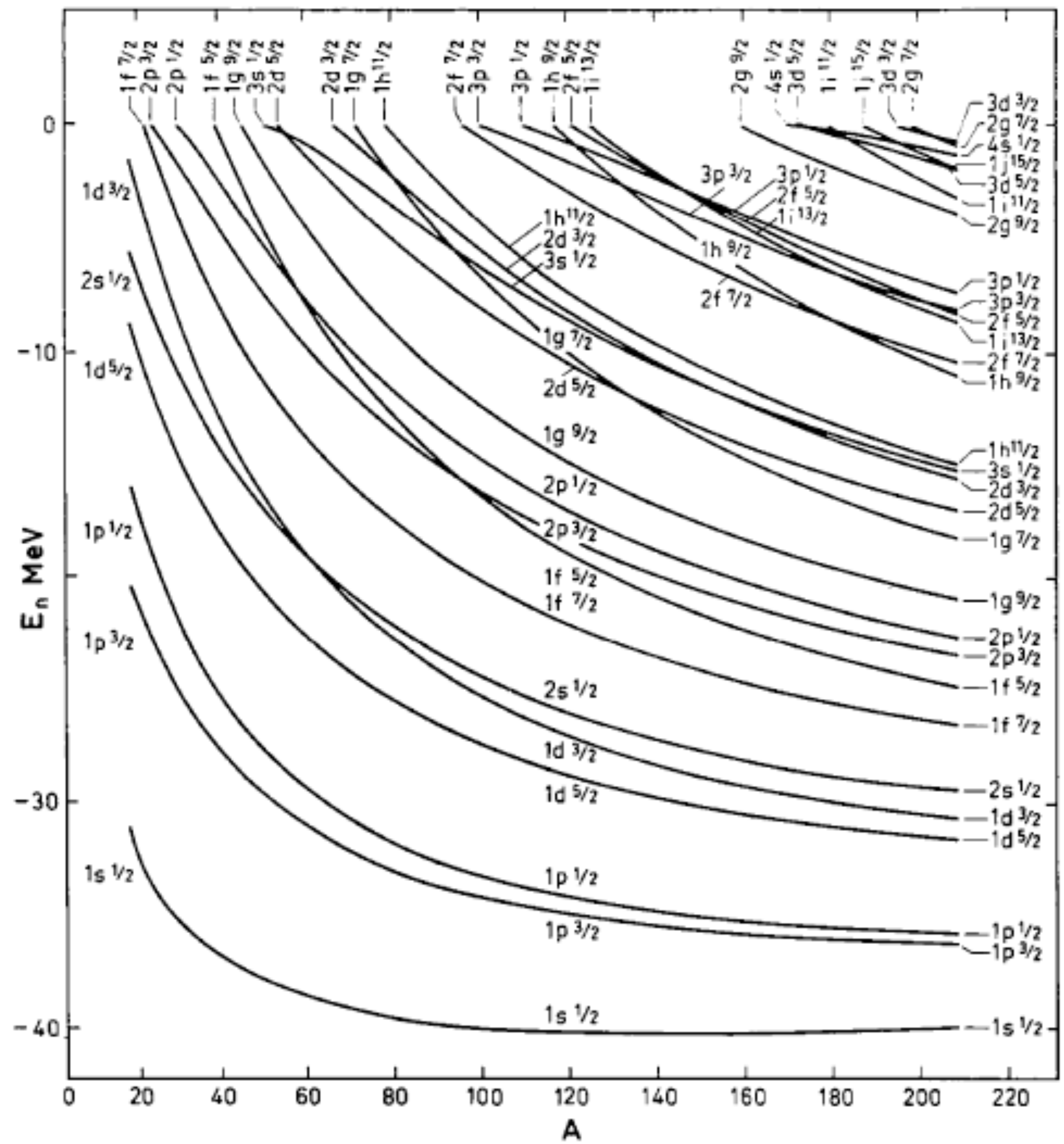
$$0i\tfrac{13}{2}$$

- Noticeably shifted
- Correct magic numbers!



Neutron levels as a function of A

- Phenomenological!
- Calculated from Woods-Saxon plus spin-orbit



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