CISS07 8/29/2007

Comprehensive treatment of correlations at different energy scales in nuclei using Green's functions

Lecture 1: 8/28/07	Propagator description of single-particle motion and the link with experimental data
Lecture 2: 8/29/07	From Hartree-Fock to spectroscopic factors < 1: inclusion of long-range correlations
Lecture 3: 8/29/07	Role of short-range and tensor correlations associated with realistic interactions
Lecture 4: 8/30/07	Dispersive optical model and predictions for nuclei towards the dripline
Adv. Lecture 1: 8/30/07	Saturation problem of nuclear matter & pairing in nuclear and neutron matter
Adv. Lecture 2: 8/31/07	Quasi-particle density functional theory

Wim Dickhoff
Washington University in St. Louis

Outline

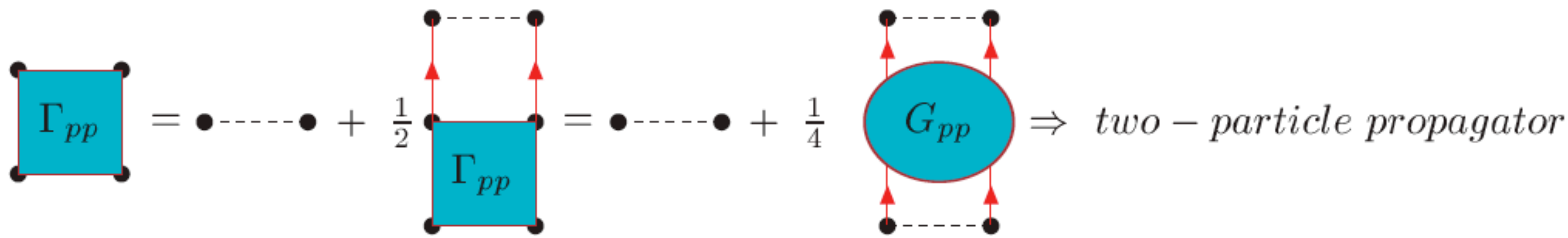
- SRC for free particles
- Ladders in the medium
- Self-energy and Dyson equation
- Nuclear matter simplifications
- Self-consistent Green's functions in nuclear matter & results
- SRC in finite nuclei: where are the high-momentum nucleons
- Summary of sp strength in closed-shell nuclei
- Other nuclei
- N very different from Z
- Nuclear matter with isospin polarization

Short-range correlations for two free particles

Solve the Schrödinger equation or the equivalent "*T*"-matrix

$$\langle k\ell|\Gamma^{JST}_{pp}(k_0)|k'\ell'\rangle = \langle k\ell|V^{JST}|k'\ell'\rangle + \frac{m}{2\hbar^2}\sum_{\ell''}\int_0^{\infty}\frac{dq}{(2\pi)^3}q^2\langle k\ell|V^{JST}|q\ell''\rangle\frac{1}{k_0^2-q^2+i\eta}\langle q\ell''|\Gamma^{JST}_{pp}(k_0)|k'\ell'\rangle$$

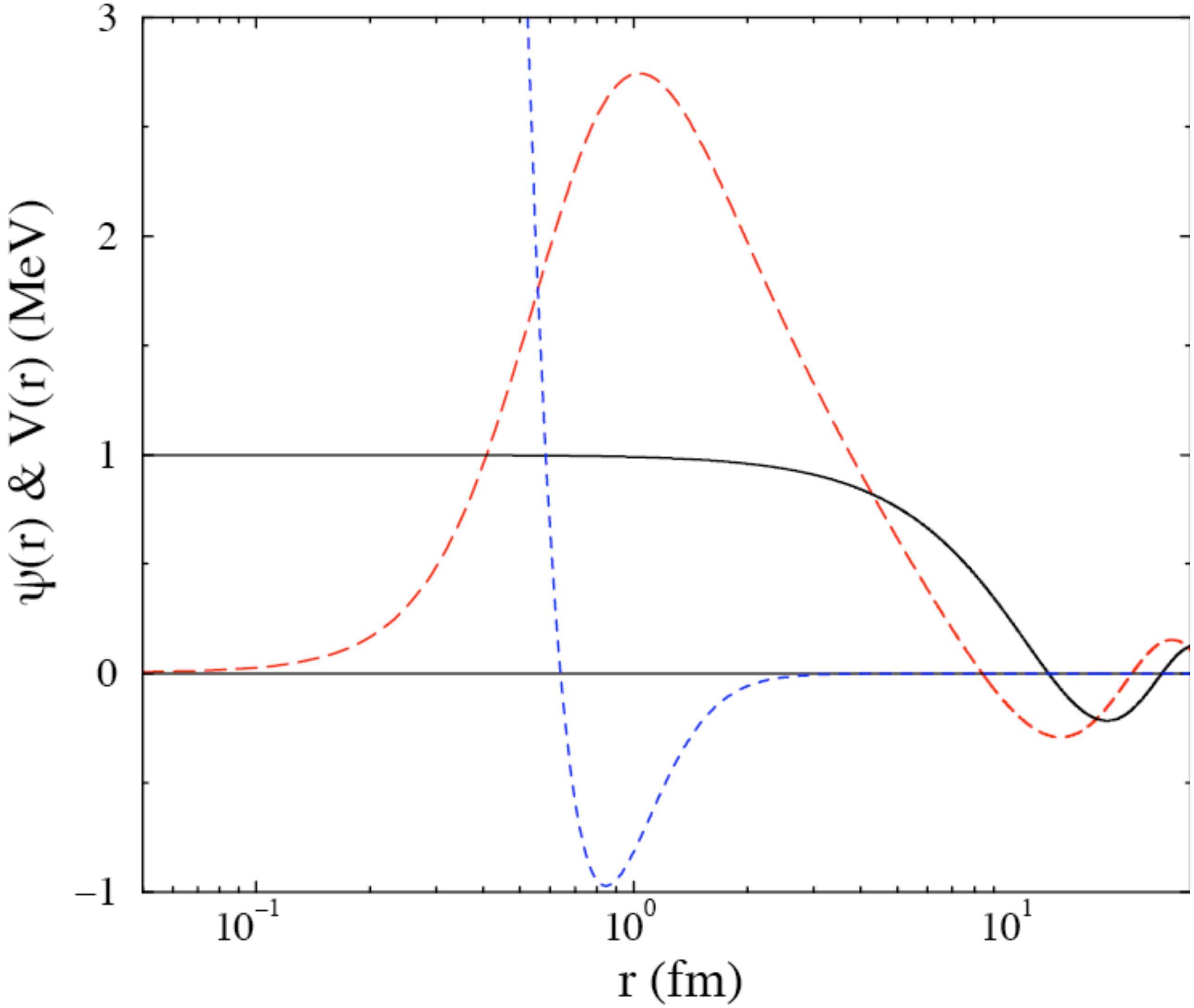
Effective interaction



Sum of ladder diagrams

Sum of ladder diagram takes care of SRC
Also in the medium!

Relative wave function and potential



Reid $^1S_0$
$k_0 = 0.25\ \mathrm{fm}^{-1}$

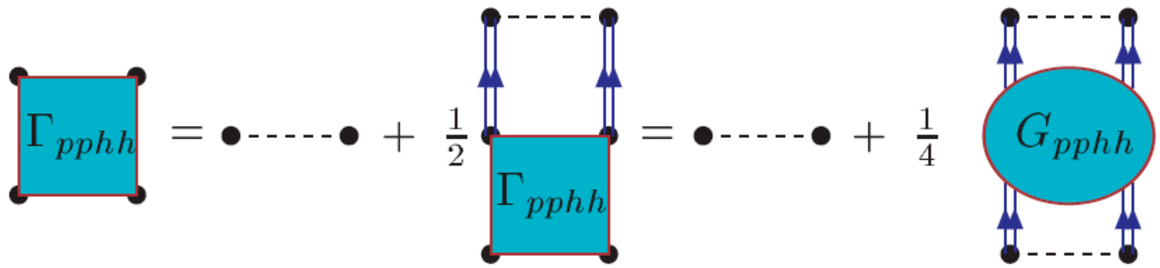
V(r)/100

$\sin(k_0 r)/k_0 r$

correlated wf

Ladder diagrams in the medium (options)

Ladders in the medium



$\Rightarrow$ *self − energy calculation*

$$\langle k\ell|\Gamma^{JST}_{pphh}(K,E)|k'\ell'\rangle = \langle k\ell|V^{JST}|k'\ell'\rangle + \frac{1}{2}\sum_{\ell''}\int_0^\infty \frac{dq}{(2\pi)^3} q^2 \langle k\ell|V^{JST}|q\ell''\rangle G^f_{pphh}(q;K,E)\langle q\ell''|\Gamma^{JST}_{pphh}(K,E)|k'\ell'\rangle$$

G^f_{pphh} has different form depending on the level of sophistication

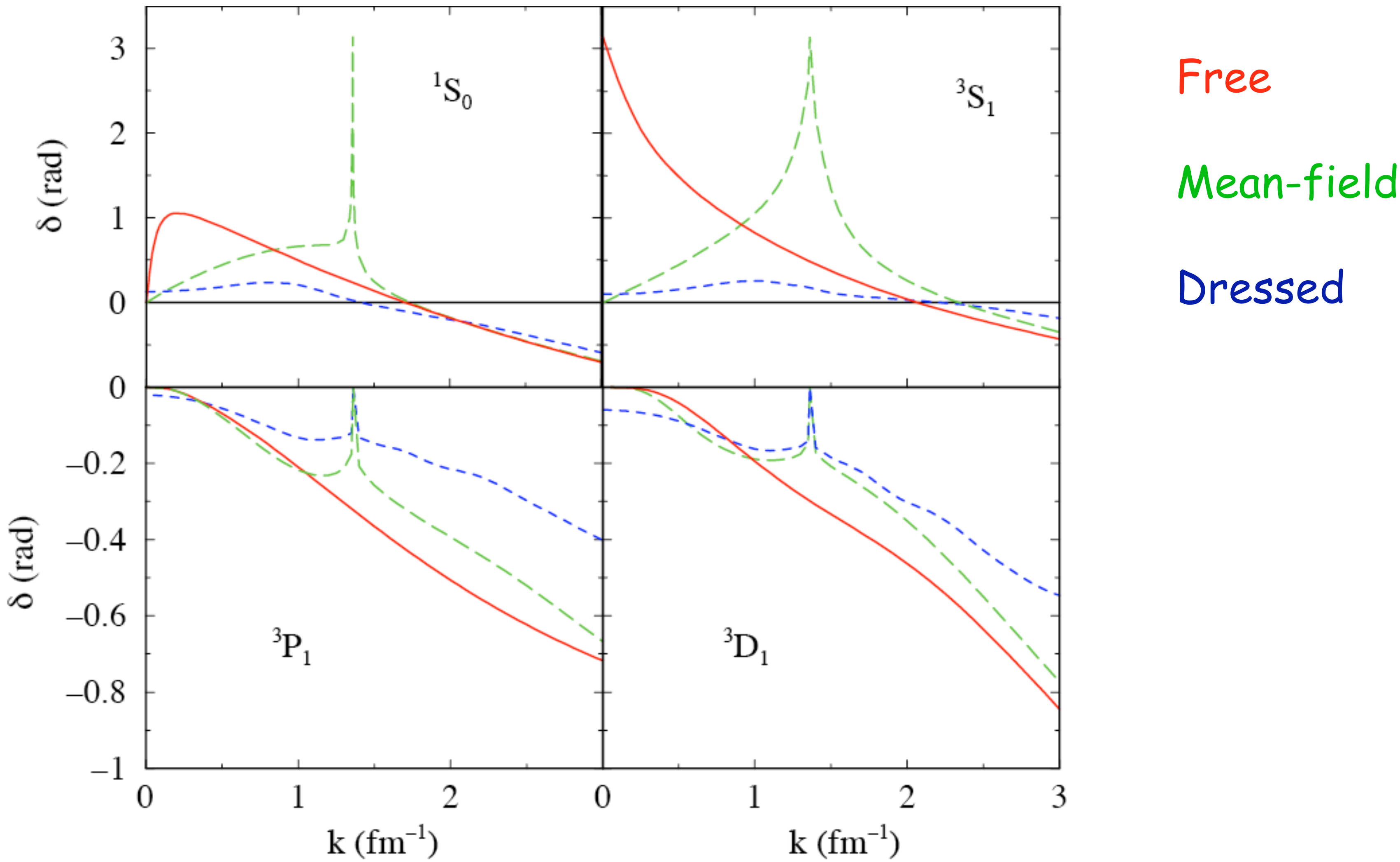
Nuclear matter:

$$G^f_{BG}(k_1,k_2;E) = \frac{\theta(k_1-k_F)\theta(k_2-k_F)}{E-\varepsilon(k_1)-\varepsilon(k_2)+i\eta}$$ Bethe-Goldstone

$$G^f_{GF}(k_1,k_2;E) = \frac{\theta(k_1-k_F)\theta(k_2-k_F)}{E-\varepsilon(k_1)-\varepsilon(k_2)+i\eta} - \frac{\theta(k_F-k_1)\theta(k_F-k_2)}{E-\varepsilon(k_1)-\varepsilon(k_2)-i\eta}$$ Galitskii-Feynman

$$G^f_{pphh}(k_1,k_2;E) = \int_{\varepsilon_F}^{\infty} dE_1 \int_{\varepsilon_F}^{\infty} dE_2 \frac{S_p(k_1;E_1)S_p(k_2;E_2)}{E-E_1-E_2+i\eta} - \int_{-\infty}^{\varepsilon_F} dE_1 \int_{-\infty}^{\varepsilon_F} dE_2 \frac{S_h(k_1;E_1)S_h(k_2;E_2)}{E-E_1-E_2-i\eta}$$ SCGF

Phase shifts for dressed nucleons



PRC**60**, 064319 (1999) also PRC**58**, 2807 (1998)

Dyson equation and spectral functions in nuclear matter (some simplifications)

$$G(k;E) = G^{(0)}(k;E) + G^{(0)}(k;E)\Sigma(k;E)G(k;E)$$

Dyson equation

$$= \frac{1}{E - \varepsilon(k) - \Sigma(k;E)}$$

$$G^{(0)}(k;E) = \frac{\theta(k-k_F)}{E-\varepsilon(k)+i\eta} + \frac{\theta(k_F-k)}{E-\varepsilon(k)-i\eta}$$

Noninteracting sp propagator

$$S_p(k;E) = -\frac{1}{\pi}\frac{\mathrm{Im}\,\Sigma(k;E)}{\left(E-\varepsilon(k)-\mathrm{Re}\,\Sigma(k;E)\right)^2 + \left(\mathrm{Im}\,\Sigma(k;E)\right)^2}$$

particle spectral function

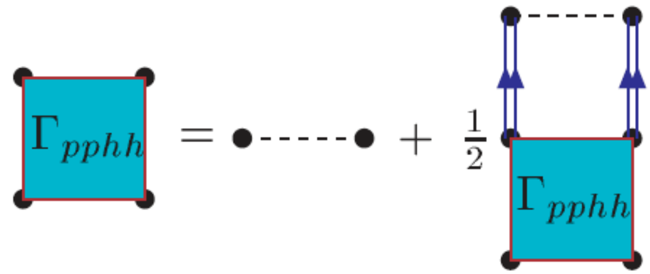
$$S_h(k;E) = \frac{1}{\pi}\frac{\mathrm{Im}\,\Sigma(k;E)}{\left(E-\varepsilon(k)-\mathrm{Re}\,\Sigma(k;E)\right)^2 + \left(\mathrm{Im}\,\Sigma(k;E)\right)^2}$$

hole spectral function

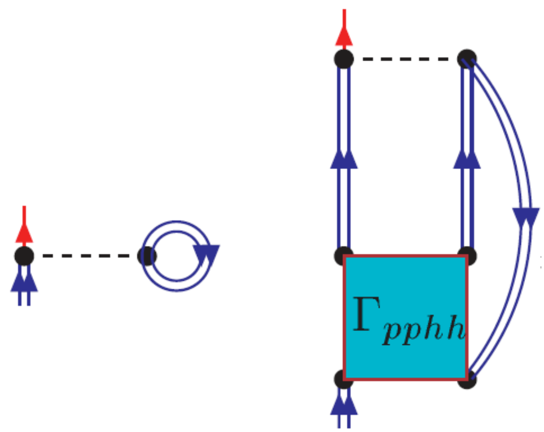
$$G(k;E) = \int_{\varepsilon_F}^{\infty} dE' \frac{S_p(k;E')}{E-E'+i\eta} + \int_{-\infty}^{\varepsilon_F} dE' \frac{S_h(k;E')}{E-E'-i\eta}$$

numerator sp strength
denominator where

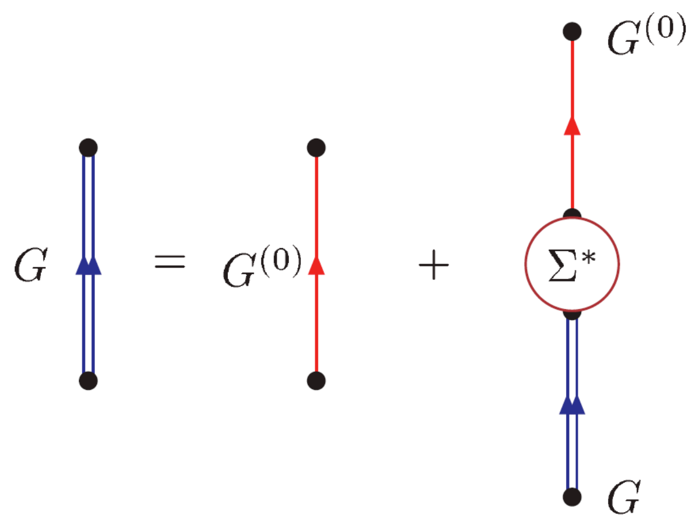
Self-consistency



Interaction



Self-energy



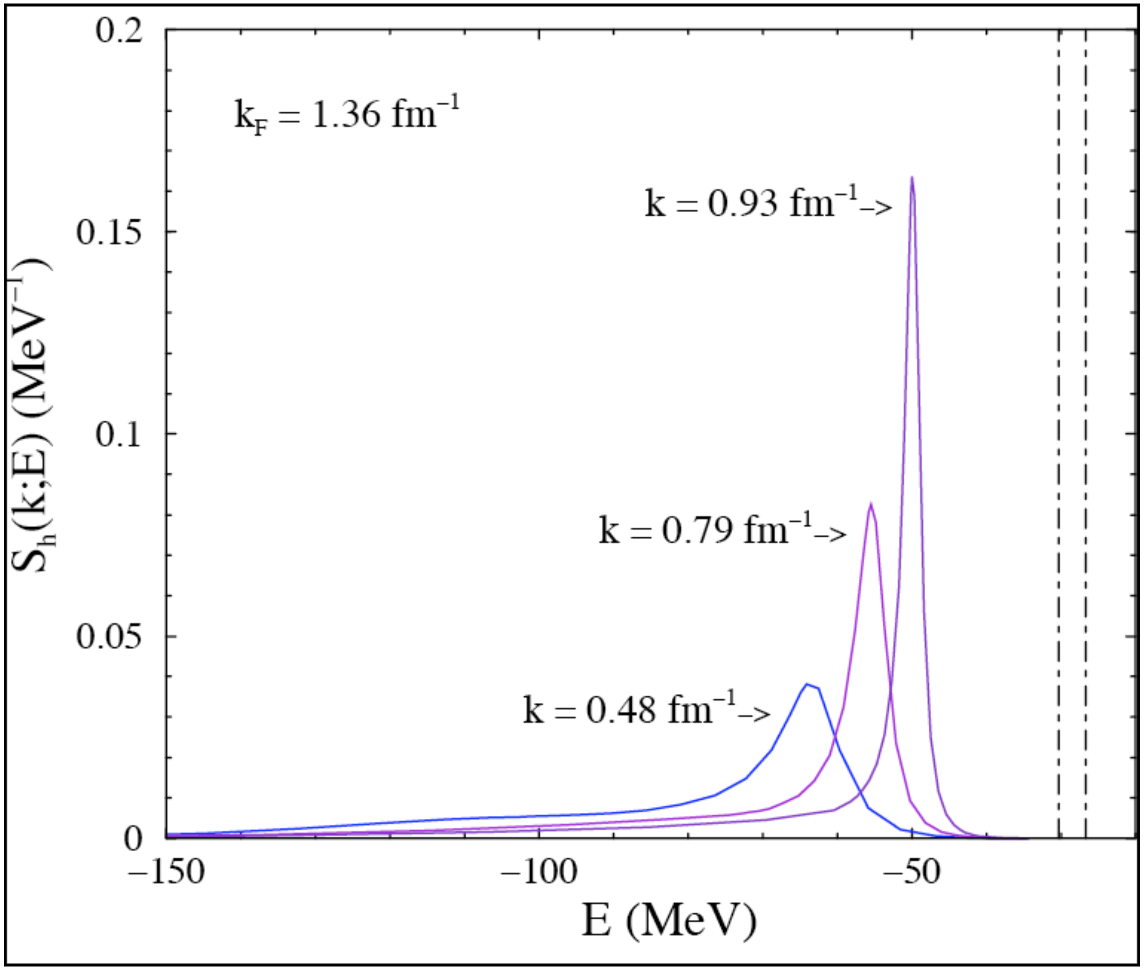
Dyson equation

Groups

- St. Louis ⇒ Complete self-consistency for spectral functions

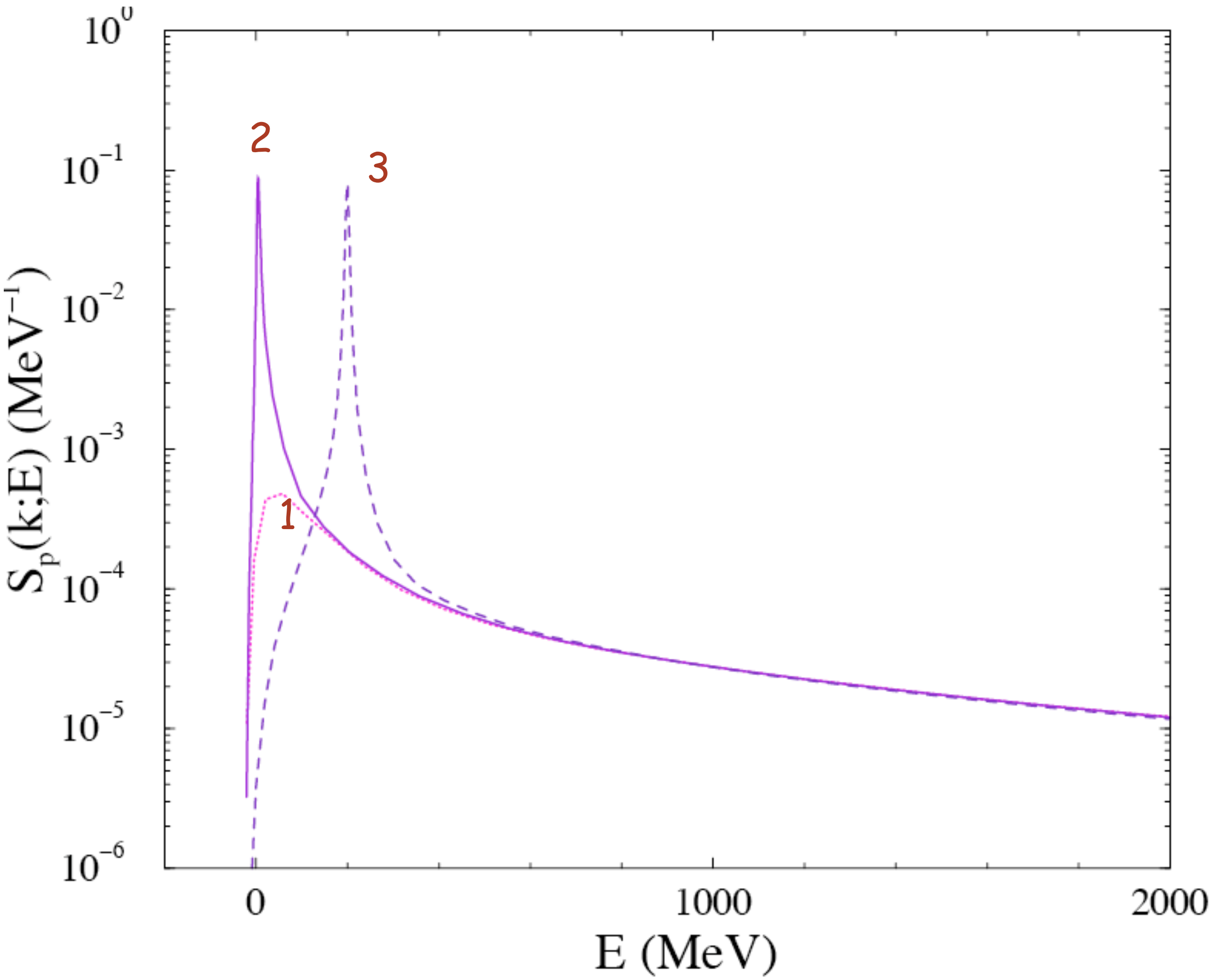
  (Ramos, Vonderfecht, Gearhart, Roth/Stoddard)
- Ghent ⇒ Discrete method **(Van Neck,** Dewulf)
- Cracow ⇒ Separable & soft interactions **(Bozek, Czerski,** Soma)
- Tübingen ⇒ Finite temperature & soft interactions **(Müther,** Frick)
- Barcelona ⇒ Finite temperature & soft interactions **(Polls, Ramos,** Rios)
- Giessen ⇒ Interaction related to cross section **(Lenske et al)**

Illustrative results for mean-field input



Strength distribution as in nuclei …
peak strength +10% at lower energy + global depletion!!

Where does the strength go?



All tails the same!
⇒ SRC

k_1 = 0.79 fm^{-1}
k_2 = 1.74 fm^{-1}
k_3 = 3.51 fm^{-1}

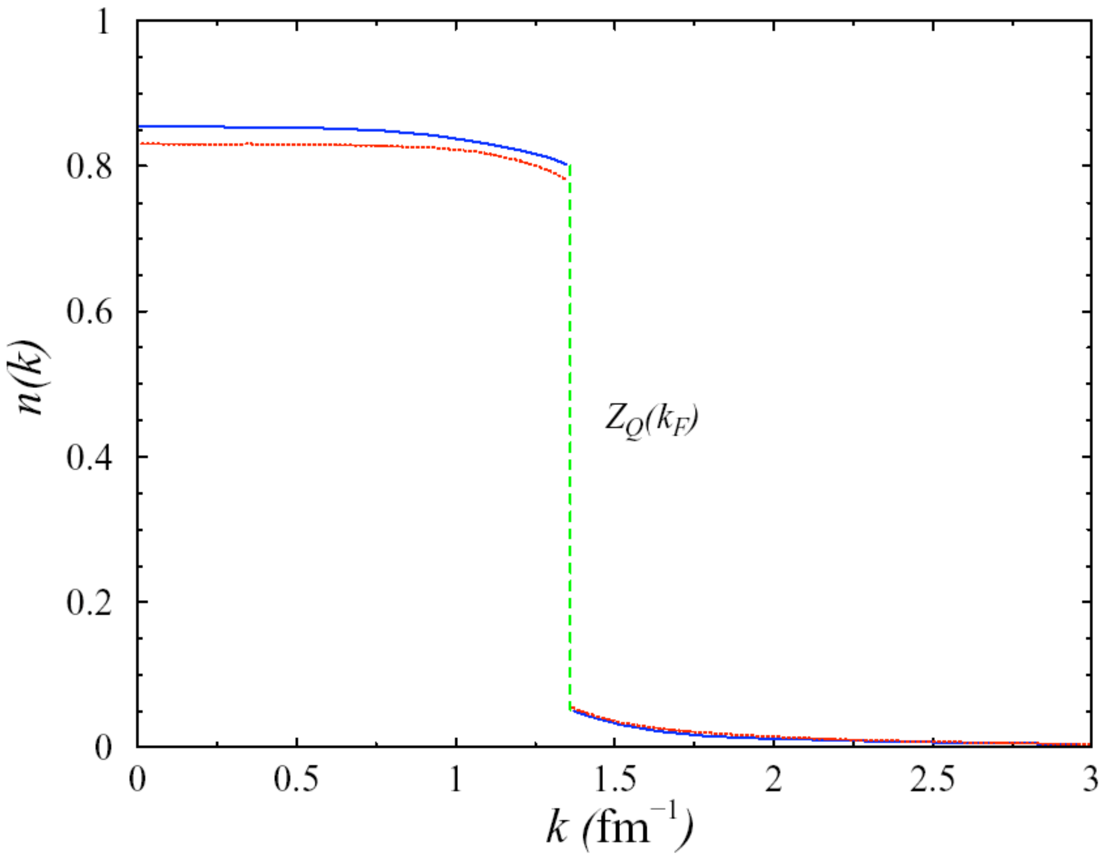
$k < k_F$: 17% > ε_F with 13% above 100 MeV (7% above 500 MeV)

Without tensor force only 10.5% above ε_F

Short-range correlations in nuclear matter and n(k)

$n(k = 0)$ = 0.83 / 0.85 ⇒ finite nuclei

$$n(k) = \int_{-\infty}^{\varepsilon_F} dE\, S_h(k;E)$$



Reid soft core
k_F = 1.36 fm^{-1}

Old prediction!
New result

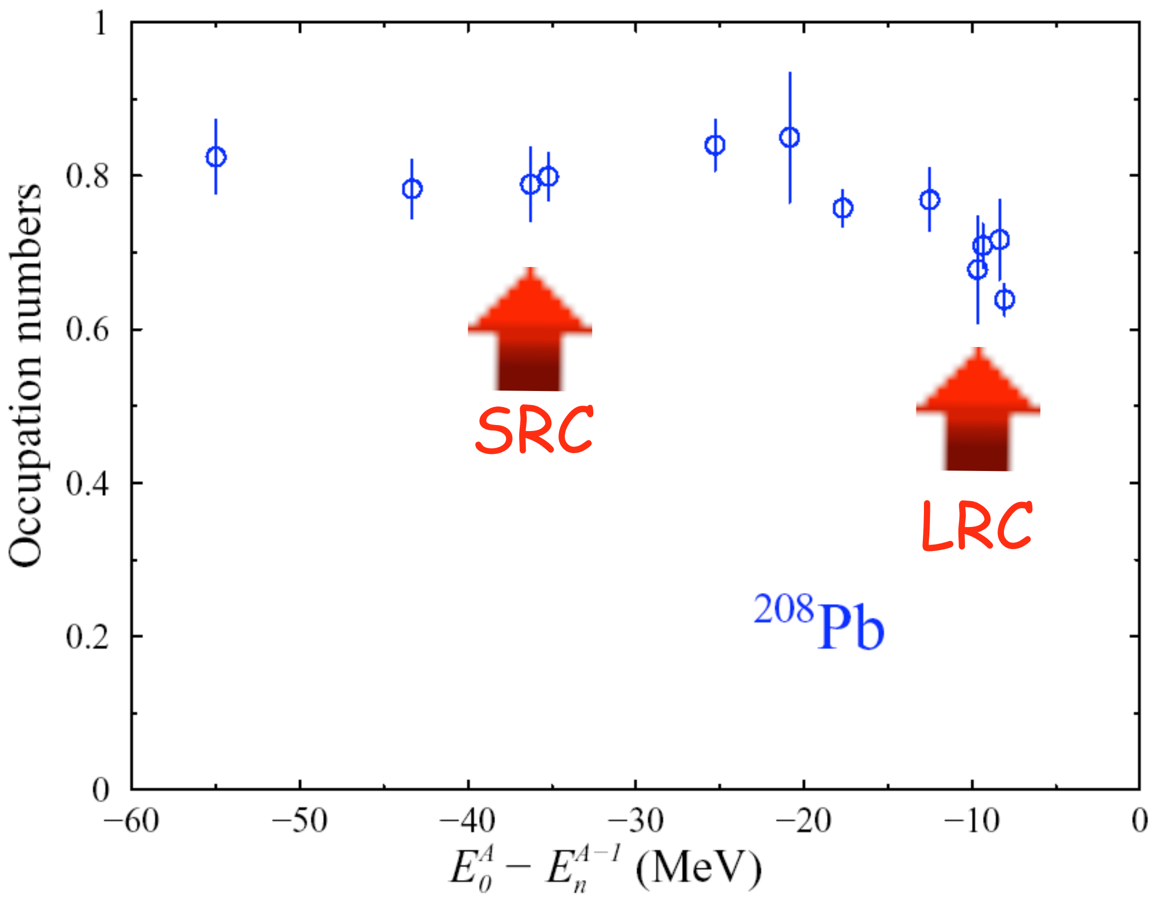
$Z_Q(k_F)$=0.72

$Z_Q(k_F)$=0.75

B.E.Vonderfecht et al. Nucl. Phys. A555, 1 (1993)
E.R.Stoddard, thesis (self-consistent ladders)

M. van Batenburg (thesis, 2001) & L. Lapikás from ^{208}Pb (e,e´p) ^{207}Tl

Occupation of deeply-bound proton levels from **EXPERIMENT**



Up to 100 MeV missing energy and 270 MeV/c missing momentum

Covers the whole mean-field domain for the FIRST time!!

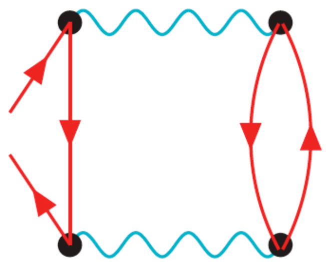
Confirmation of theory

Two effects associated with short-range correlations

- Depletion of the Fermi sea
- Admixture of high-momentum components to replace depleted strength

Location of high-momentum components

high momenta



require specific intermediate states

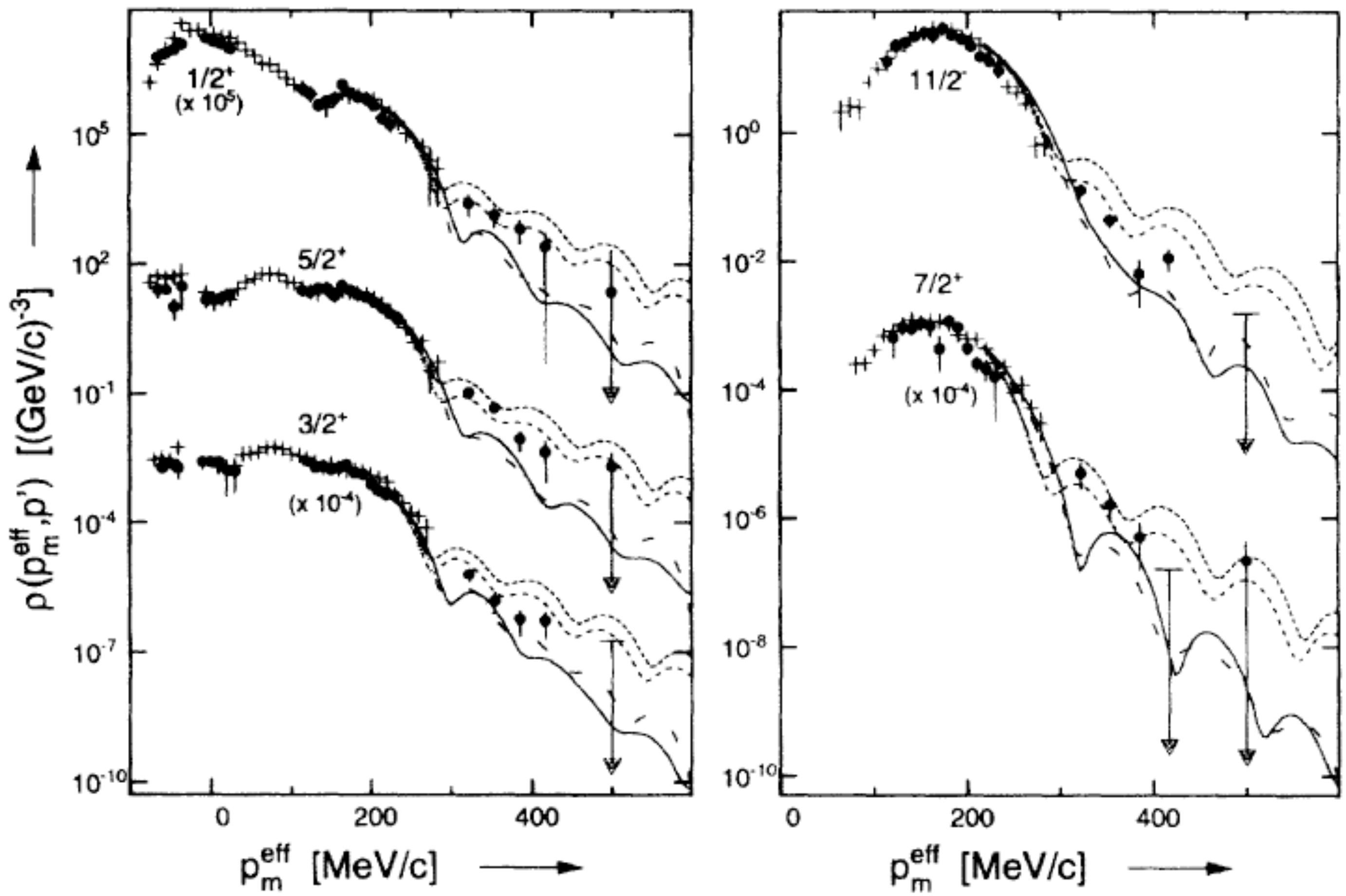
External line ***k*** (large).
Intermediate holes < k_F, say total momentum ~ 0.
Momentum conservation: intermediate particle -***k***

$\Rightarrow$ Energy intermediate state ~ $\langle \varepsilon_{2h} \rangle - \varepsilon(\boldsymbol{k})$

$\Rightarrow$ the higher *k* the more negative the location of its strength

$\Rightarrow$ no high-momentum components near ε_F

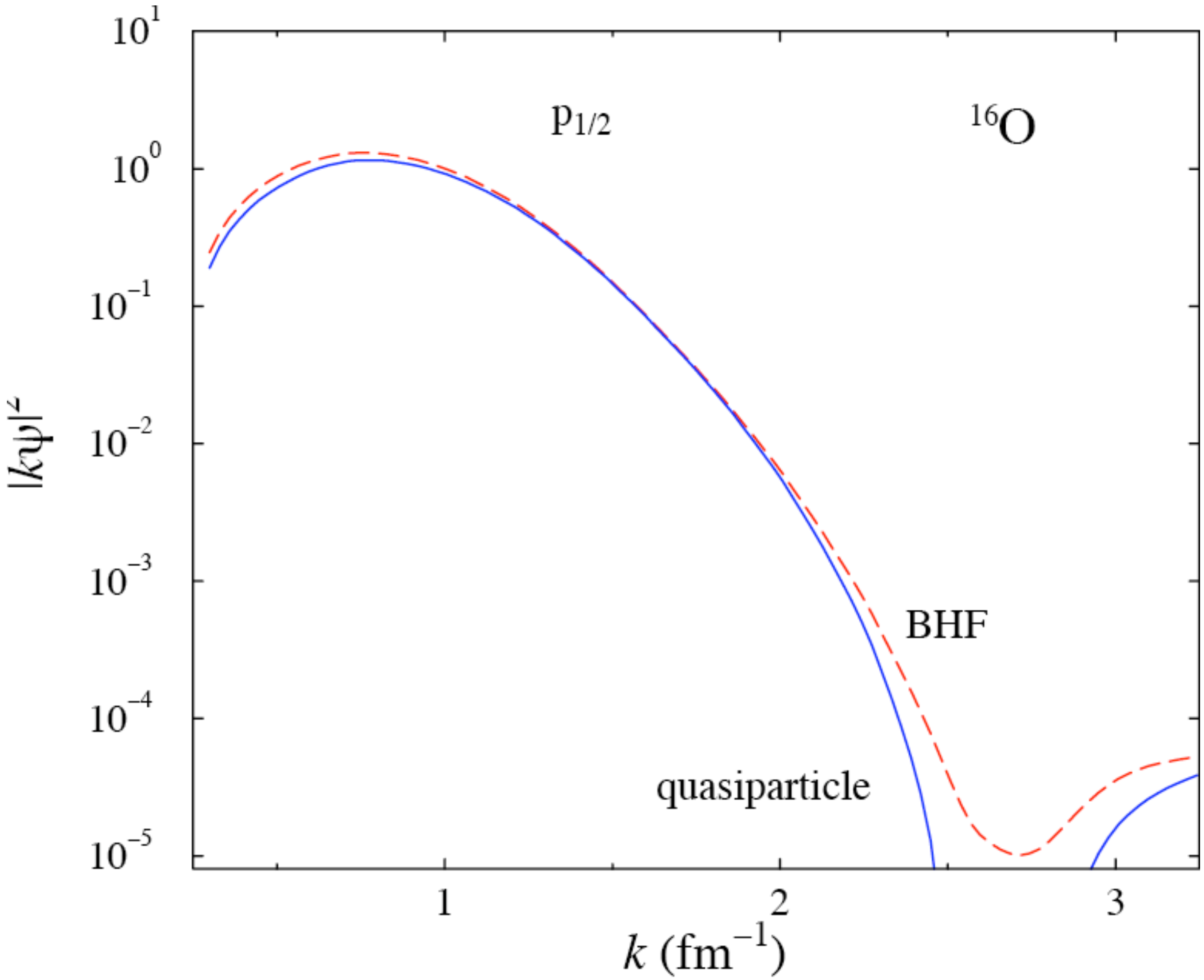
High-momenta near ε_F?



NO!

I. Bobeldijk *et al.*, Phys. Rev. Lett. **73**, 2684 (1994)

SRC (only) calculated in ^{16}O

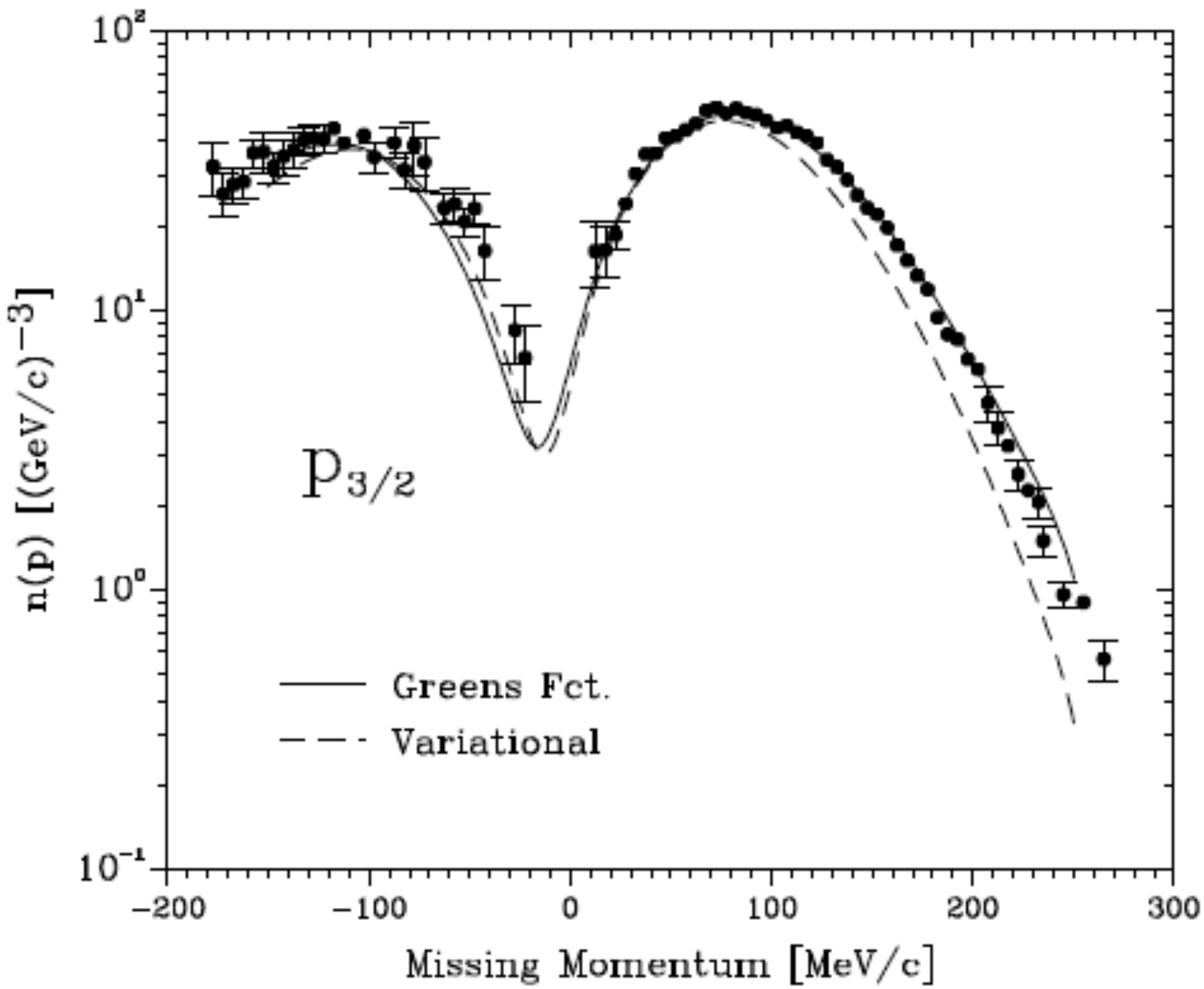


No enhancement of high k near ε_F

Not observed experimentally either!
PRL**73**,2684(1994)
PLB**344**,85(1995)

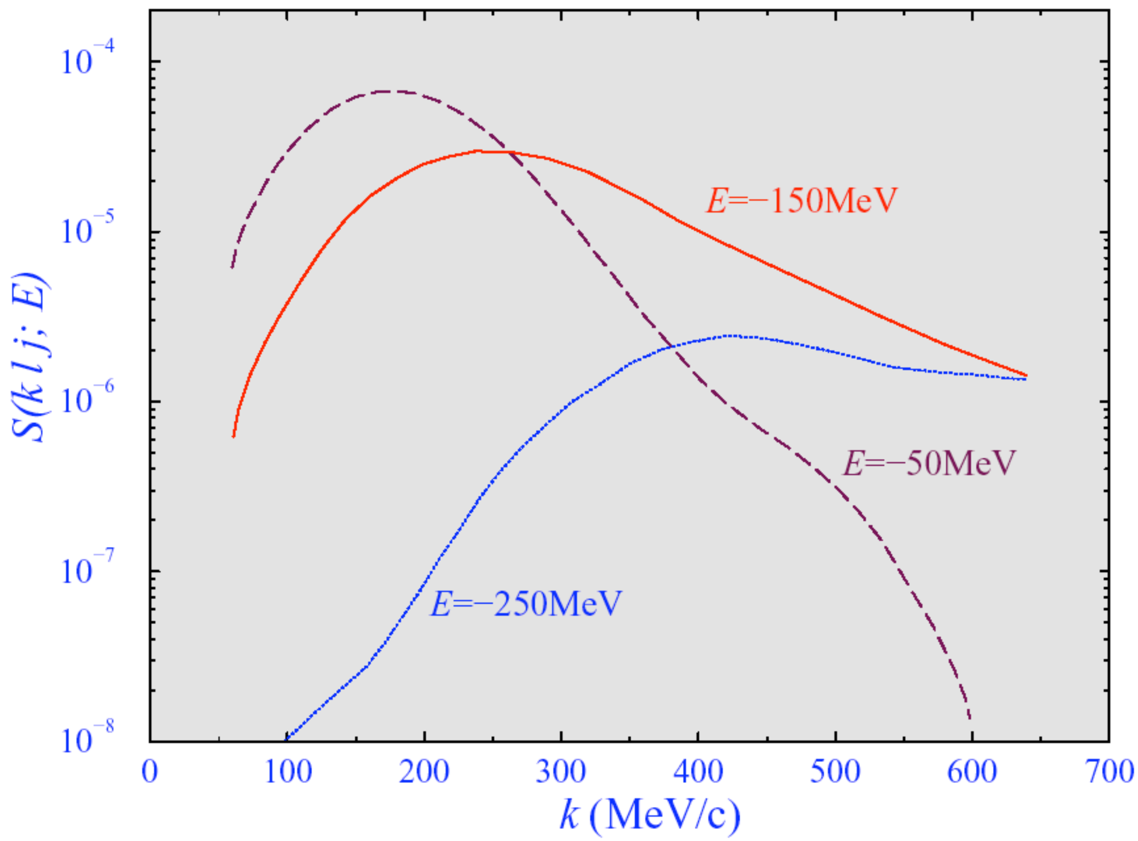
Strength depleted by 10% due to SRC

Quality of quasihole wave function



PRC**55**, 810 (1997)

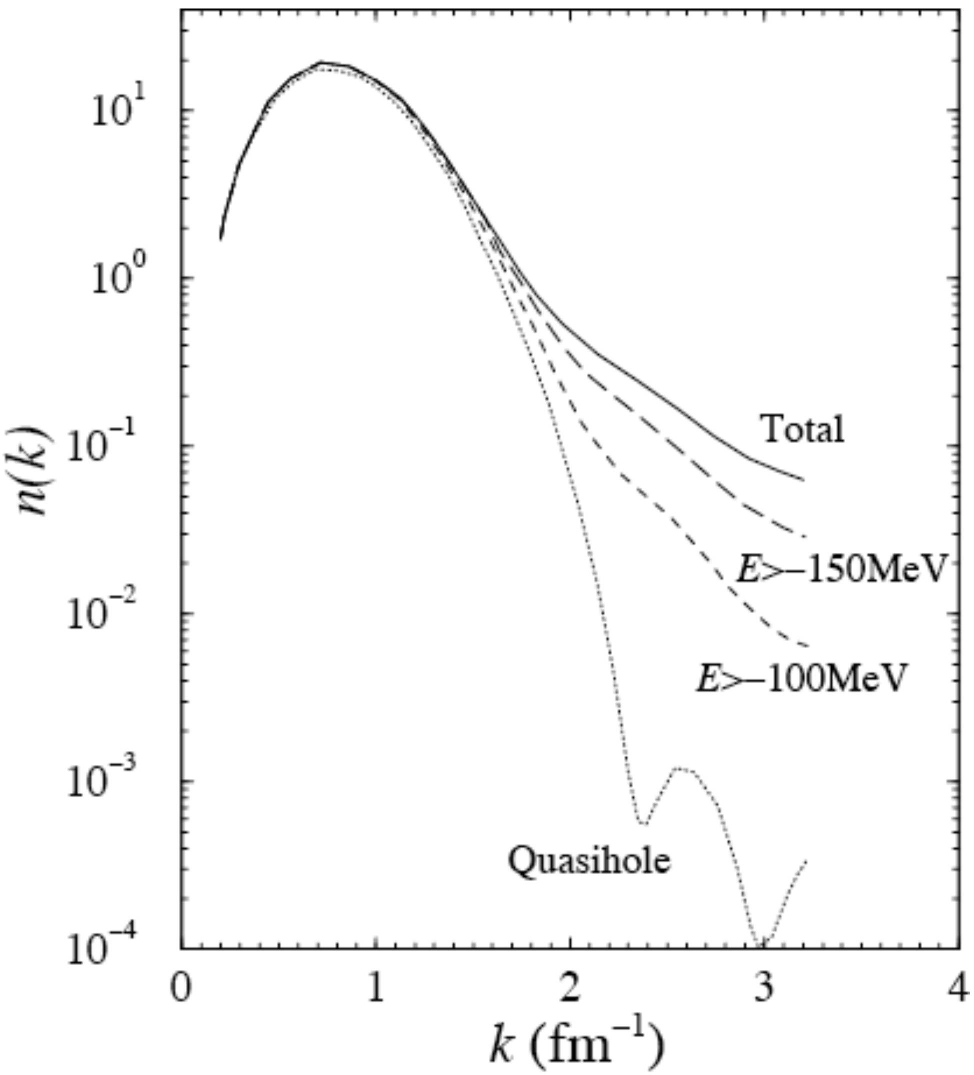
Prediction of high-momentum components



$p_{1/2}$ spectral function at fixed energies in ^{16}O

Phys. Rev. C49, R17 (1994)

Momentum distribution ^{16}O

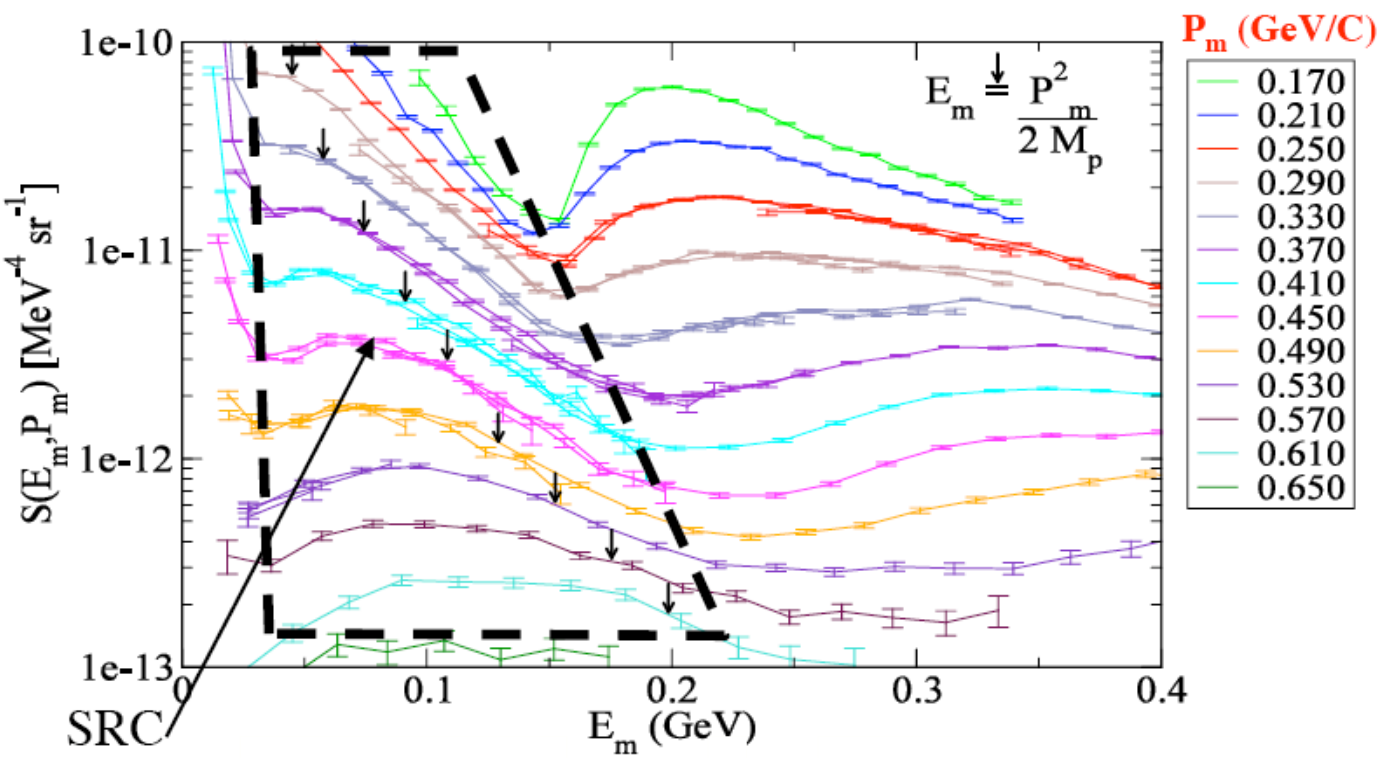


Confirms expectation:

High momentum nucleons can be found at large negative energies

What are the rest of the protons doing?

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.

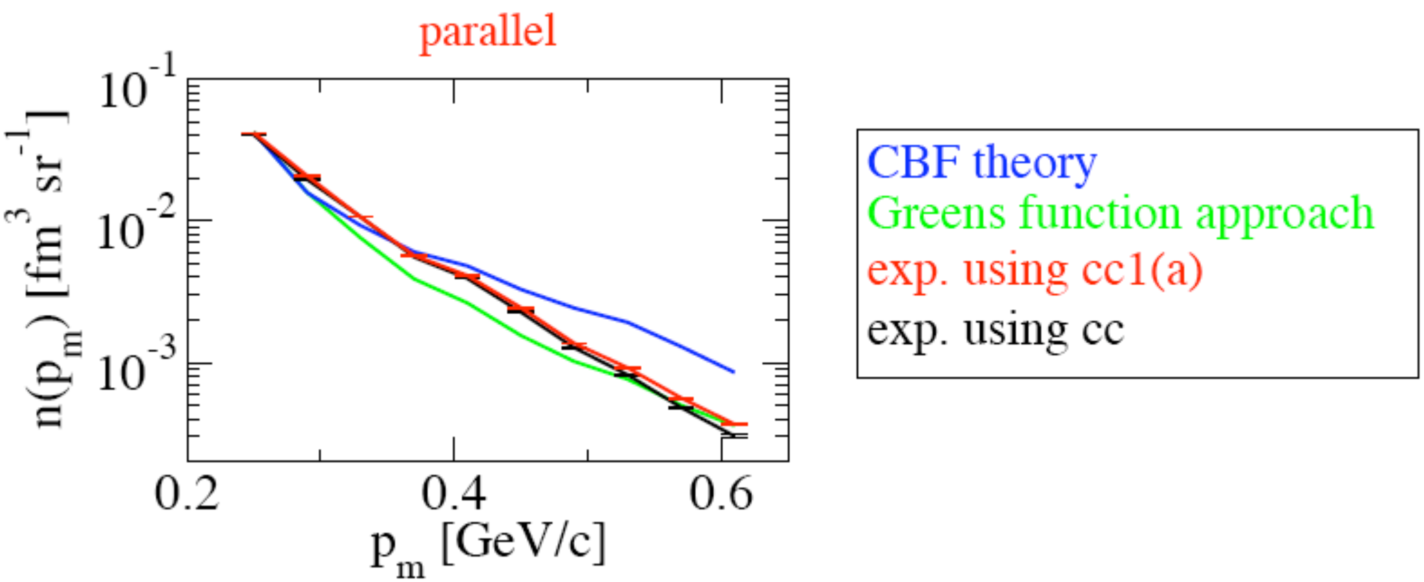


^{12}C

- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)
  $\Rightarrow$ 0.6 protons for ^{12}C

Integrated strength ⇒ n(k)

momentum dependence



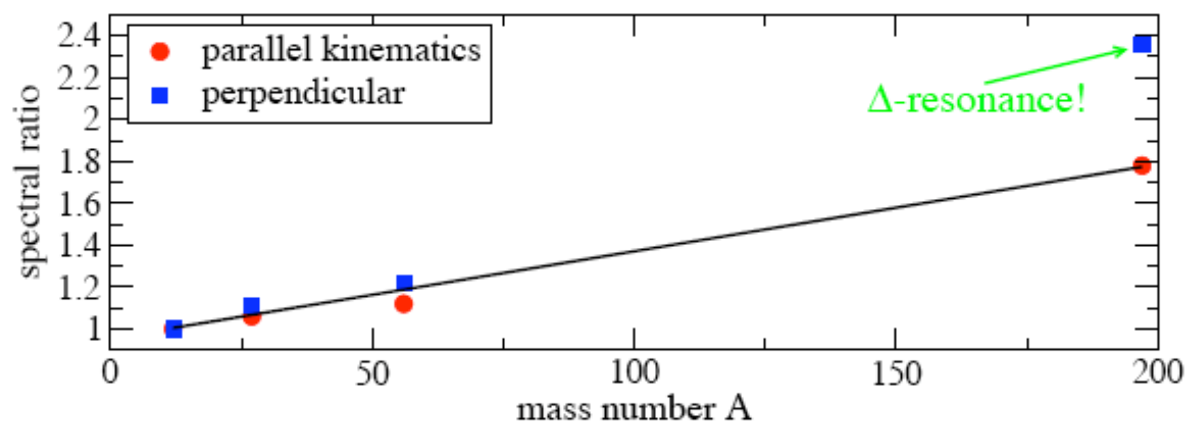
→ theory and experiment ± agree

From: Sick, ECT* workshop July 2007

Asymmetry dependence?

ratio to C of correlated strength
find ratio ~ 1 as expected

Ratio Al, Fe, Au to C spectral function
integrated over correlated region



enhancement for Au

not yet understood

consequence of n-p correlations as $N > Z$??, rescattering ??

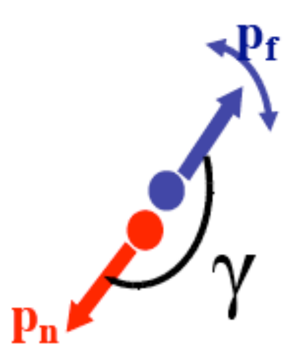
would like to get $S(k, E)$ for N≠Z

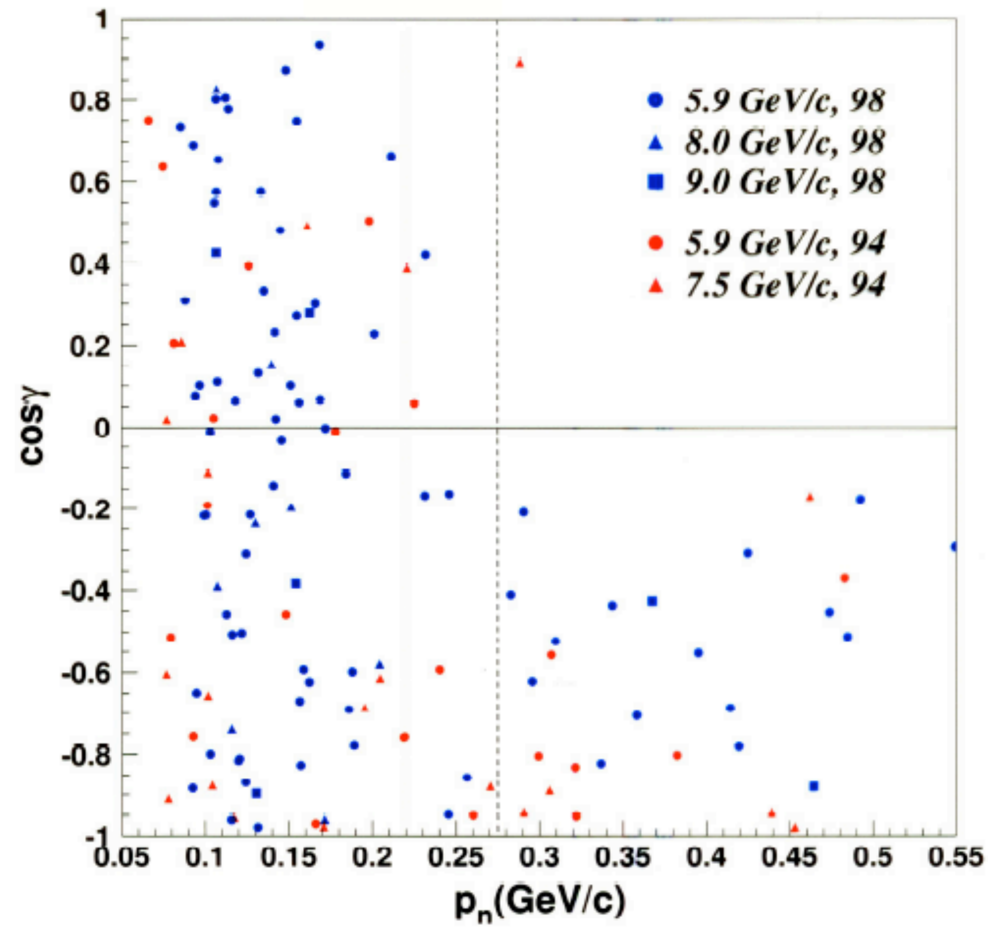
Data from BNL

Brookhaven EVA Collaboration Result

A. Tang *et al.*, Phys. Rev. Lett. **90** (2003) 042301.

$^{12}C(p,2p+n)$ Reaction





$p_f = p_1 + p_2 - p_0$

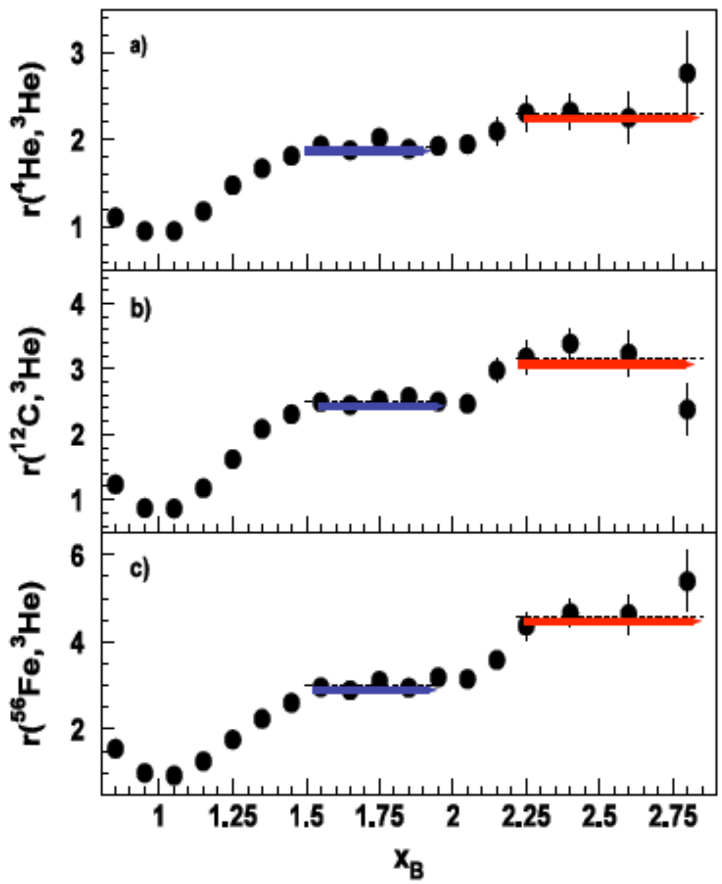
p_0 = incident proton

p1 and p2 are detected

Electron scattering results JLab

New CLAS A(e,e') Result

K. Sh. Egiyan et al., Phys. Rev. Lett. 96, 082501 (2006)



The probabilities for 3-nucleon SRC are smaller by one order of magnitude relative to the 2N SRC.

The observed "scaling" means that the electrons probe the high-momentum nucleons in the 3-nucleon phase, and the scaling factors determine the per-nucleon probability of the 3N-SRC phase in nuclei with A>3 relative to ^{3}He.

Less than 1% of total.

Combination of data & analysis

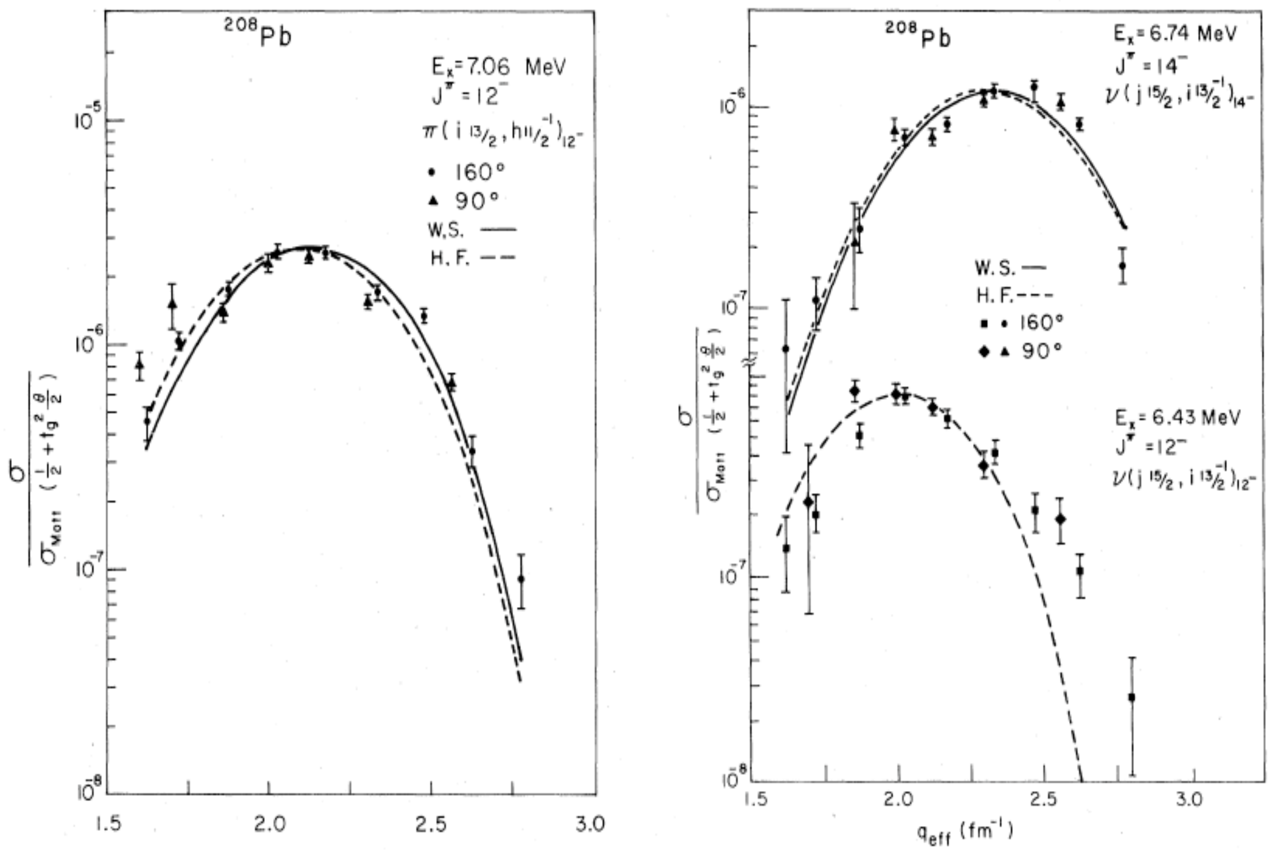
From the (e,e'), (e,e'p), and (e,e'pN) Results

- 80 +/- 5% single particles moving in an average potential
  - 60 – 70% independent single particle in a shell model potential
  - 10 – 20% shell model long range correlations
- 20 +/- 5% two-nucleon short range correlations
  - from (e,e'pp) approx. 1% pp SRC
  - exact ratio for (e,e'pn) / (e,e'pp) not final, but preliminary result gives approx. 18% pn SRC
  - combining preliminary results we can deduce approx. 1% nn SRC
- less than 1% multi-nucleon correlations

E. Piatsetzky et al., PRL97, 162504 (2006)

From D.W. Higinbotham, JLab

Consequences of correlated sp strength



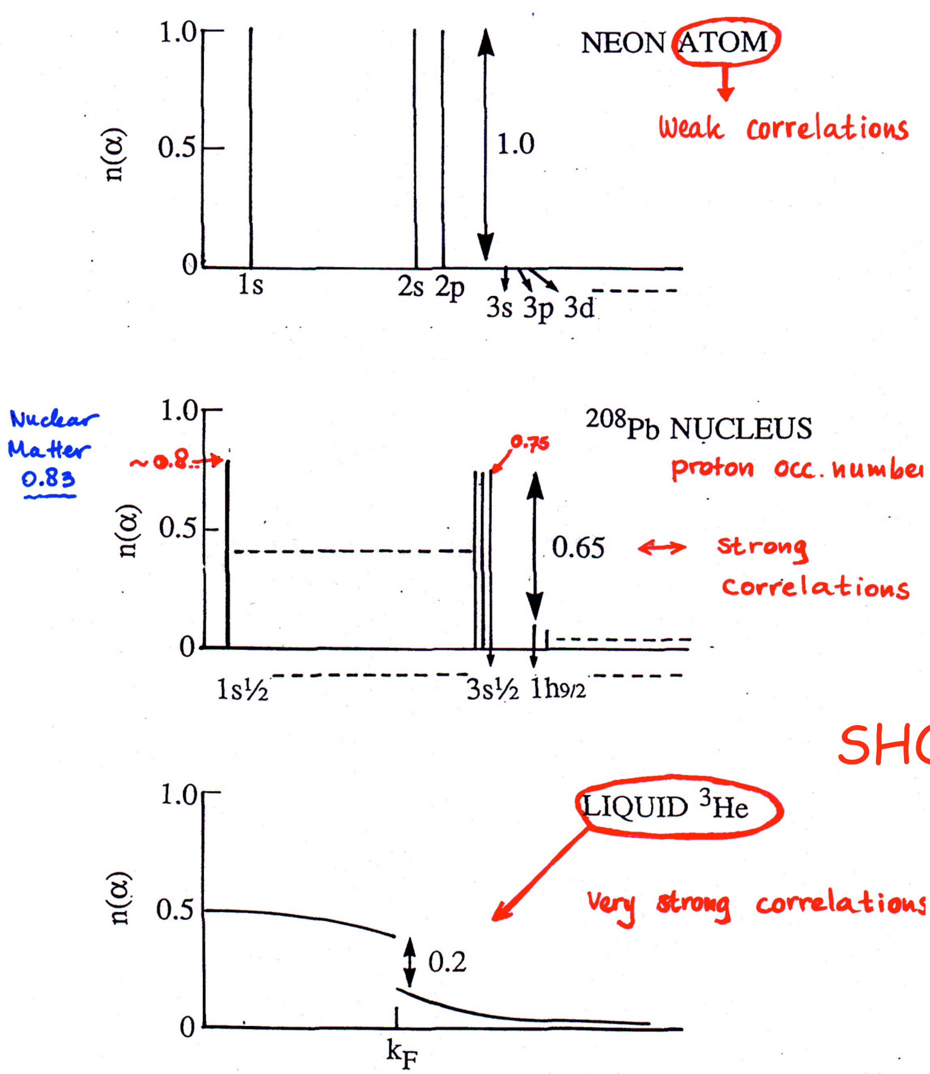
M12 and M14 transitions in (e,e´) only 50% of ph estimate

⇒ $Z_h * Z_p$

Data:

PRC20, 497(1979)

Occupation Numbers



Phys. Rep. 242 ('94) 119

Charge density in ^{208}Pb interior closely linked to nuclear saturation density

In turn, this charge density is predominantly determined by

SHORT-RANGE CORRELATIONS

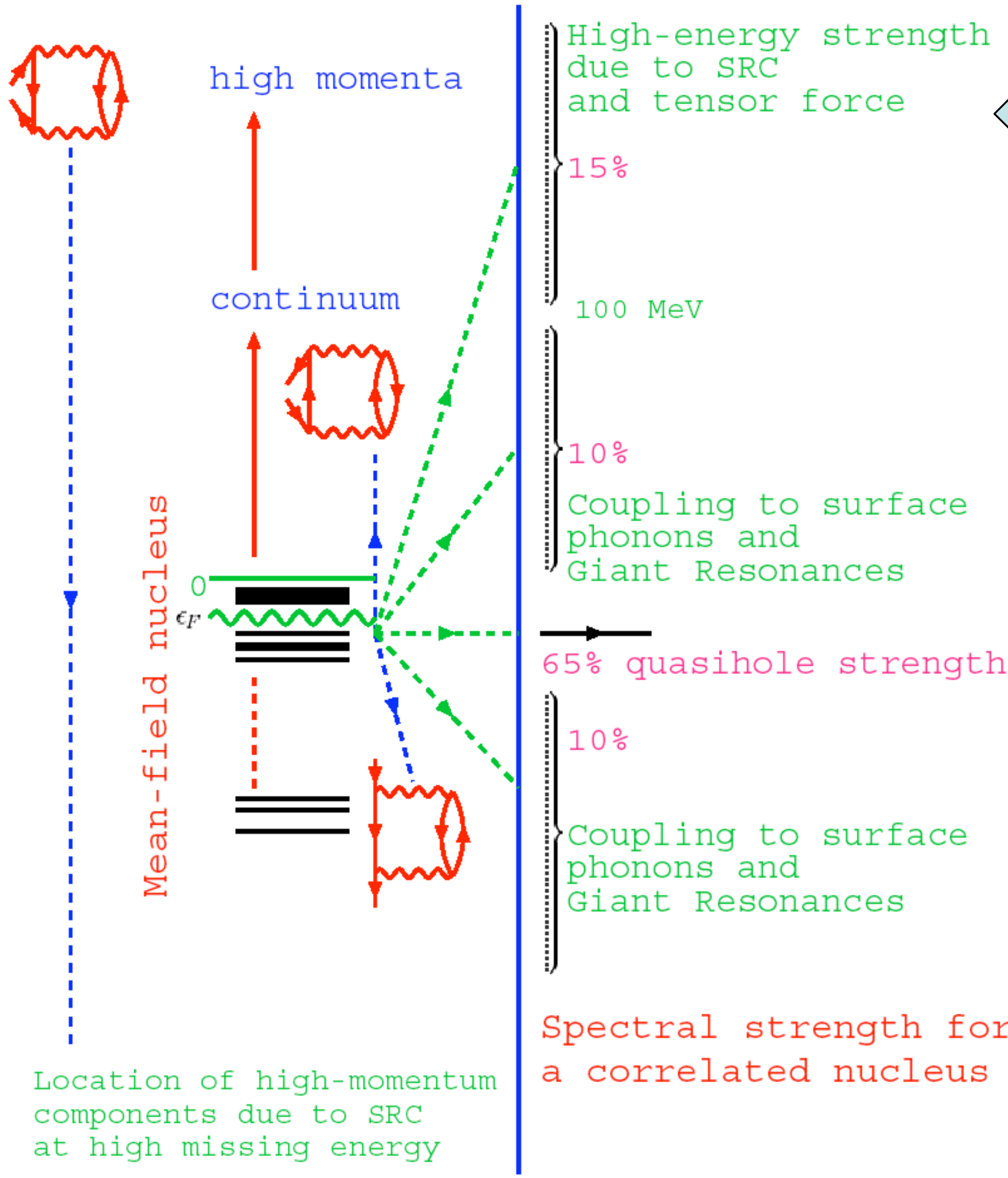
We now essentially know what all the protons are doing in the ground state of a "closed-shell" nucleus !!!

- Unique for a correlated many-body system
- Information available for electrons in atoms (Hartree-Fock)
- Not for electrons in solids
- Not for atoms in quantum liquids
- Not for quarks in nucleons

$\Rightarrow$ Demonstrates the value of the study of the nucleus for its intrinsic interest as a quantum many-body problem!

Location of single-particle strength in nuclei

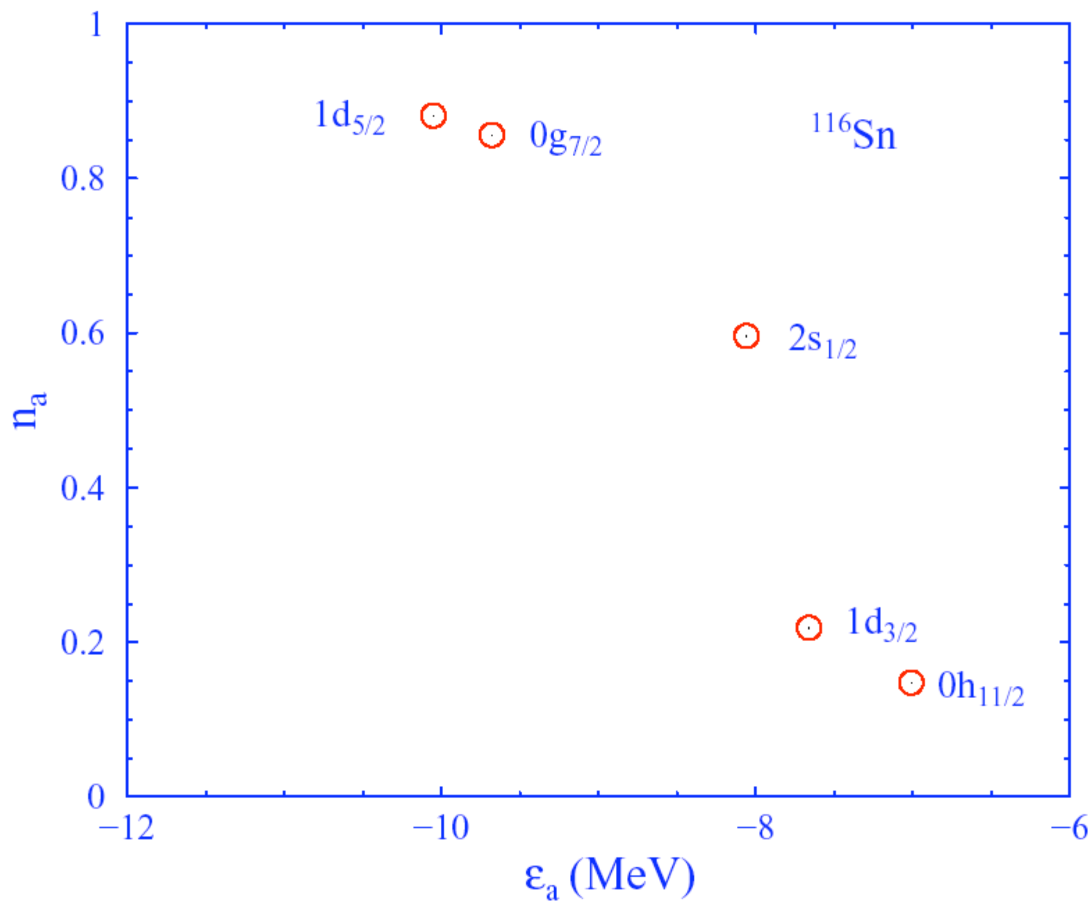
SRC



SRC

What about open-shell systems?

Semi-magic nuclei
Green's function calculation



SRC	the same
GRs	similar

only difference near ε_F

removal & addition probabilities similar size for $2s_{1/2}$!!

$\Rightarrow$ pairing

Deformation?

^{142}Nd(e,e´p) Z=60; N=82 compare with ^{146}Nd(e,e´p) Z=60; N=86
Nucl. Phys. **A560**, 811 (1993)

E_x ^{141}Pr	J^π	S_{exp}
0.000	$5/2^+$	0.23
0.145	$7/2^+$	0.39
1.118	$11/2^-$	0.05
1.298	$1/2^+$	0.03

E_x ^{145}Pr	J^π	S_{exp}
0.000	$7/2^+$	0.19
0.063	$5/2^+$	0.17
0.189	$5/2^+$	0.03
0.348	$3/2^+$	0.02
0.555	$7/2^+$	0.03

Wave functions in both nuclei are the same!

Systems with N very different from Z?

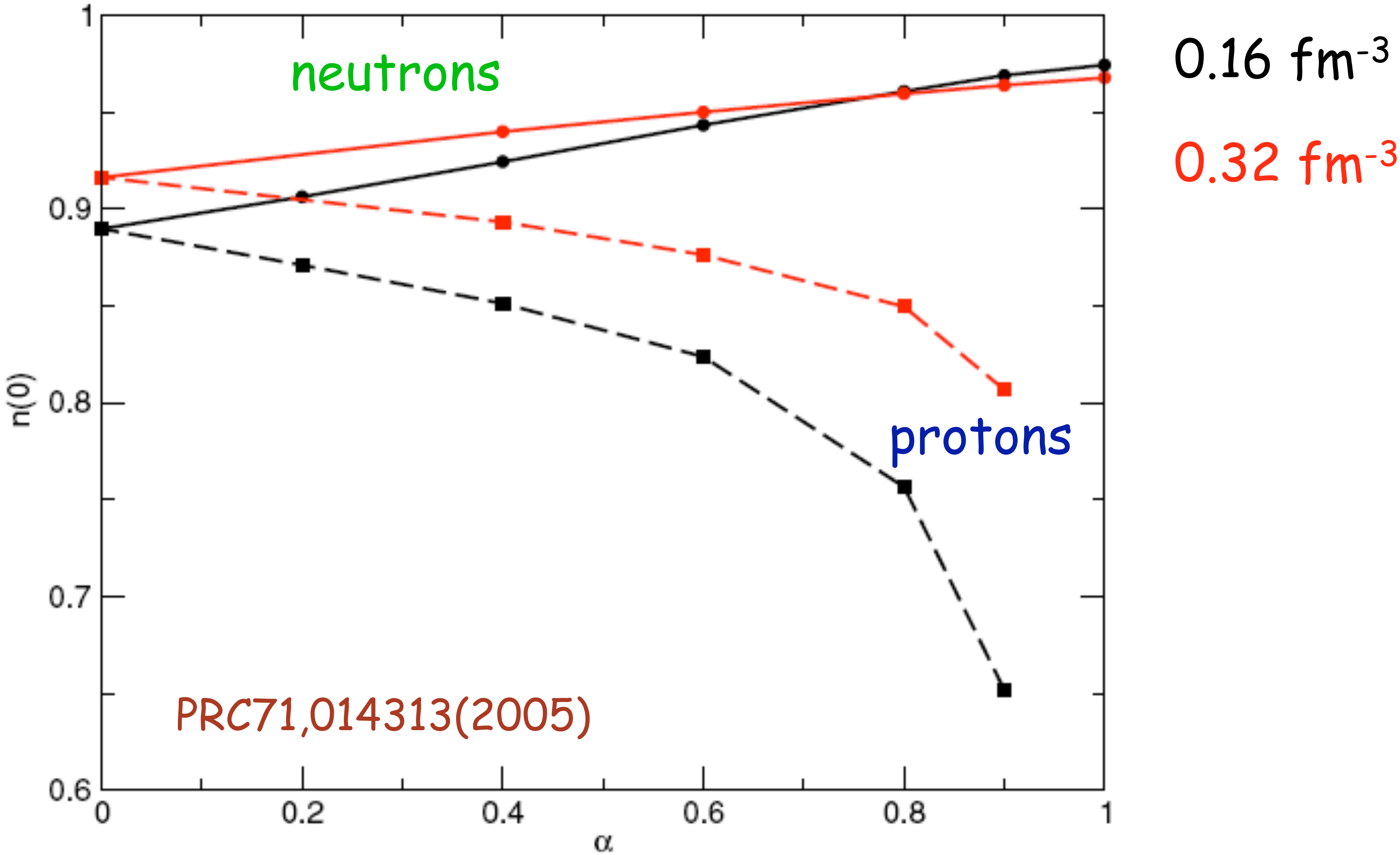
SRC still the same (tensor force disappears for n and "increases" for p for N>Z) (see PRC71,014313(2005))

Collectivity of excited states could be reduced

- So less fragmentation
  - and removal of sp strength becomes
  - more like mean-field (+ SRC+ whatever is left of tensor force for n but perhaps strong effect for p!)

Continuum effects (soft dipoles ...)

SCGF for isospin-polarized nuclear matter



asymmetry