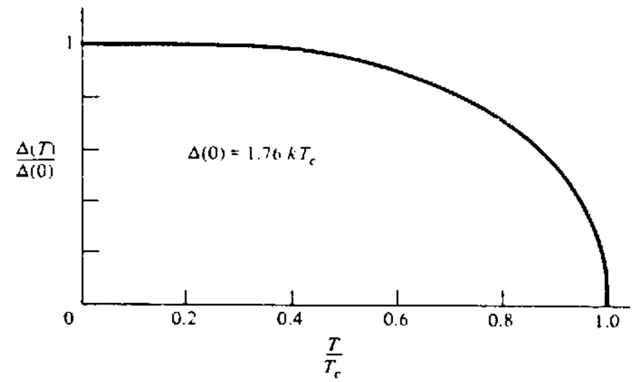
Pairing in nuclear matter

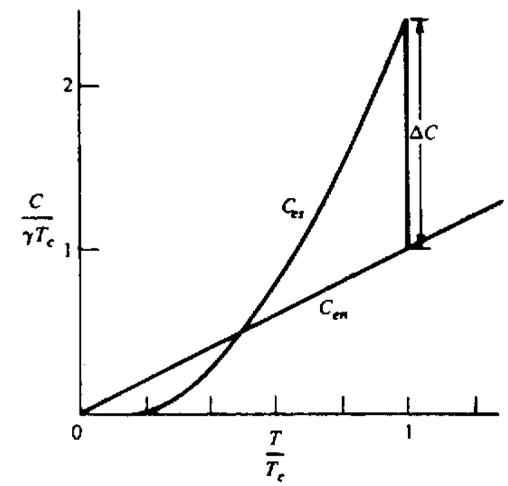
- Connection with neutron stars, very briefly...
- Hints from experimental data
- What is different in the medium...
- Cooper problem
- Gap equation and BCS for nuclear and neutron matter
- Spectroscopic factors in nuclei --> consequences
- Inclusion of realistic nucleon propagators
- Results
- Brief comparison with other many-body calculations

Pairing in neutron stars...

- 1959 Migdal suggests pairing in neutron stars before they are even observed...
- 1969 Vela and Crab pulsars exhibit sudden spin-ups (glitches)
- Relaxation to constant rate of slowing down too slow to be explained in terms of viscous processes of normal matter --> glitches --> superfluidity (Pines)

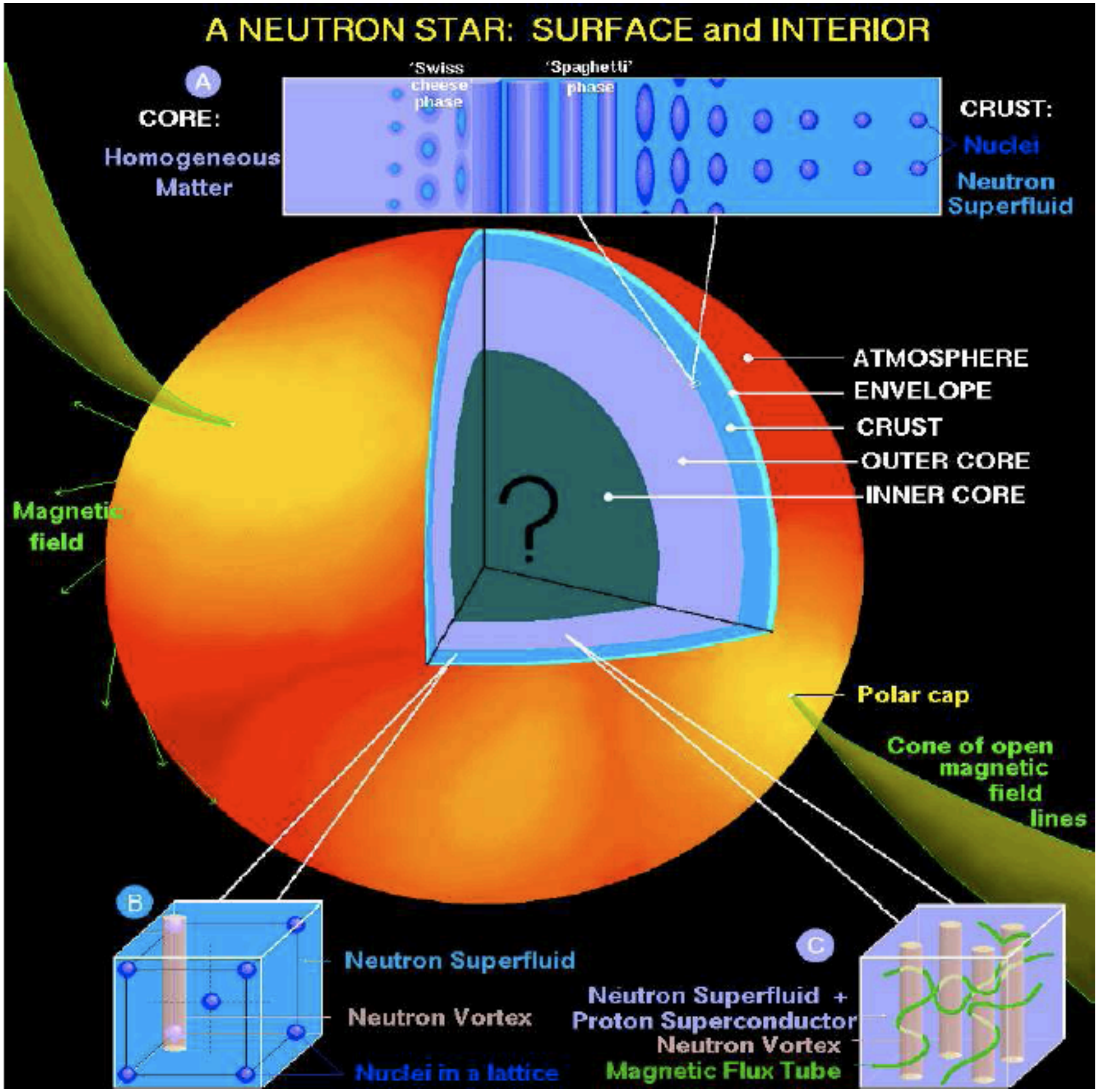


- Critical information: pairing gap as a function of temperature
- BCS yields--->



- Lots and lots of BCS calculations of neutron matter
- Also calculations of pairing in symmetric matter --> puzzle

Lots of things to consider



Dany Page UNAM

Pairing N*

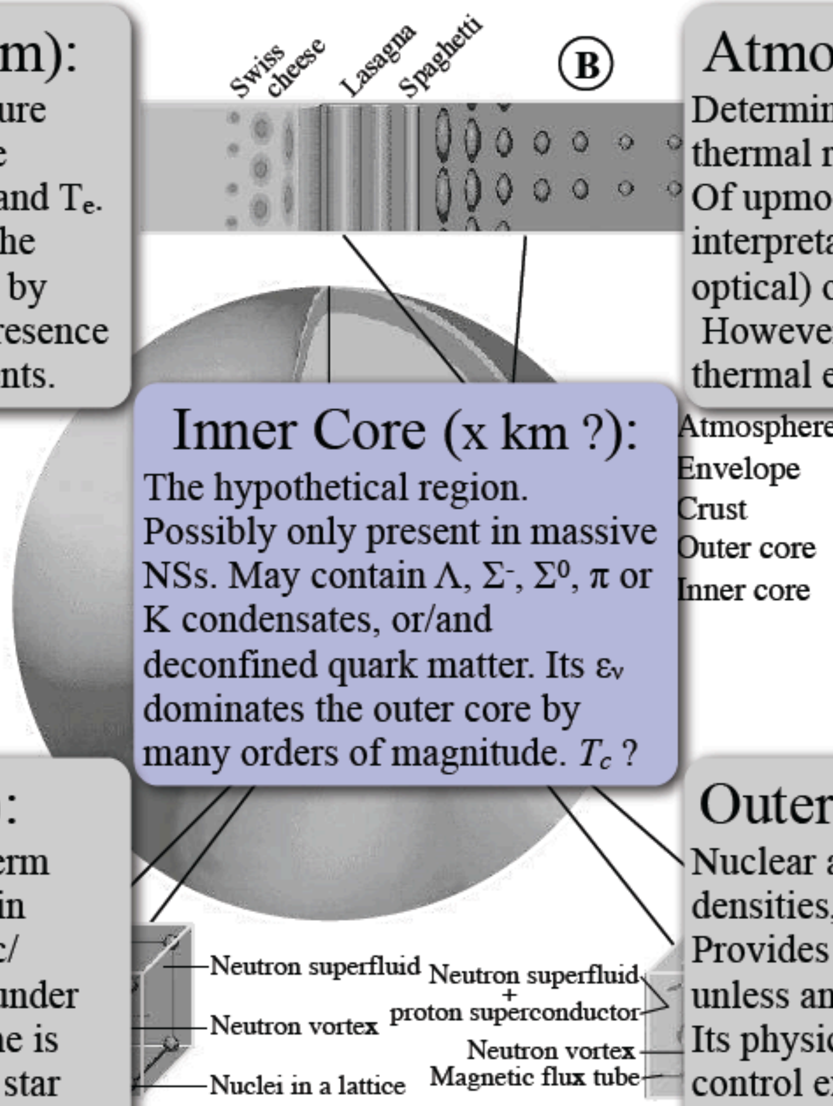
Reminder

Envelope (100 m):

Contains a huge temperature gradient: it determines the relationship between T_{int} and T_e. Extremely important for the cooling, strongly affected by magnetic fields and the presence of “polluting” light elements.

Atmosphere (10 cm):

Determines the shape of the thermal radiation (the spectrum). Of upmost importance for interpretation of X-ray (and optical) observation.
However it as NO effect on the thermal evolution of the star.



Inner Core (x km ?):

The hypothetical region. Possibly only present in massive NSs. May contain Λ, Σ^-, Σ^0, π or K condensates, or/and deconfined quark matter. Its ε_ν dominates the outer core by many orders of magnitude. T_c ?

Crust (1 km):

Little effect on the long term cooling. BUT: may contain heating sources (magnetic/ rotational, pycnonuclear under accretion). Its thermal time is important for very young star and for quasi-persistent accretion

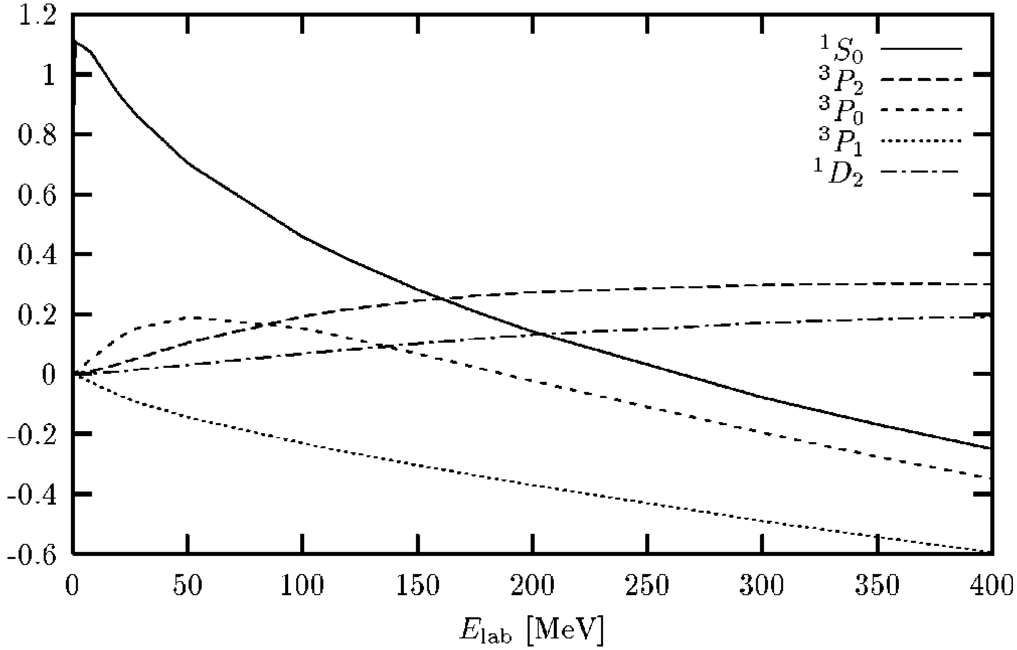
Outer Core (10-x km):

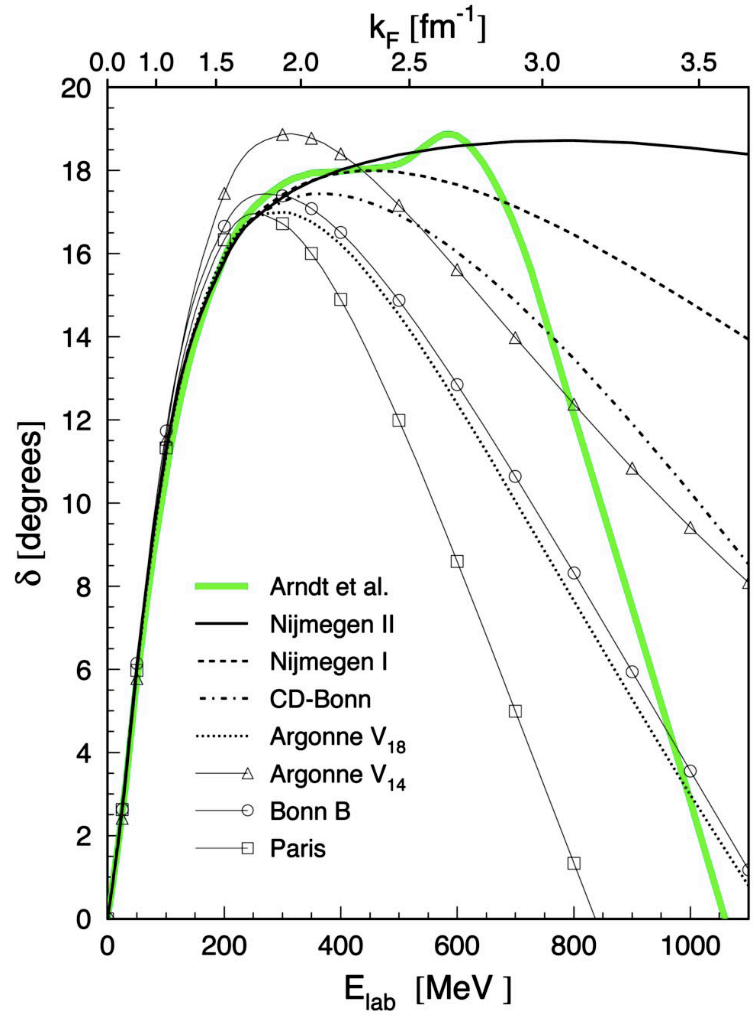
Nuclear and supranuclear densities, containing n, p, e & μ. Provides about 90% of c_v and ε_ν unless an inner core is present. Its physics is basically under control except pairing T_c which is essentially unknown.

“Dense Matter in Compact Stars: Theoretical Developments and Observational Constraints”, Page D. & Reddy S., 2006ARNPS..56..327P

NN interaction and phase shifts for T=1

- L+S+T --> odd (Pauli)
- T=1 --> L+S even
- Attraction: positive phase shift





- --> low density $^1S_0$ dominates with $^3P_2$ possibly at higher density

Review: e.g. Dean & Hjorth-Jensen, Rev.Mod.Phys.75, 607 (2003)

Low-density --> phase shifts almost enough

- BCS solution

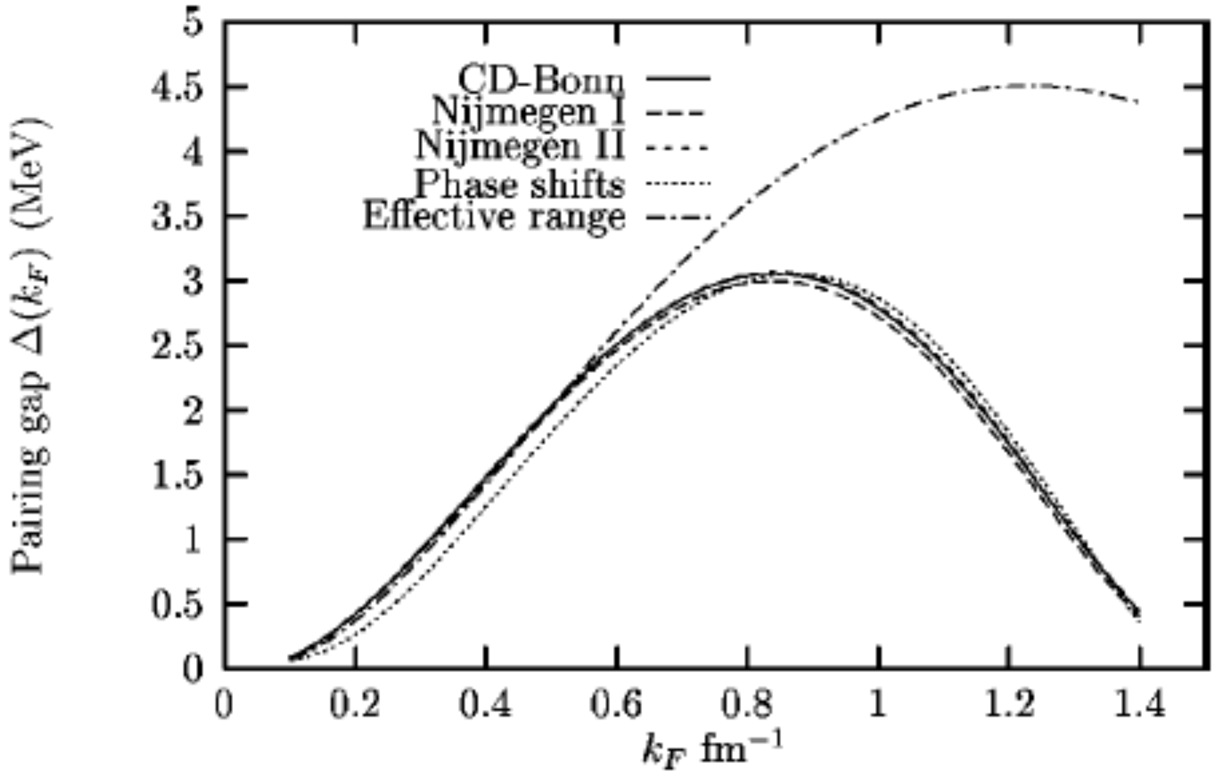


FIG. 6. $^1S_0$ energy gap in neutron matter with the CD-Bonn, Nijmegen I, and Nijmegen II potentials. In addition, we show the results obtained from phase shifts only, Eqs. (31)–(33), and the effective range approximation of Eq. (35). From Elgarøy and Hjorth-Jensen, 1998.

Pairing in nuclei: like nucleons (but angular momentum)

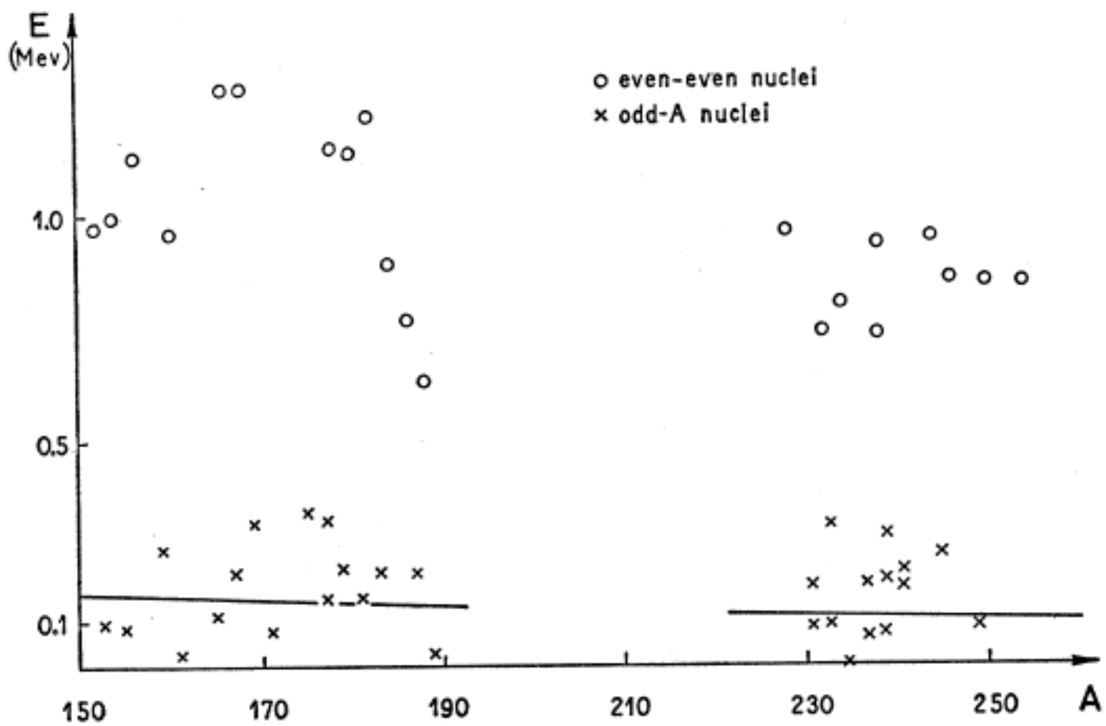
- "Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State", Bohr, Mottelson, Pines, 1958 Phys. Rev. 110, 936

EXCITATION SPECTRA OF NUCLEI 937

FIG. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-A nuclei (see reference 1).

The figure contains all the available data for nuclei with $150<A<190$ and $228<A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd-A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

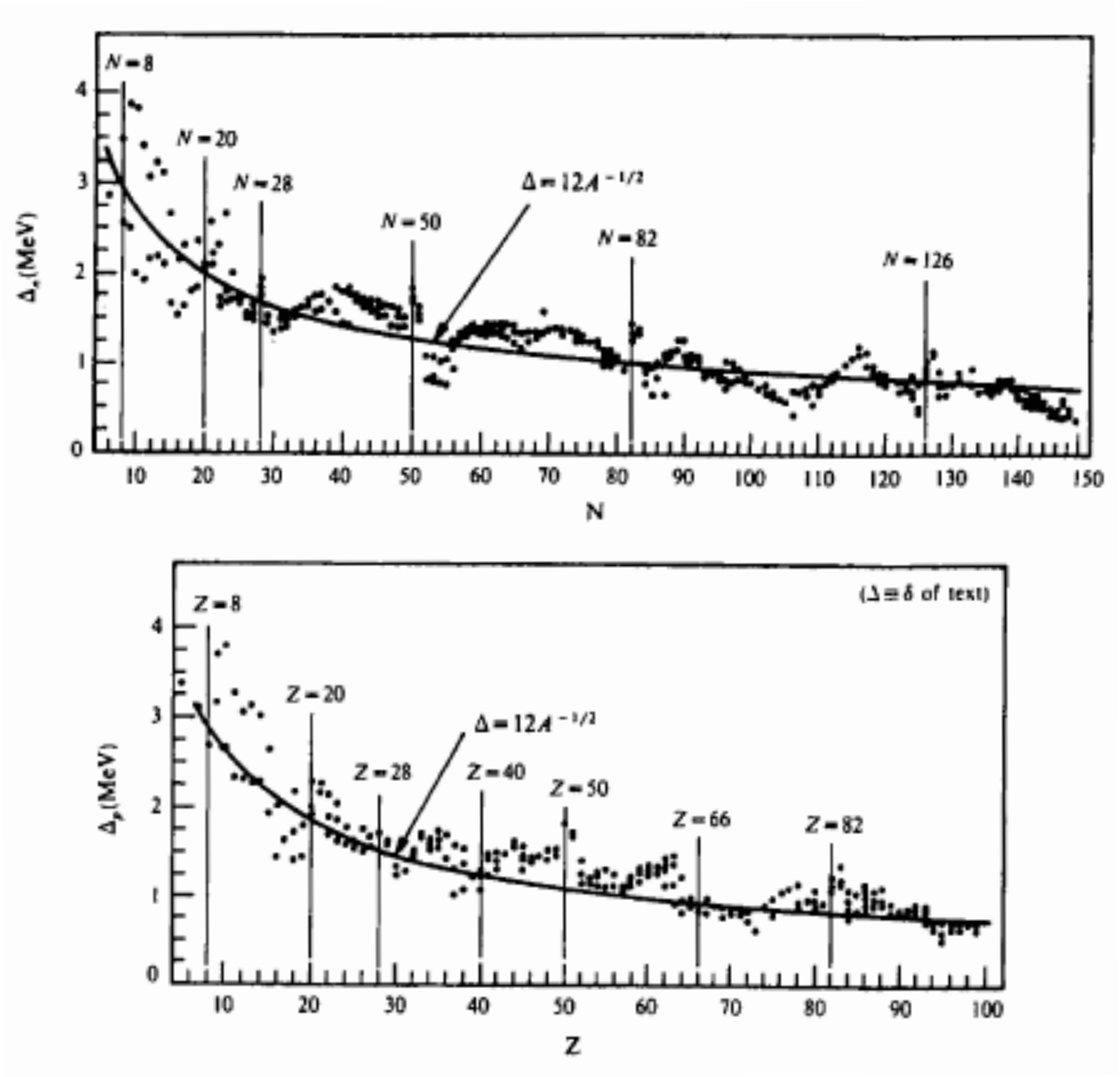
We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.



Semi-empirical mass formula

- Pairing contribution

$$\Delta = \frac{12}{A^{1/2}}$$



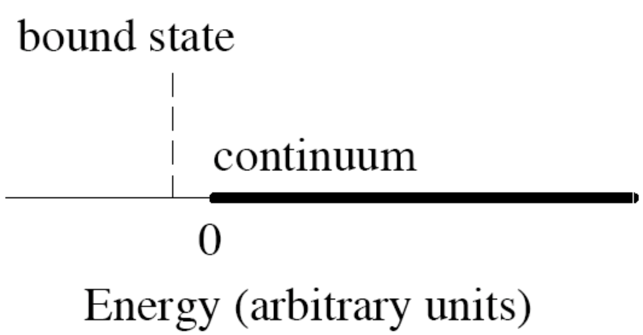
Pairing N*

Appearance of bound-pair states

- Reminder of appearance of bound states for free particles
- Write eigenvalue equation in wave vector space

$$\psi_n(\boldsymbol{k}; m_\alpha m_{\alpha'}) = \frac{1}{E_n - \hbar^2 \boldsymbol{k}^2/m} \frac{1}{2} \sum_{m_\gamma m_{\gamma'}} \int \frac{d^3 q}{(2\pi)^3} \langle \boldsymbol{k} m_\alpha m_{\alpha'} | V | \boldsymbol{q} m_\gamma m_{\gamma'} \rangle \psi_n(\boldsymbol{q}; m_\gamma m_{\gamma'})$$

- Two electrons or two ^{3}He atoms with spin $\frac{1}{2}$ have antisymmetry requirement $\ell + S$ even
- For $\ell = 0$ spin $S = 0$
- For $\ell = 1$ spin $S = 1$ and so on
- In this basis $\psi_n(k; \ell S) = \dfrac{1}{E_n - \hbar^2 \boldsymbol{k}^2/m} \dfrac{1}{2} \displaystyle\int \frac{dq\ q^2}{(2\pi)^3} \langle k | V^{\ell S} | q \rangle \psi_n(q; \ell S)$
- Visualize appearance of bound state

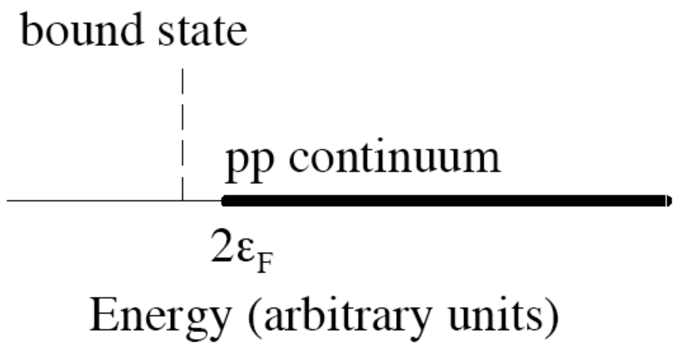


In the medium --> Cooper problem

- Two particles on top of the Fermi sea
- Most favorable total wave vector zero

$$G_{pp}^{(0)}(\boldsymbol{K}=0,q;E)=\frac{\theta(q-k_F)}{E-2\varepsilon(q)+i\eta}$$

- Similar to free space



- Eigenvalue equation

$$\psi_C(k;\ell S)=\frac{\theta(k-k_F)}{E_C-2\varepsilon(k)}\,\frac{1}{2}\int\frac{dq\;q^2}{(2\pi)^3}\,\langle k|\,V^{\ell S}\,|q\rangle\,\psi_C(q;\ell S)$$

- Subscript C for Cooper
- Use separable interaction to illustrate properties

Cooper problem

- Interaction $\langle k| V^{\ell S} |q\rangle = \lambda_\ell w_\ell(k) w_\ell^*(q)$
- S implied
- Substitute -------------------> $\psi_C(k;\ell S) = \mathcal{N} \dfrac{\theta(k-k_F) w_\ell(k)}{E_C - 2\varepsilon(k)}$
- with $\mathcal{N} = \frac{1}{2}\lambda_\ell \int \frac{dq\ q^2}{(2\pi)^3} w_\ell^*(q)\psi_C(q;\ell S)$
- Amplitude substituted in eigenvalue equation yields

$$\frac{1}{\lambda_\ell} = \frac{1}{2}\int \frac{dq\ q^2}{(2\pi)^3} \frac{\theta(q-k_F)|w_\ell(q)|^2}{E_C - 2\varepsilon(q)}$$

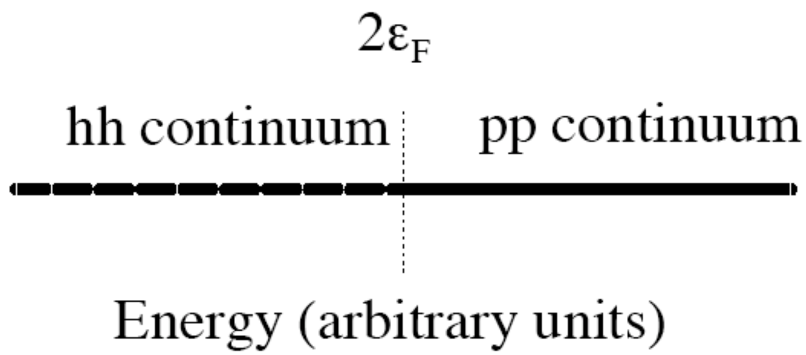
- Right side negative definite for energy below pp continuum, diverging to $-\infty$ when approaching this limit
- So always solution for attractive interaction!
- None for repulsive interaction
- Peculiarity: bound state resides in hh continuum...

Inclusion of hh propagation

- Attempt to include hh propagation in eigenvalue equation

$$\begin{aligned}\psi_C(k;\ell S) &= \frac{\theta(k-k_F)}{E_C-2\varepsilon(k)}\,\frac{1}{2}\int\frac{dq\;q^2}{(2\pi)^3}\,\langle k|\,V^{\ell S}\,|q\rangle\,\psi_C(q;\ell S)\\ &- \frac{\theta(k_F-k)}{E_C-2\varepsilon(k)}\,\frac{1}{2}\int\frac{dq\;q^2}{(2\pi)^3}\,\langle k|\,V^{\ell S}\,|q\rangle\,\psi_C(q;\ell S)\end{aligned}$$

- Visualize unperturbed spectrum
- No "room" for bound states
- Either pp or hh



- Not possible to have discrete (real) eigenvalues for an attractive interaction
- Instead yields complex eigenvalues signaling instability of starting point (pairing instability)

Bound-pair states

- Consider original propagator equation
- Cannot legitimately eliminate noninteracting propagator
- Unless there is a *GAP* in the sp spectrum at k_F
- Add auxiliary sp potential with a constant shift Δ below k_F
- Implies gap of 2Δ between pp and hh continuum
- Now a legitimate eigenvalue problem can be obtained
- Use separable interaction to get transition amplitudes

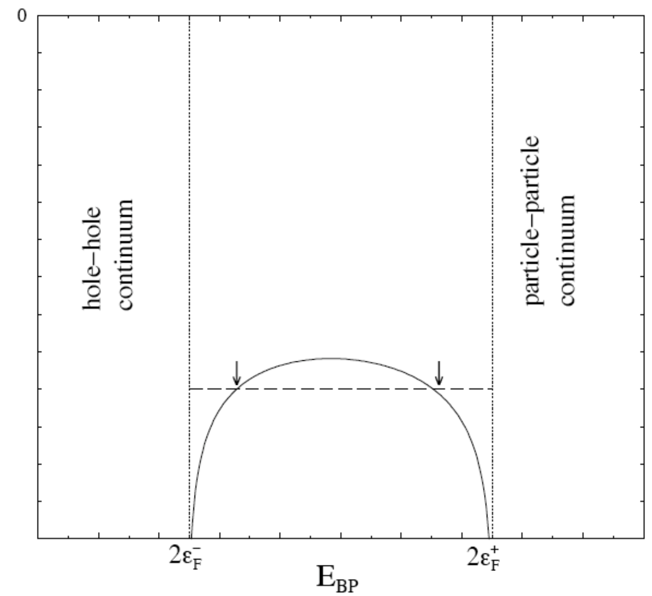
$$\psi_{BP}(k;\ell S) = \mathcal{N}\frac{\theta(k-k_F)w_\ell(k)}{E_{BP}-2\varepsilon(k)} \qquad \psi_{BP}(k;\ell S) = -\mathcal{N}\frac{\theta(k_F-k)w_\ell(k)}{E_{BP}-2\varepsilon(k)}$$

- and eigenvalue problem

$$\frac{1}{\lambda_\ell} = \frac{1}{2}\int\frac{dq\ q^2}{(2\pi)^3}\frac{\theta(q-k_F)|w_\ell(q)|^2}{E_{BP}-2\varepsilon(q)} - \frac{1}{2}\int\frac{dq\ q^2}{(2\pi)^3}\frac{\theta(k_F-q)|w_\ell(q)|^2}{E_{BP}-2\varepsilon(q)}$$

Graphical illustration

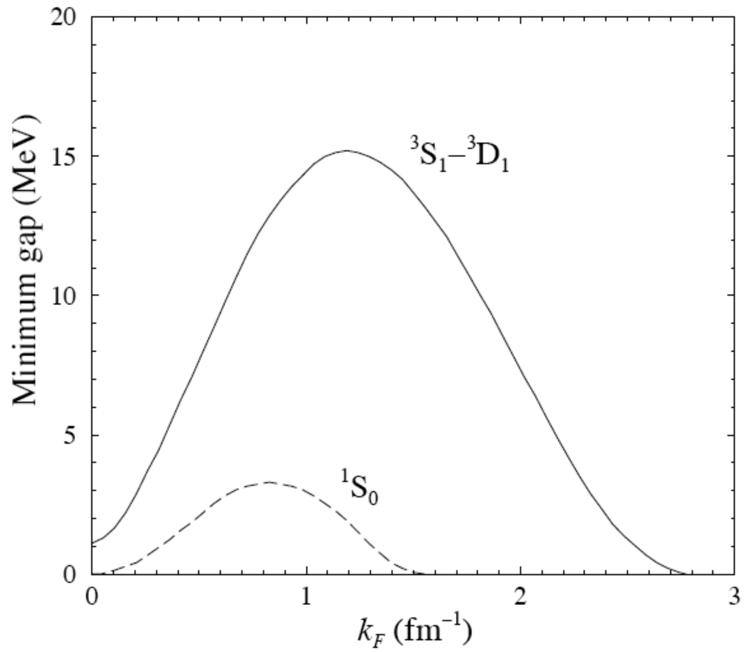
- Plot right side of $$\frac{1}{\lambda_\ell} = \frac{1}{2}\int \frac{dq\ q^2}{(2\pi)^3}\frac{\theta(q-k_F)|w_\ell(q)|^2}{E_{BP}-2\varepsilon(q)} - \frac{1}{2}\int \frac{dq\ q^2}{(2\pi)^3}\frac{\theta(k_F-q)|w_\ell(q)|^2}{E_{BP}-2\varepsilon(q)}$$
- as a function of E_{BP} between pp and hh continuum
- Both terms yield negative contributions diverging near respective boundaries
- Only solutions for attraction indicated for one choice by horizontal dashed line
- Even true for very small coupling constant
- Stronger attraction -> complex eigenvalues



Can always get real eigenvalues by increasing gap!

Bound-pair states in nuclear matter N=Z

- Free space interaction generates deuteron bound state
- Scattering phase shifts indicate strong attraction in the medium
- Relevant eigenvalue problem (with gap in sp spectrum)



$$
\begin{aligned}
\psi_{BP}(k;(\ell S)JT) &= \frac{\theta(k-k_F)}{E_{BS}-2\varepsilon(k)}\,\frac{1}{2}\sum_{\ell'}\int\frac{dq\;q^2}{(2\pi)^3}\,\langle k\ell|\,V^{JST}\,|q\ell'\rangle\,\psi_{BP}(q;(\ell' S)JT) \\
&- \frac{\theta(k_F-k)}{E_{BS}-2\varepsilon(k)}\,\frac{1}{2}\sum_{\ell'}\int\frac{dq\;q^2}{(2\pi)^3}\,\langle k\ell|\,V^{JST}\,|q\ell'\rangle\,\psi_{C}(q;(\ell' S)JT)
\end{aligned}
$$

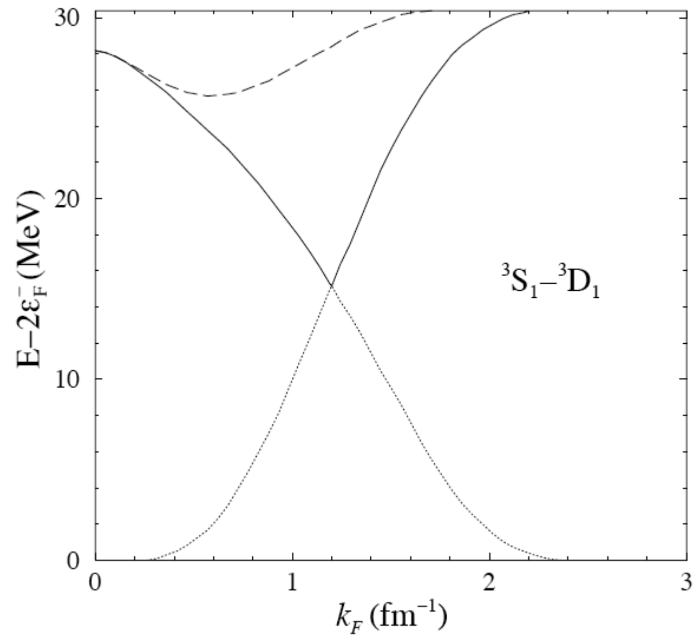
- Gap required to avoid pairing instability sensitive function of density both for $^3S_1$-$^3D_1$ and $^1S_0$

Note zero density limit deuteron channel

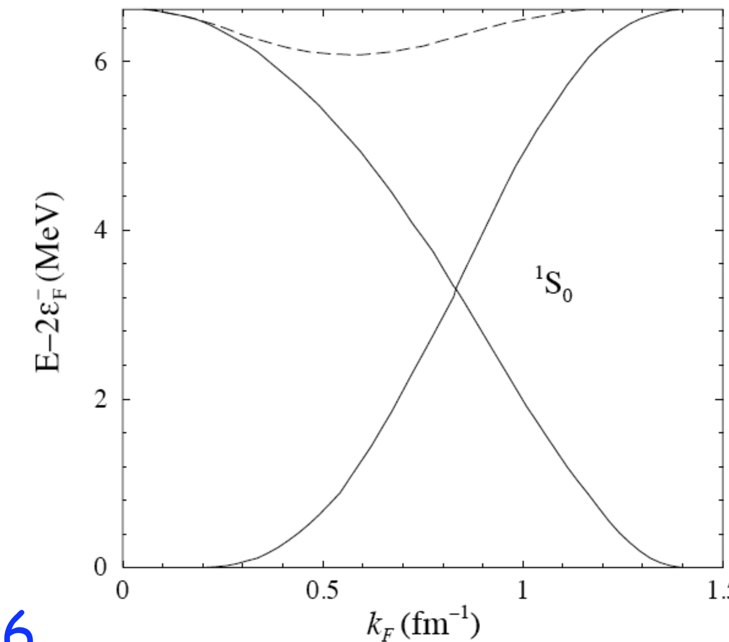
Pairing N*

Bound-pair eigenvalues

- Gap required to high density
- Deuteron attraction greater than $^1S_0$
- Maximum sp gap ~ 15 MeV at k_F=1.2fm^{-1}
- Keep this gap for all densities to study eigenvalues

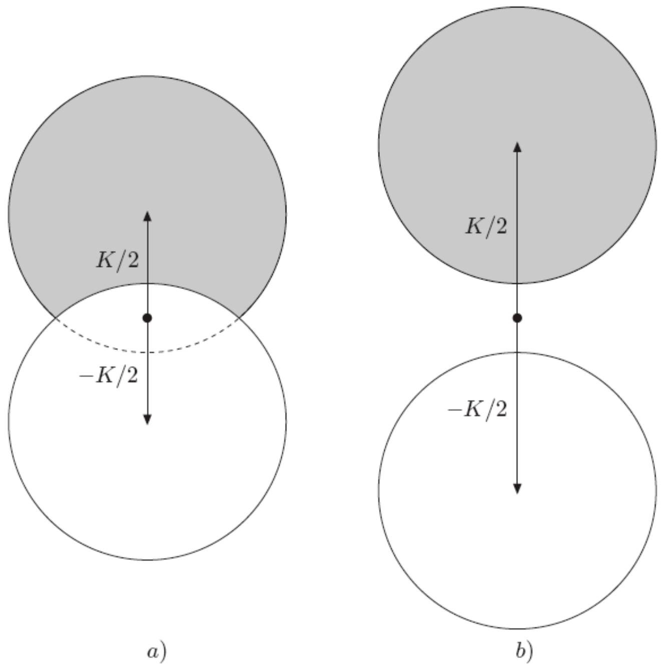


- Similarly for $^1S_0$ (> 3MeV gap)
- Also Cooper eigenvalue
- BCS approximately matches these results --> include gap in spectrum self-consistently --> gap equation



Phase space and Pauli principle

- Introduces total wave vector dependence illustrated in figure
- a) total wave vector < $2k_F$
- b) $>2k_F$
- Constraint by step functions
- Outside both spheres: pp
- Inside both: hh
- Most phase space for $|\mathbf{K}|=0$
- Extremely relevant for possible bound states...



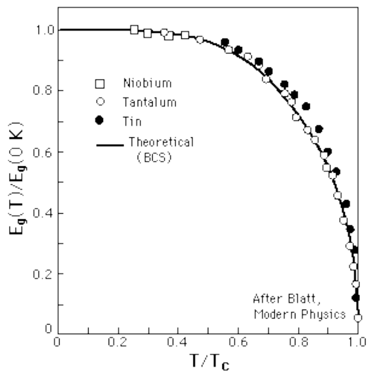
Pairing N*

Other systems

- Superconductivity in metals
  - resistance to electric current drops below critical temperature
  - current in superconducting ring persists without dissipation
  - 1911 discovery --> 1957 explanation
  - problem: convert repulsive Coulomb --> attractive interaction
  - isotope effect (critical T depends of mass of ions) --> electron-phonon interaction important
  - e-e interaction through exchange of lattice vibrations
  - Fröhlich interaction

$$(\boldsymbol{p}_1\boldsymbol{p}_2|\,V(E)\,|\boldsymbol{p}_3\boldsymbol{p}_4) = \delta_{\boldsymbol{p}_1+\boldsymbol{p}_2,\boldsymbol{p}_3+\boldsymbol{p}_4}\frac{1}{V}\gamma^2\frac{\Omega_{\boldsymbol{Q}}^2}{E^2-\Omega_{\boldsymbol{Q}}^2}\theta(\Omega_D-\Omega_{\boldsymbol{Q}})$$

    - phonon spectrum; electron-phonon coupling; energy transfer; Debye frequency (maximal allowed in discrete lattice)
    - attractive for $|E| < \Omega_{\boldsymbol{Q}} < \Omega_D$ can overcome screened Coulomb but only in a domain of ~10^{-2} eV around Fermi energy
    - gaps tiny (example static approximation) ~10^{-4} eV



Superfluidity in ^3He

- Transition below 3mK
- Pair state with L=1 and S=1
- Anisotropic superfluid (in metals S=0 isotropic)

Neutron stars

- BCS with free NN interaction for neutrons
- low density $^1S_0$ pairing perhaps $^3P_2$-$^3F_2$ at higher density
- also $^1S_0$ proton superconductivity (beta-equilibrium)

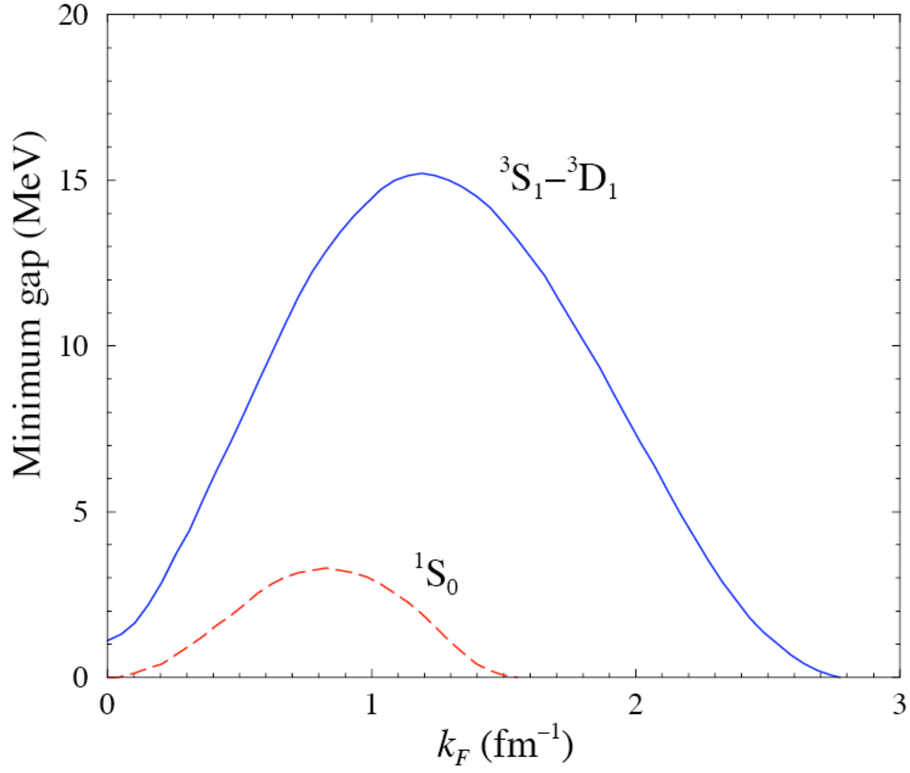
Some pairing issues in infinite matter

- Gap size in nuclear matter & neutron matter
- Density & temperature range of superfluidity
- Resolution of $^3S_1$-$^3D_1$ puzzle (size of pn pairing gap)
- Influence of short-range correlations (SRC)
- Influence of polarization contributions
- Relation of infinite matter results & finite nuclei

Review: e.g. Dean & Hjorth-Jensen, RMP75, 607 (2003)

Results from:
H. Müther and WHD
Pairing properties of nucleonic matter employing dressed nucleons.
Phys. Rev. C72, 054313 (2005)

Puzzle related to gap size in $^3S_1$-$^3D_1$ channel



Mean-field particles

Early nineties: BCS gaps ~ 10 MeV

Alm et al. Z.Phys.A337,355 (1990)
Vonderfecht et al. PLB253,1 (1991)
Baldo et al. PLB283, 8 (1992)

Dressing nucleons is expected to reduce pairing strength as suggested by in-medium scattering

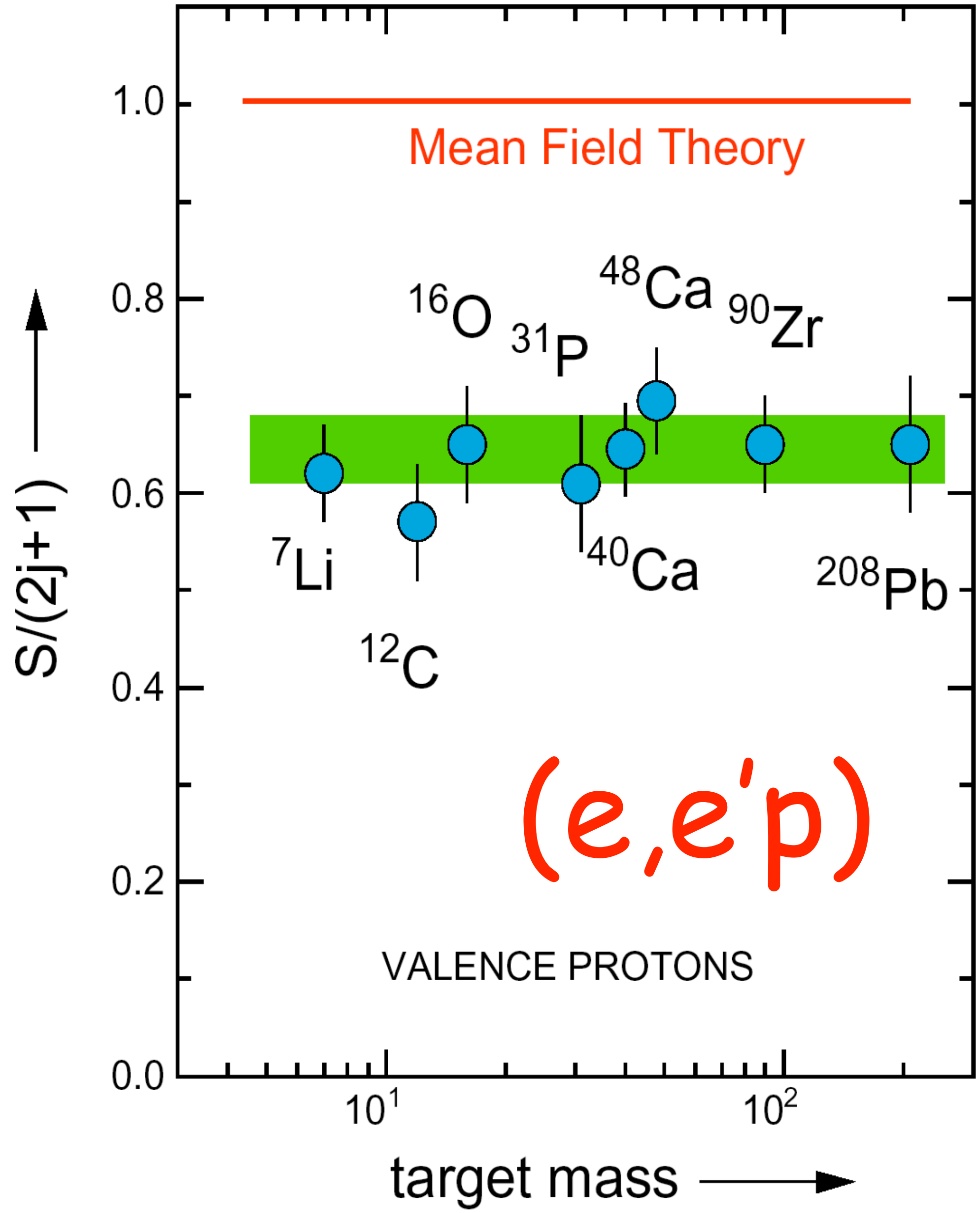
Removal probability for valence protons from NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)

S ≈ 0.65 for valence protons
Reduction ⇒ both SRC and LRC

Weak probe but propagation in the nucleus of removed proton using standard optical potentials to generate distorted waves --> associated uncertainty ~ 5-10%

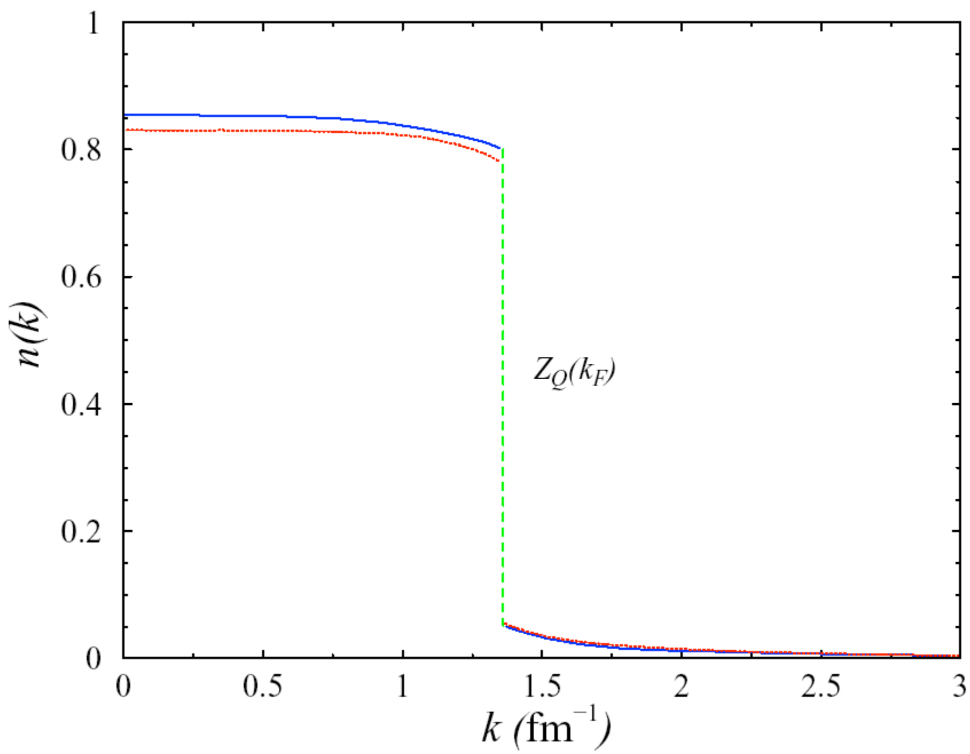
Why: details of the interior scattering wave function uncertain since non-locality is not constrained (so far)



Pairing N*

Results from Nuclear Matter (N=Z)

2nd generation (2000)



Momentum distribution: only minor changes when self-consistency is included

Occupation in nuclei: Depleted similarly!

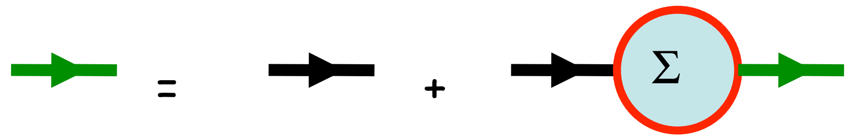
Thesis Libby Roth Stoddard (2000)

Pairing N*

Green´s function and Γ-matrix approach (ladders)

Single-particle Green´s function $G(k,t_1,t_2)=-i\langle T\, c_k(t_1)c_k^+(t_2)\rangle$

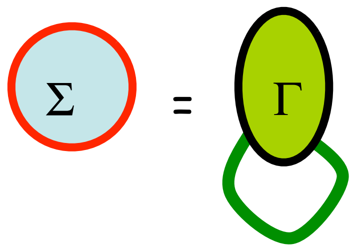
Dyson equation:



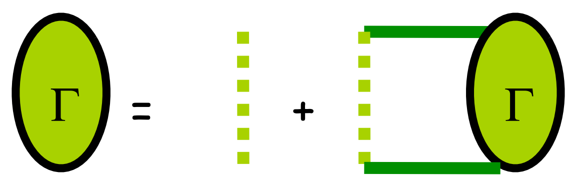
$$G(k,\omega)=G^{(0)}(k,\omega)+G^{(0)}(k,\omega)\Sigma(k,\omega)G(k,\omega)$$

$$G(k,\omega) \;=\; \frac{1}{\omega-k^2/2m-\Sigma(k,\omega)} \quad\Rightarrow\quad S(k,\omega)=-2\,\mathrm{Im}\,G(k,\omega)$$

Self-energy



, Γ-matrix

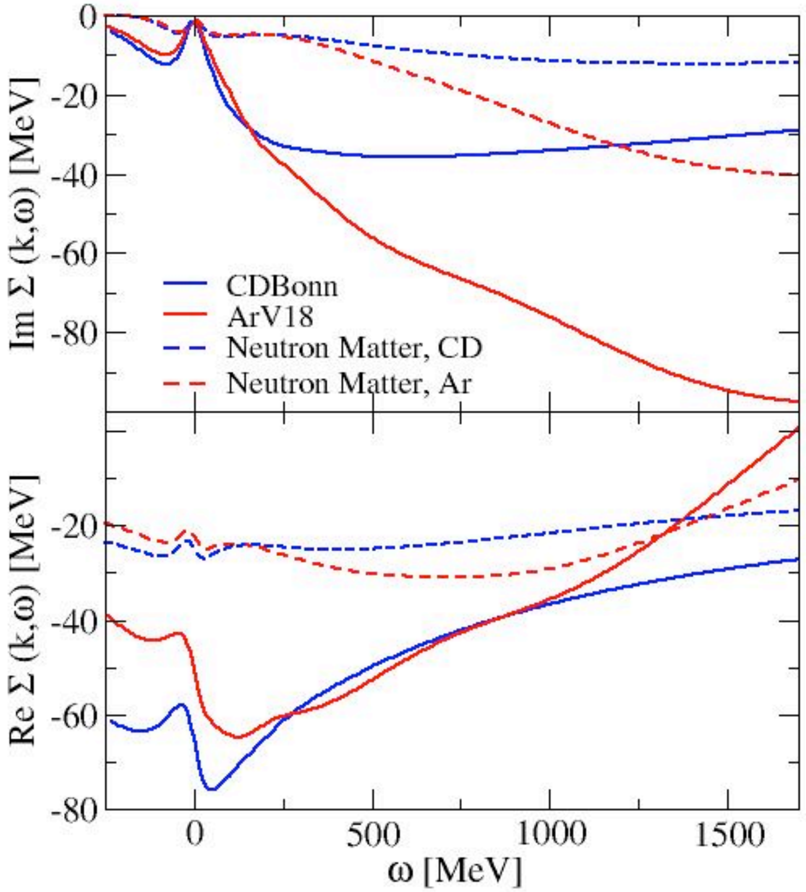


- Pairing instability possible
- Finite temperature calculation can avoid this

Pairing N*

Self-energy

$$G(k,\omega) = \frac{1}{\omega - k^2/2m - \Sigma(k,\omega)} \quad \Rightarrow \quad S(k,\omega) = -2\,\mathrm{Im}\,G(k,\omega)$$



Real and imaginary part of the retarded self-energy

- $k_F = 1.35\ \mathrm{fm}^{-1}$
- $T = 5$ MeV
- $k = 1.14\ \mathrm{fm}^{-1}$

Note differences due to NN interaction

Pairing N*

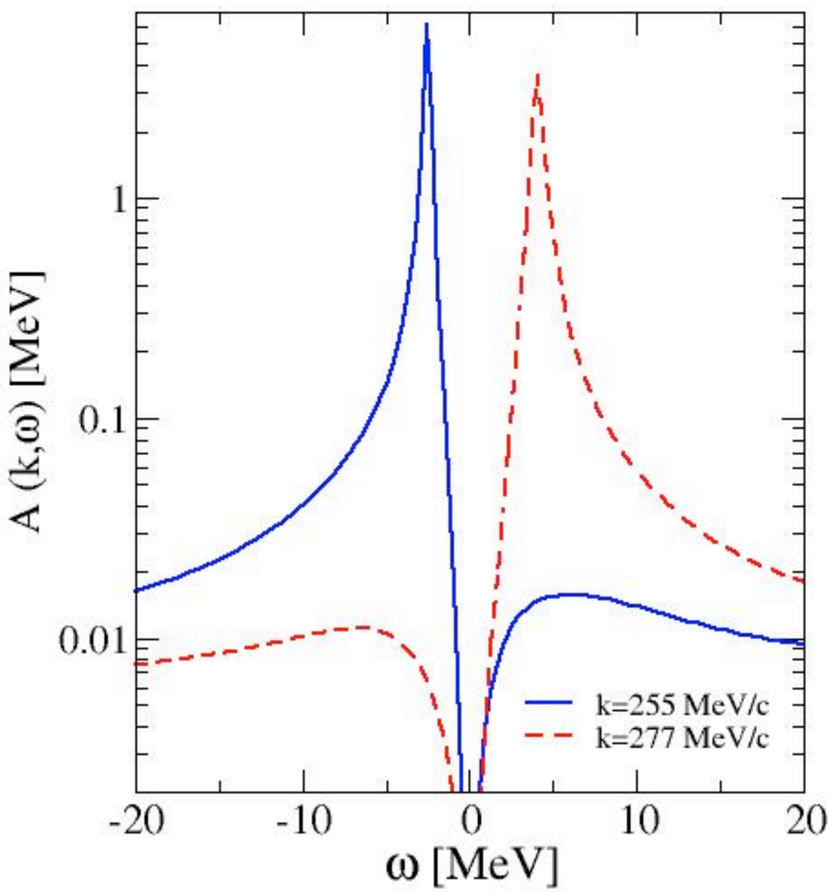
Spectral functions

•Strength above and below the Fermi energy as in BCS

• But broad distribution in energy

• BCS not just a cartoon of SCGF but both features must be considered in a consistent way

• CDBonn interaction at “T=0“



Pairing N*

BCS: a reminder

NN correlations on top of Hartree-Fock: $\varepsilon_k, \quad c_k^+$

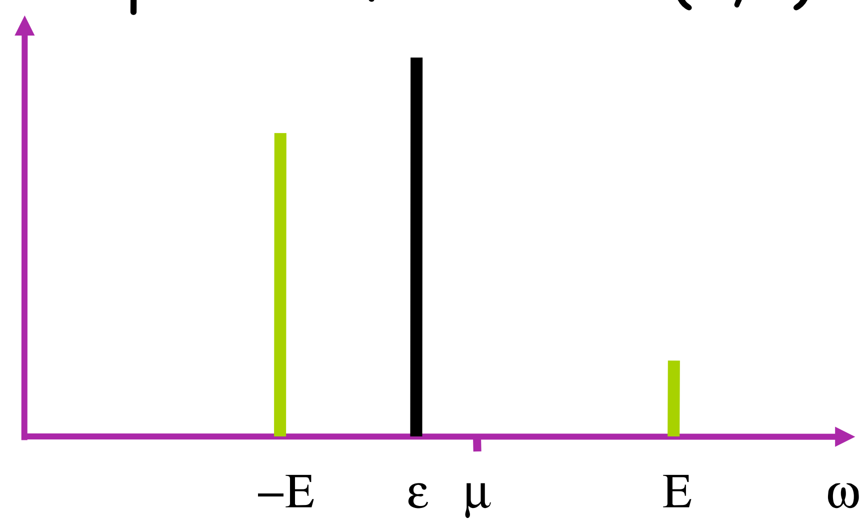
Bogoliubov transformation $\quad a_k^+ = u_k c_k^+ + v_k c_{\bar{k}}$

with $\begin{matrix} u_k^2 \\ v_k^2 \end{matrix} = \frac{1}{2}\left[1 \pm \frac{\varepsilon_k - \mu}{\sqrt{(\varepsilon_k - \mu)^2 + \Delta(k)^2}}\right]$, $E(k) = \sqrt{(\varepsilon_k - \mu)^2 + \Delta(k)^2}$

Gap equation

$$\Delta(k) = \int k'^2 dk' < k, \bar{k} \,|\, V \,|\, k', \bar{k}' > \frac{\Delta(k')}{-2E(k)}$$

Spectral function S(k,ω)



Pairing N*

Solution of the gap equation

$$\Delta(k) = \sum_{k'} <k,\bar{k}|V|k',\bar{k}'> \frac{\Delta(k')}{\omega - 2E(k)}$$ with $$E(k) = \sqrt{(\varepsilon_k - \mu)^2 + \Delta(k)^2}$$ and ω=0

Define: $$\delta(k) = \frac{\Delta(k)}{\omega - 2E(k)}$$

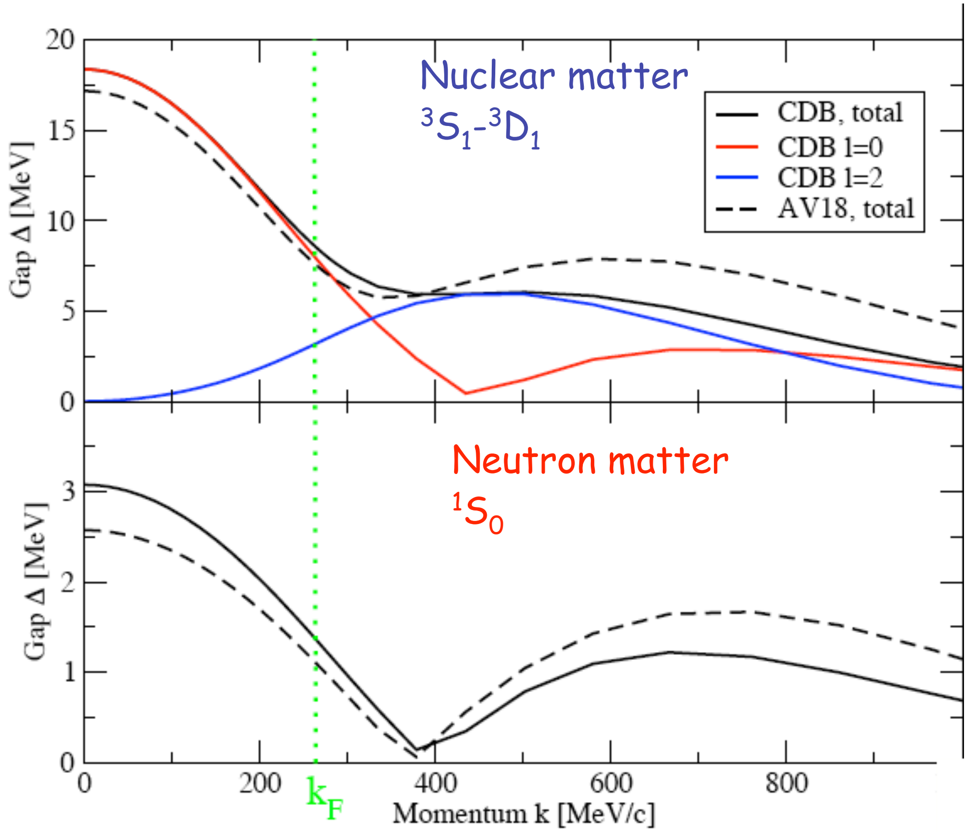
$$\begin{pmatrix} 2E(k)+<k|V|k>, & \ldots, & <k|V|k'> \\ \vdots & \ddots & \vdots \\ <k'|V|k>, & \cdots, & 2E(k')+<k'|V|k'> \end{pmatrix} \begin{pmatrix} \delta(k) \\ \vdots \\ \delta(k') \end{pmatrix} = \omega \begin{pmatrix} \delta(k) \\ \vdots \\ \delta(k') \end{pmatrix}$$

Eigenvalue problem for a pair of nucleons at ω=0

Steps of the calculation:

- Assume Δ(k) and determine E(k)
- Solve eigenvalue equation and evaluate new Δ(k)
  - If lowest eigenvalue ω<0 enhance Δ(k) (resp. δ(k))
  - If lowest eigenvalue ω>0 reduce Δ(k)
- Repeat until convergence

Gaps from BCS for realistic interactions



- momentum dependence $\Delta(k)$
- different NN interactions
- very similar to pairing gaps in finite nuclei for like particles...!?
- for np pairing no strong empirical evidence...?!

Early nineties: BCS gaps ~ 10 MeV

Alm et al. Z.Phys.A337,355 (1990)
Vonderfecht et al. PLB253,1 (1991)
Baldo et al. PLB283, 8 (1992)

T = 0
Mean-field particles

Beyond BCS in the framework of SCGF

Generalized Green's functions: Extend $G(k,t_1,t_2) = -i\langle T\, c_k(t_1) c_k^+(t_2)\rangle$

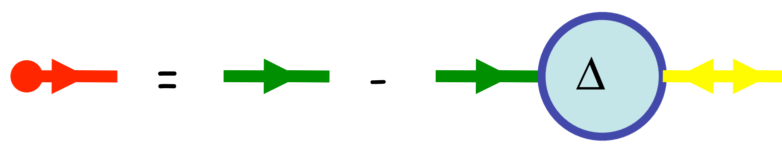
Anomalous propagators

$$G(k,t_1,t_2) = \begin{pmatrix} -i\langle Tcc^+\rangle & -i\langle Tcc\rangle \\ i\langle Tc^+c^+\rangle & i\langle Tc^+c\rangle \end{pmatrix} = \begin{pmatrix} G & F \\ F^+ & \overline{G} \end{pmatrix}$$

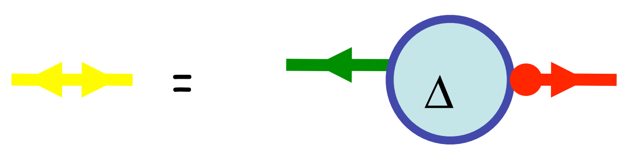
Generalized Dyson equation: Gorkov equations

$$\begin{pmatrix} \omega - t_k - \Sigma(k,\omega) & -\Delta(k,\omega) \\ -\Delta^+(k,\omega) & \omega + t_k + \Sigma(k,\omega) \end{pmatrix} \begin{pmatrix} G_{pair}(k,\omega) & F(k,\omega) \\ F^+(k,\omega) & \overline{G}_{pair}(k,\omega) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Leads to e.g.

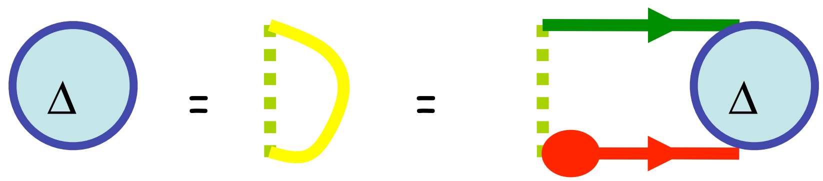


$G_{pair} \quad = \quad G \quad - \quad G \quad \Delta \quad F$



G includes all normal self-energy terms

Anomalous self-energy: Δ & generalized Gap equation



$$\Delta(k) \;=\; \int k'^2 dk' \langle k|V|k'\rangle \int d\omega \int d\omega' \frac{1-f(\omega)-f(\omega')}{-\omega-\omega'} S(k',\omega)\, S_{pair}(k',\omega') \quad \Delta(k')$$

Fermi function $\quad f(\omega)=\dfrac{1}{e^{\beta\omega}+1}$

If we replace $S(k,\omega)$ by "HF" approx. and $S_{pair}(k,\omega)$ by BCS:

⇨ Usual Gap equation

If we take $S_{pair}(k,\omega) = S(k,\omega)$:

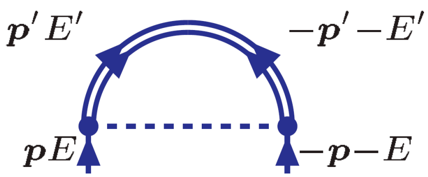
⇨ Corresponds to the homogeneous solution of Γ-matrix eq.

With $S_{pair}(k,\omega)$:

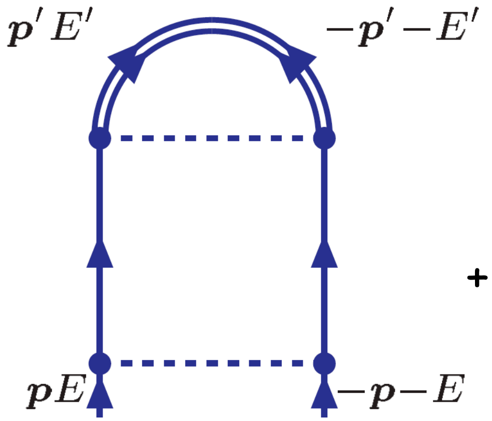
⇨ The above and self-consistency

Pairing N*

Consistency of Gap equation (anomalous self-energy) and Ladder diagrams



Iteration of Gorkov equations for anomalous propagator generates $\Rightarrow$



+ ...

... and all other ladder diagrams at
total momentum and energy zero (w.r.t. 2μ)
plus anomalous self-energy terms in normal part of propagator

So truly consistent with inclusion of ladder diagrams at other total momenta and energies

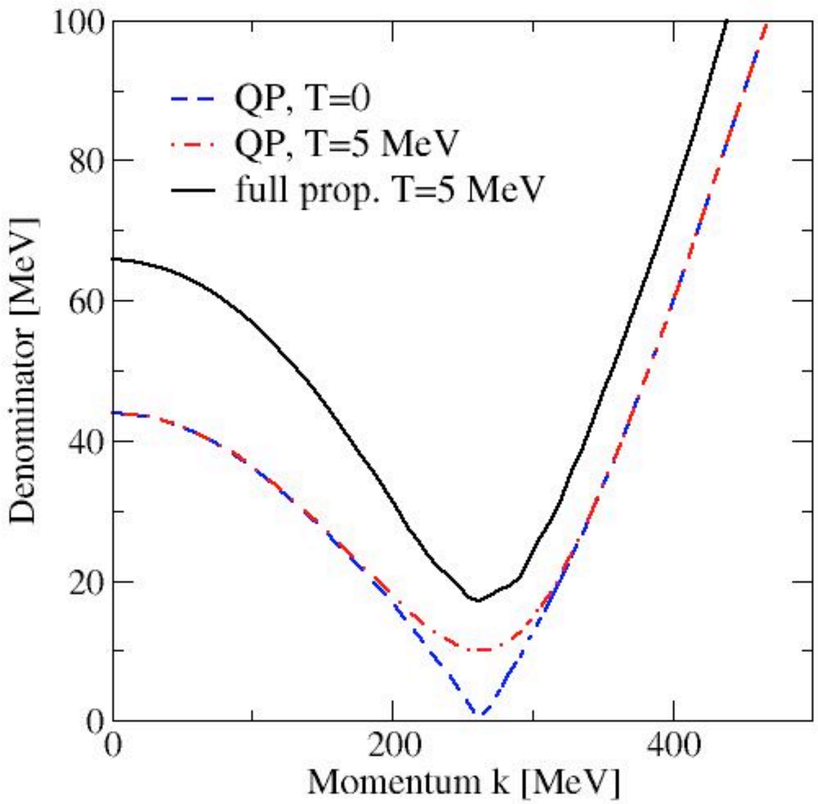
Pairing N*

Features of generalized gap equation

$$\Delta(k) \;=\; \int k'^2 dk' \langle k|V|k'\rangle \int d\omega \int d\omega' \frac{1-f(\omega)-f(\omega')}{-\omega-\omega'} S(k',\omega)\, S_{\cancel{pair}}(k',\omega') \quad \Delta(k')$$

$$-\frac{1}{2\tilde{\chi}_{k'}}$$

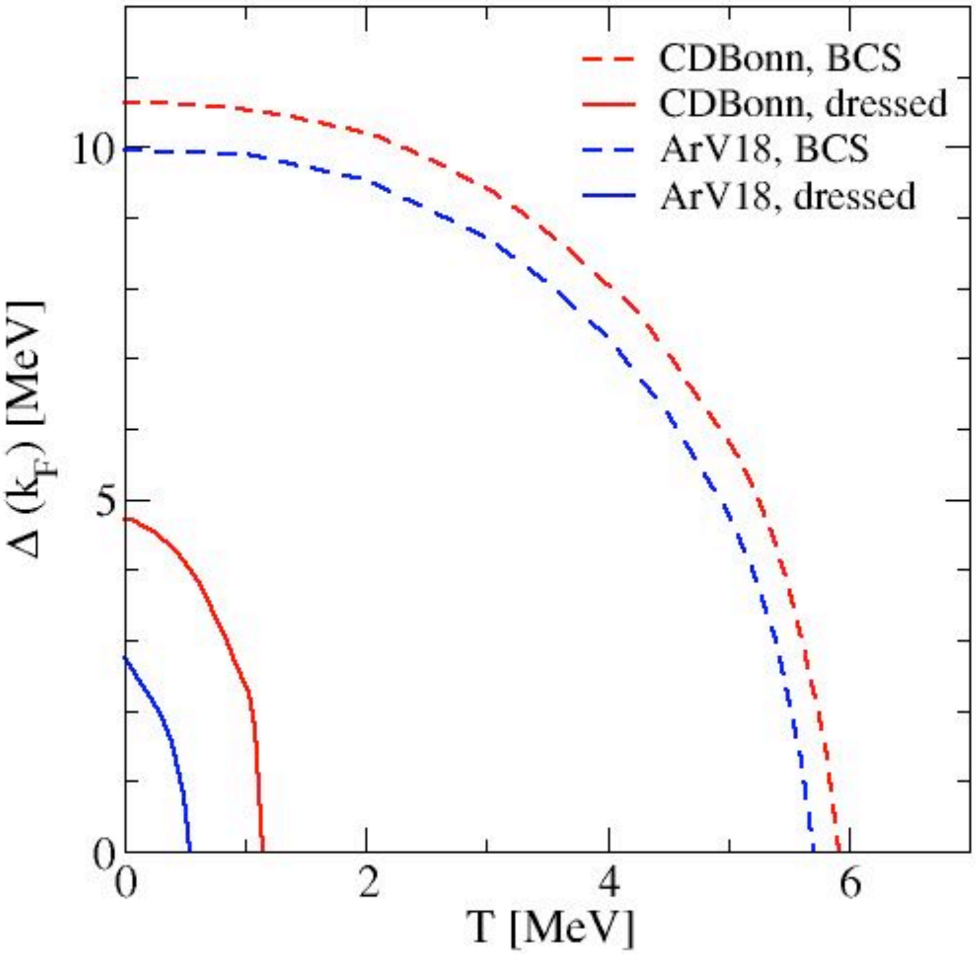
$\tilde{\chi}_k$



Dashed:
Spectral strength only at 1 energy
Dashed-dot:
Effect of temperature (5 MeV)
Solid:
Includes complete strength distribution due to SRC

Related studies by
Baldo, Lombardo, Schuck et al.
use BHF self-energy

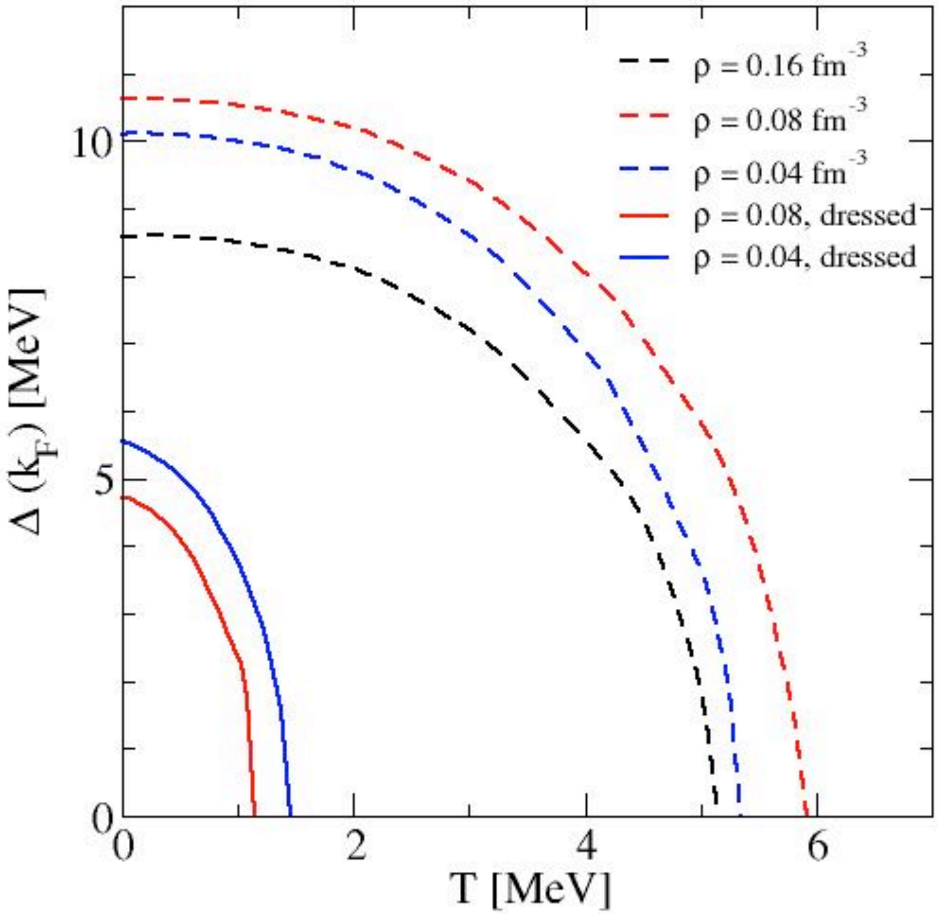
Pairing N*



CDBonn yields stronger pairing than ArV18

Pairing N*

Proton-neutron pairing in symmetric nuclear matter $^3S_1$-$^3D_1$



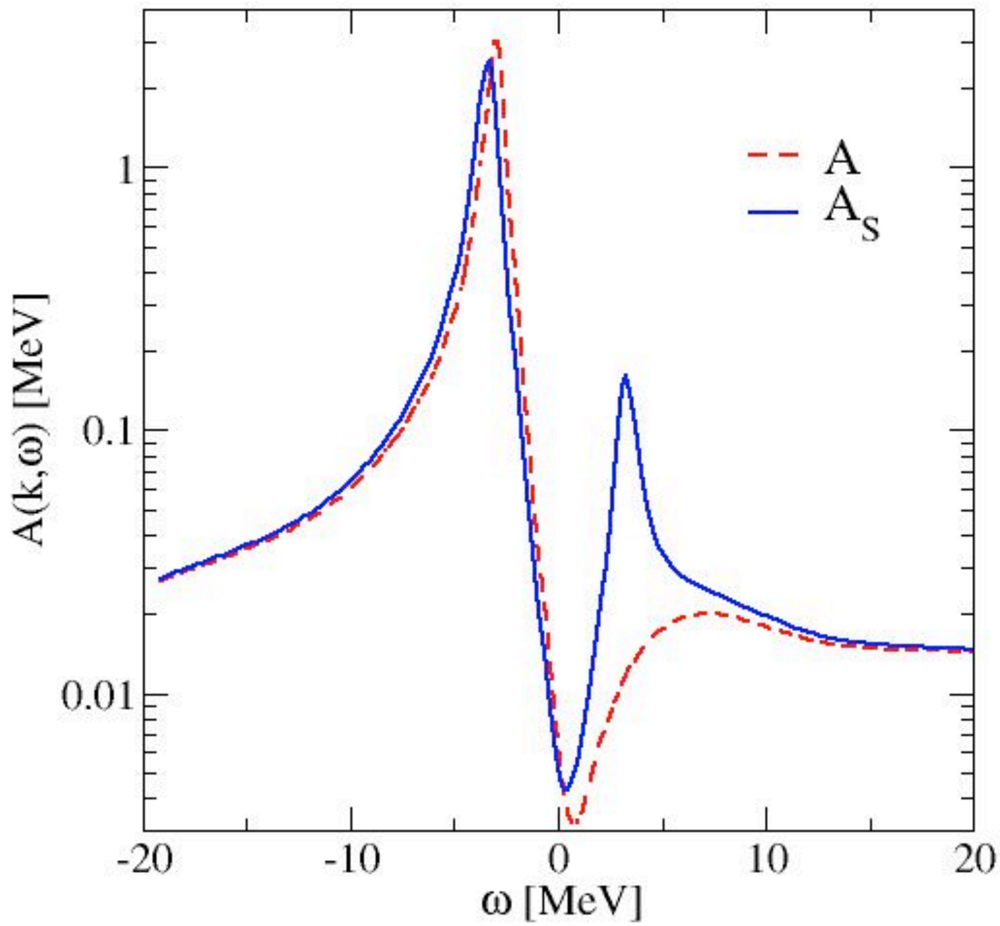
Using CDBonn

Dashed lines:
quasiparticle poles

Solid lines:
dressed nucleons

No pairing at saturation density!!!!

Pairing and spectral functions



Spectral functions

$S(k,\omega)$ dashed = $A(k,\omega)$

$S_{pair}(k,\omega)$ solid = $A_S(k, \omega)$

$\rho = 0.08\ fm^{-3}$

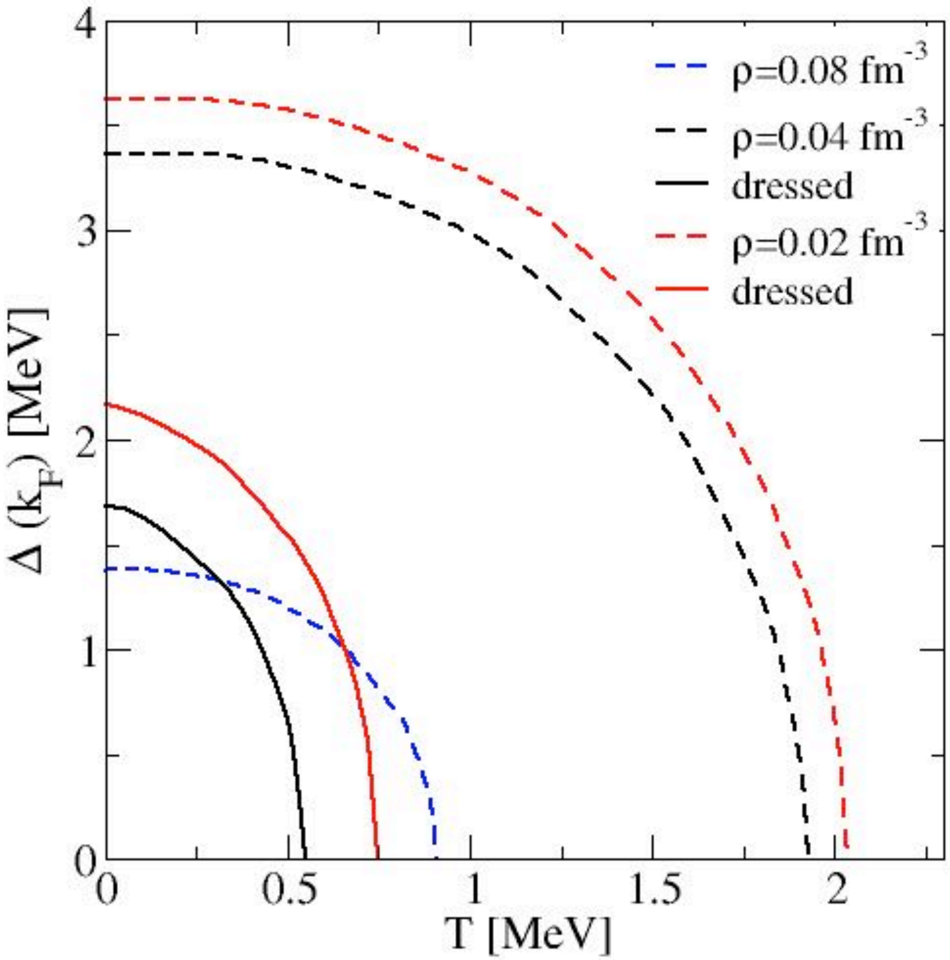
T = 0.5 MeV

k = 193 MeV/c 0.9 k_F

Expected effect

Pairing N*

Pairing in neutron matter --> $^1S_0$



Dressing effects weaker, but non-negligible

CDBonn

Pairing N*

Possible effect of polarization (higher-order corrections to interaction)

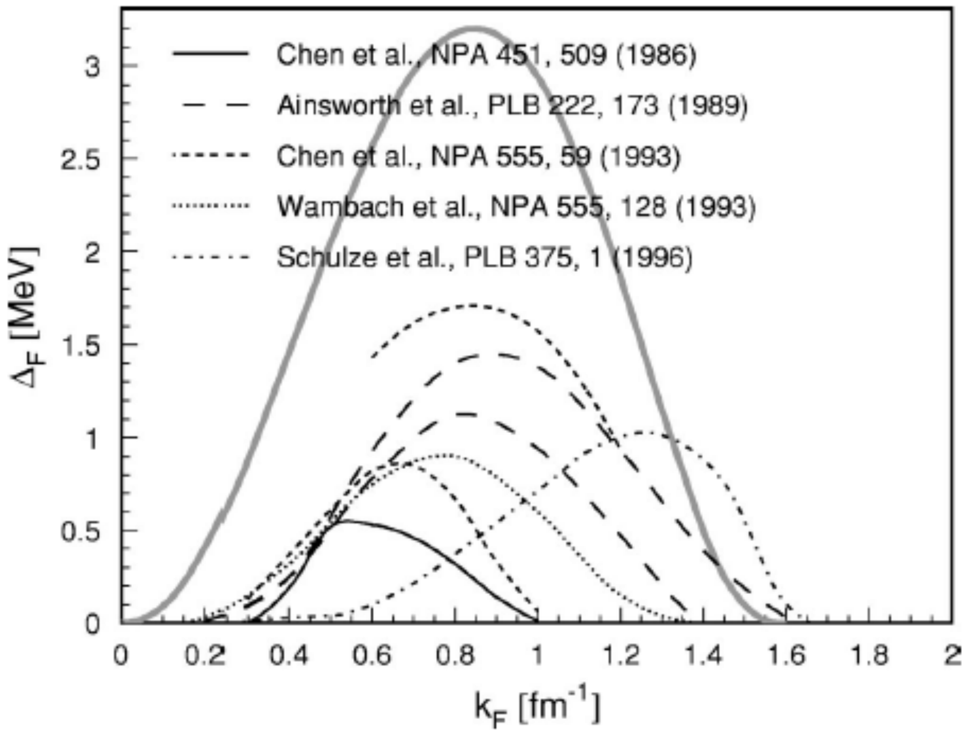
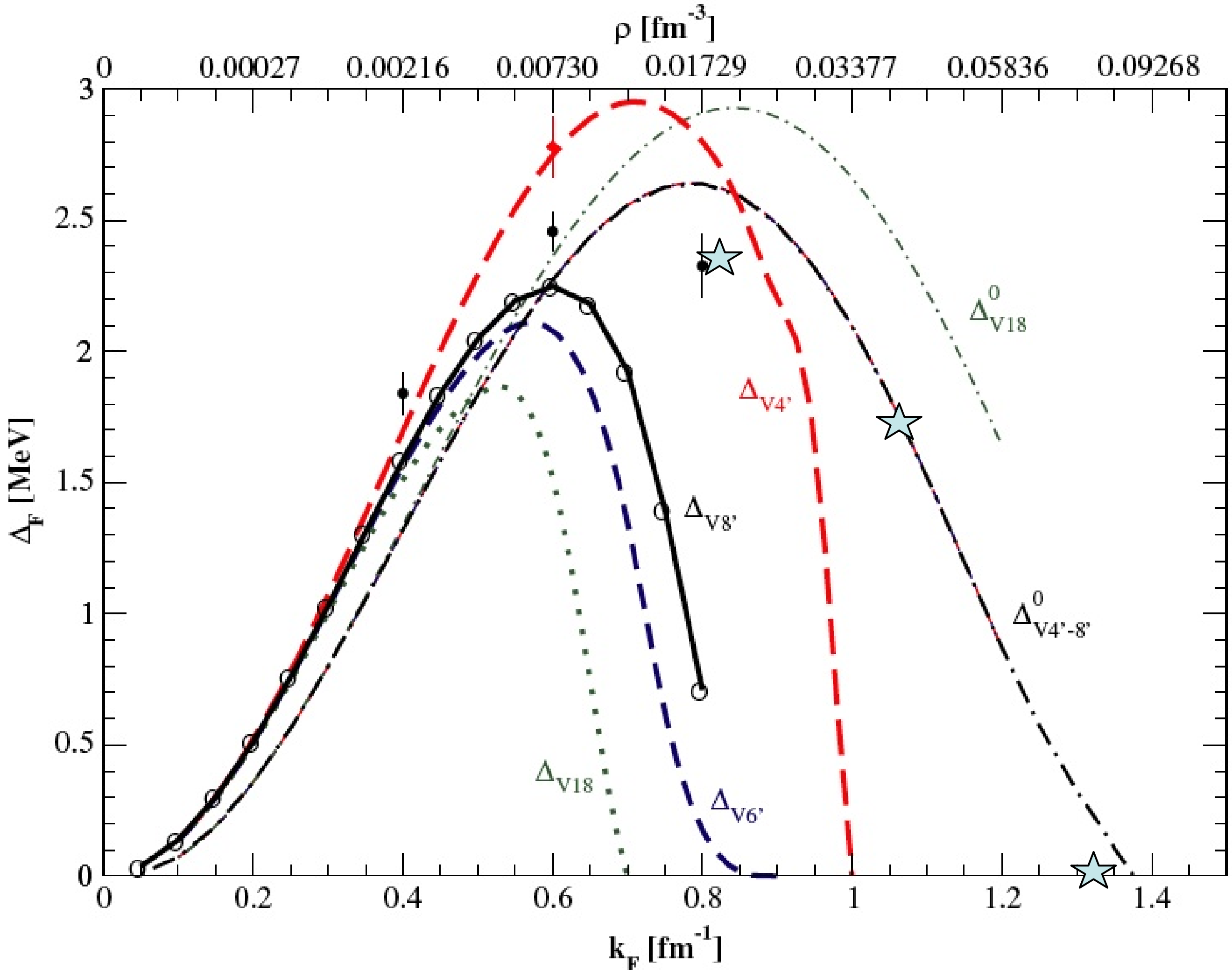


FIG. 14. The $^1S_0$ gap in pure neutron matter predicted in several publications taking account of polarization effects. From Lombardo and Schulze, 2001.

- However...

Comparison for neutron matter with CBF & Monte Carlo PRL95,192501(2005)



☆ ⇒ SCGF

Relevant for high density

- $^3P_2$-$^3F_2$

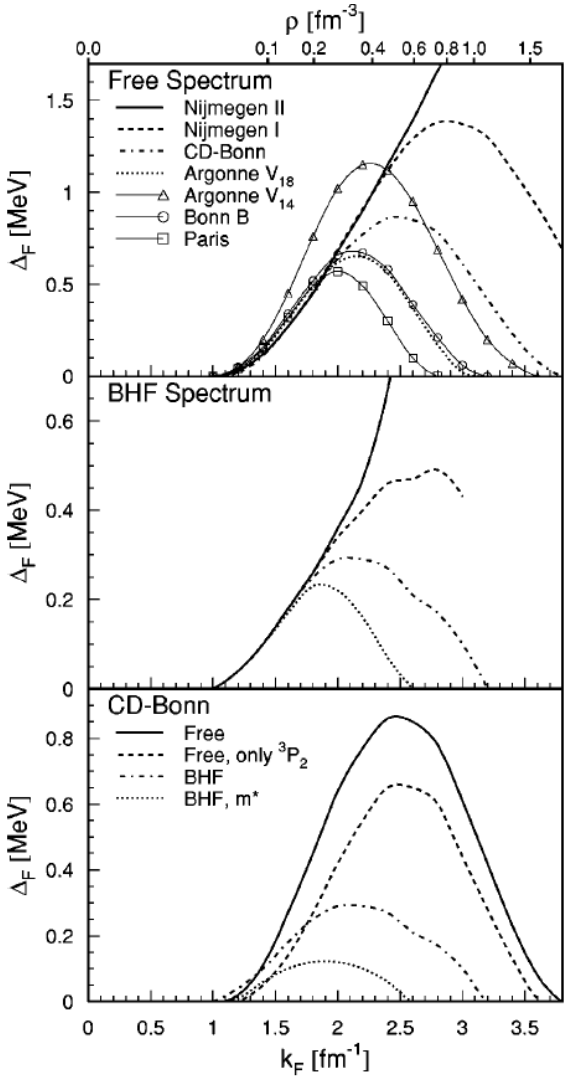


FIG. 7. Top panel: The angle-averaged $^3P_2$-$^3F_2$ gap in neutron matter depending on the Fermi momentum, evaluated with free single-particle spectrum and different nucleon-nucleon potentials. Middle panel: The gap evaluated with Brueckner-Hartree-Fock spectra. Bottom panel: The gap with the CD-Bonn potential in different approximation schemes. From Baldo, Elgarøy, *et al.*, 1998.